Diffusion, mobility fluctuation, and island models of flicker noise

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Connections and equivalences among the diffusion, mobility fluctuation, and island theories of the flicker noise are shown, and the usual division in two model categories, based on the carrier-number and -mobility fluctuations, is so removed. To this end, the diffusion-noise theory, as well as the previous island model, is applied to conducting media containing localized states, the drift-velocity cross-correlation functions of the hemimicroscopic motion of each carrier, which at random is trapped and released by the islands, are computed, and, finally, the voltage noise spectrum at the device terminals is achieved as a sum of Lorentzian spectra by means of the Wiener-Khintchine theorem and the impedance-field method. It partly coincides with the one yielded, in the case of defect sizes smaller than the Debye length, by the island model through the charge-conservation principle, the Langevin method, the Schottky theorem, and the dipole-current and impedance-field methods applied directly to each island. For both approaches, in order to obtain the total noise spectrum in the flickerlike form $1/f^\gamma$, the defect relaxation times, which the diffusion theory cannot yield, and the computation methods of the island model are to be employed. The diffusion-noise model, instead, by the computation of the cross-correlation functions of the hemimicroscopic motion allows one to ascribe the flicker-noise origin also to the hemimicroscopic mobility fluctuation of each carrier due just to the stochastic process of its trapping and releasing by the islands during the drift displacement. Therefore the general validity of the island model, previously shown by its ability to yield an unitary synthesis of the flicker, burst, and generation-recombination noises, is now further extended and strengthened by its capability to account for and to contain other noise-analysis methods, such as the diffusion-noise and mobility-fluctuation models.

I. INTRODUCTION

Most models of the flicker noise, according also to recent surveys,[1–5] may be classified in two categories based on the carrier-number or -mobility fluctuation. Other approaches that put the origin of the phenomenon in the temperature fluctuation[6] or in the transport mechanisms, and that are not comprised in such a classification, seem to be laid to rest.[2–5,7,8]

The recent island model,[9–11] which yields an unitary approach and synthesis of the flicker, burst, and generation-recombination noises and which contains also, as very particular case, the McWhorter model,[12] is to be included in the category of the carrier-number fluctuations.

The aim of this paper is to remove such a model division in two classes and to show that the island approach accounts for and contains the mobility-fluctuation theory.[1,2,5] To this end, as it has been done for the island model,[9–11] the diffusion-noise theory[13,14] is applied to conducting media containing localized states, the cross-correlation functions of the drift-velocity components of the hemimicroscopic motion of each carrier, which at random is trapped and released by the islands, are computed, and then by means of them, of the Wiener-Khintchine theorem, and of the impedance-field method (IFM), the voltage-noise spectrum at the device terminals is achieved as sum of Lorentzian spectra.

Such a spectrum partly coincides with that obtained, in the case of defect sizes smaller than the Debye length, by means of the island model through the charge-conservation principle, the Langevin method, the Schottky theorem, and the dipole-current and impedance-field methods referred to each island.

In both approaches, in order to carry out the total voltage-noise spectrum in the flickerlike form $1/f^\gamma$ from the sum of the shotlike spectra, the defect-relaxation times, which cannot be yielded at all by the diffusion-noise theory, and the computational procedures of the island model are then to be used.

By computing the cross-correlation function of the hemimicroscopic motion, the diffusion-noise approach instead allows one to ascribe the cause of the $1/f^\gamma$ noise to the hemimicroscopic-mobility fluctuation of each carrier due to the stochastic process of its trapping and releasing by the islands along the drift motion. It is, in this way, justified theoretically how the mobility-fluctuation may be the origin of the flicker noise, as it has been postulated by several authors[1,2,5] on the basis of the Hooge empirical formula which, on the other hand, is accounted for in a direct and complete way by the island model.[11]

Therefore, the island model not only yields a unified approach of three important excess noises such as the flicker, burst, and generation-recombination noises, but it also accounts for and includes other significant analysis methods proposed to explain the flicker noise, like the mobility-fluctuation model and/or the diffusion-noise theory applied to media containing localized states.

Moreover, the spectrum computation according the island model and IFM is further developed by means of the reciprocal theorem.[13] A new expression of the noise variance, given as a function of the local direct power density, is so achieved.

II. DIFFUSION-NOISE THEORY

A. Microscopic and hemimicroscopic motions

The aim of this section is to compute, by means of the diffusion-noise theory and the IFM, the spectral density S_V of the voltage fluctuations between two probe points M' and N' of a conducting medium. To this end let us consider the case where each charge carrier may be, at random, subject to either of two conditions $c=a$ and $c=b$ characterized by the carrier velocity

$$\vec{v}_c(t)=\langle\vec{v}_c\rangle_t+\vec{u}_c(t)\ , \tag{2.1}$$

where

$$\langle\vec{v}_c\rangle_t=\mu_c\vec{\mathscr{E}} \tag{2.2}$$

is the microscopic drift velocity due to an electric field $\vec{\mathscr{E}}$, μ_c is the corresponding microscopic mobility, and $\vec{u}_c(t)$ is the fluctuation about $\langle\vec{v}_c\rangle_t$ due to any scattering process that does not change the carrier condition c.

At any time t the carrier velocity $\vec{v}(t)$ may be written in the form

$$\vec{v}(t)=\vec{v}_b+(\vec{v}_a-\vec{v}_b)\sum_i(-1)^iW(t-t_i)\ , \tag{2.3}$$

where $W(t-t_i)$ is the unit-step function, $(t_{i+1}-t_i)>0$ is a stochastic variable, and $i=-\infty,\ldots,-1,0,1,\ldots,\infty$. Therefore, if the carrier remains, on the average, in the a and b conditions during the time fractions α and β, respectively, the time average value $\langle\vec{v}\rangle_t$ and the fluctuation $\vec{U}(t)$ of $\vec{v}(t)$ become

$$\langle\vec{v}\rangle_t=\alpha\langle\vec{v}_a\rangle_t+\beta\langle\vec{v}_b\rangle_t\ , \tag{2.4}$$

$$\vec{U}(t)=\vec{U}_\Delta(t)+\vec{u}(t)\ , \tag{2.5}$$

where

$$\vec{u}(t)=\vec{u}_b(t)+[\vec{u}_a(t)-\vec{u}_b(t)]\times\sum_i(-1)^iW(t-t_i) \tag{2.6}$$

is the microscopic fluctuation about $\langle v_a\rangle_t$ or $\langle v_b\rangle_t$, and

$$\vec{U}_\Delta(t)=\left[-\alpha+\sum_i(-1)^iW(t-t_i)\right]\vec{\Delta}\ , \tag{2.7}$$

with

$$\vec{\Delta}=\langle\vec{v}_a\rangle_t-\langle\vec{v}_b\rangle_t\ , \tag{2.8}$$

is the hemimicroscopic fluctuation of the velocity about $\langle\vec{v}\rangle_t$ due to the random transitions of the carrier between the two conditions a and b, characterized by the different drift velocities $\langle\vec{v}_a\rangle_t$ and $\langle\vec{v}_b\rangle_t$.

B. Diffusion-impedance field noise formula

According to the diffusion-noise theory and the IFM,[13,14,5] the carrier random motion $\vec{U}(t)$ superimposed to the steady noiseless motion $\langle\vec{v}\rangle_t$ is the cause of the device noise, and the spectrum S_V of the voltage fluctuation between two arbitrary probe points N' and M' is given by the diffusion-impedance field formula,

$$S_V=\sum_{j=1}^{3}\sum_{k=1}^{3}\int 4q^2\bar{n}D_{jk}\frac{\partial Z}{\partial x_j}\frac{\partial Z^*}{\partial x_k}d^3x\ , \tag{2.9}$$

where q is the electron charge, $\bar{n}$ is the time average value of the carrier density, and the impedance $Z=Z(N',\vec{r},f)$ is defined by $Z\equiv\delta V(M'N',\vec{r})/\delta I$, δV being the phasor of ac voltage appearing between N' and M' when a small ac current of phasor δI and frequency f is injected in the point $\vec{r}$ and it is extracted from M'; Z^* is the complex conjugate value of Z.

The diffusion coefficient $D_{jk}=D_{jk}(\omega)$ at $\omega=2\pi f$, according to the Wiener-Khintchine theorem, is given by

$$D_{jk}=\frac{1}{2}\int_{-\infty}^{\infty}\langle U_j(t)U_k(t+s)\rangle e^{j\omega s}ds\ , \tag{2.10}$$

where U_j (U_k) is the $\vec{U}$ component along the x_j (x_k) axis.

C. Cross-correlation functions and noise spectra

Equations (2.9) and (2.10) reduce the problem of calculating noise into the computation of the impedance field $\vec{\nabla}Z$ and of the cross-correlation functions $\langle U_j(t)U_k(t+s)\rangle$.

When the correlation times τ_a and τ_b for $\vec{u}_a$ and $\vec{u}_b$, respectively, due to the carrier scattering in the corresponding conditions a and b, are much less than the decay time τ from a condition to the other, the approach of Shockley *et al.*[13] extended to the three-dimensional case yields

$$\langle U_j(t)U_k(t+s)\rangle=\langle u_j(t)u_k(t+s)\rangle+\langle U_{\Delta j}(t)U_{\Delta k}(t+s)\rangle\ , \tag{2.11}$$

where

$$\langle u_j(t)u_k(t+s)\rangle=\alpha\langle u_{aj}(t)u_{ak}(t)\rangle e^{-|s|/\tau_a}\delta_{jk}+\beta\langle u_{bj}(t)u_{bk}(t)\rangle e^{-|s|/\tau_b}\delta_{jk}\ , \tag{2.12}$$

$$\langle U_{\Delta j}U_{\Delta k}(t+s)\rangle=\alpha\beta\Delta_j\Delta_k e^{-|s|/\tau}\ . \tag{2.13}$$

In (2.12), $\delta_{jk}=0$ for $j\neq k$ and $\delta_{jk}=1$ for $j=k$ takes into account that u_{cj} and u_{ck} are uncorrelated for $j\neq k$. Moreover, according to the energy-equipartition theorem, holding at the thermal equilibrium, the following is also so:

$$\langle u_{cj}^2(t)\rangle=kT/m_c\ , \tag{2.14}$$

where k is the Boltzmann constant, T is the absolute temperature, and m_c is the carrier effective mass of the condition c.

Therefore, from (2.9)–(2.14) we have that the noise spectrum

$$S_V=S_{VT}+S_{V\Delta} \tag{2.15}$$

consists of a part

$$S_{VT}=\int 4q^2\bar{n}kT\left[\frac{\alpha\tau_a}{m_a(1+\tau_a^2\omega^2)}+\frac{\beta\tau_b}{m_b(1+\tau_b^2\omega^2)}\right]|\vec{\nabla}Z|^2d^3x\ , \tag{2.16}$$

which reduces to the Johnson-Nyquist thermal noise (see the Appendix), and a part

$$S_{V\Delta}=\int 4q^2\bar{n}\frac{\alpha\beta\tau}{1+\tau^2\omega^2}(\vec{\Delta}\cdot\vec{\nabla}Z)^2d^3x\ , \tag{2.17}$$

due to the hemimicroscopic fluctuation $\vec{U}_\Delta(t)$ about $\langle\vec{v}\rangle_t$.

The spectrum $S_{V\Delta}$ holds true in the form (2.17) if Z is real in the frequency band of interest for $S_{V\Delta}$ itself. If the τ values are dispersed uniformly on the sample with a distribution $D_\tau(\tau)$ and if the factor that multiplies $\tau/(1+\tau^2\omega^2)$ in (2.17) is a slowly varying function of $\vec{r}$, $S_{V\Delta}$ may be written also in the form

$$S_{V\Delta}=\int 4q^2\bar{n}\frac{\alpha\beta\tau D_\tau}{1+\tau^2\omega^2}(\vec{\Delta}\cdot\vec{\nabla}Z)^2d\tau\, d^3x\ , \tag{2.18}$$

which will be utilized later to compute the flicker noise.

III. MOBILITY-FLUCTUATION NOISE

We want to show now that the noise $S_{V\Delta}$ may be ascribed also to the mobility fluctuation of each carrier in its hemimicroscopic motion $\vec{U}_\Delta(t)$. In fact, the mean drift velocity $\langle\vec{v}\rangle_t$ of the carrier and the hemimicroscopic fluctuation $\vec{U}_\Delta(t)$ about it, according to (2.2), (2.4), (2.7), and (2.8), may be written in the form

$$\langle v\rangle_t=\bar{\mu}\vec{\mathcal{E}}\ , \tag{3.1}$$

$$\vec{U}_\Delta(t)=\Delta\mu(t)\vec{\mathcal{E}}\ , \tag{3.2}$$

where

$$\bar{\mu}=\alpha\mu_a+\beta\mu_b\ , \tag{3.3}$$

$$\Delta\mu(t)=(\mu_a-\mu_b)\left[-\alpha+\sum_i(-1)^iW(t-t_i)\right]\ , \tag{3.4}$$

are the time-average value and its random fluctuation, respectively, of the hemimicroscopic mobility $\mu(t)=\bar{\mu}+\Delta\mu(t)$ of the carrier.

Therefore, from (2.7), (2.9)–(2.11), (2.13), (2.15), (2.17), and (3.2), the spectrum $S_\mu(\vec{r})$ of the mobility fluctuation $\Delta\mu(t)$ of each carrier at $\vec{r}$ is

$$S_\mu(\vec{r})=\frac{4\alpha\beta(\mu_a-\mu_b)^2\tau}{1+\tau^2\omega^2}\,, \tag{3.5}$$

whereas its average value $\langle S_\mu\rangle=\int S_\mu D_\tau d\tau$ across the entire sample becomes

$$\langle S_\mu\rangle=4\langle\Delta\mu^2\rangle_t\int\frac{\tau D_\tau}{1+\tau^2\omega^2}d\tau\,, \tag{3.6}$$

where

$$\langle\Delta\mu^2\rangle_t=\alpha\beta(\mu_a-\mu_b)^2 \tag{3.7}$$

is the variance of $\Delta\mu(t)$.

Then, by following the computation methods of the island model,[11] i.e., by setting $\theta_\omega=\ln(\tau_0\omega)$ and $\theta_G=\ln(\tau/\tau_0)$, where τ_0 is a reference arbitrary time, and by indicating with $D_G(\theta_G)$ the θ_G distribution, the spectrum $\langle S_\mu\rangle$ from (3.6) becomes

$$\langle S_\mu\rangle=\frac{\langle\Delta\mu^2\rangle_t D_G(-\theta_\omega)}{f}\simeq\frac{f_0^\delta\langle\Delta\mu^2\rangle_t D_G(-\theta_0)}{f^\gamma}\,, \tag{3.8}$$

where f_0 is the frequency around which we consider the spectrum, $\theta_0=\ln(2\pi f_0\tau_0)$,

$$\delta=(d\ln D_G/d\theta_G)\,|_{\theta_G=-\theta_0}\,,$$

and $\gamma=1+\delta$.[11]

Therefore, for a "wide" distribution D_G of θ_G,[11] the average spectrum

$$\langle S_\mu\rangle=\beta_P\frac{\bar{\mu}^2}{f^\gamma} \tag{3.9}$$

of the hemimicroscopic-mobility fluctuation $\Delta\mu(t)$, as postulated by several authors[1,2,5] on the basis of the Hooge empirical formula,[15] becomes of the flicker type with a proportionality constant

$$\beta_P=\frac{\langle\Delta\mu^2\rangle_t D_G(-\theta_0)f_0^\delta}{\bar{\mu}^2}\,, \tag{3.10}$$

which, as a natural consequence of the present general approach and in agreement with a recent derivation of van der Ziel and van Vliet,[16] is independent of the carrier number (see, however, Sec. IV D).

Finally the spectrum $S_{V\Delta}$, in turn, from (2.2), (2.4), (2.8), (2.18), (3.1), (3.3), (3.6), and (3.7), may be set in the form

$$S_{V\Delta}=\int\frac{(\vec{J}_0\cdot\vec{\nabla}Z)^2}{\bar{n}}\left[\frac{\mu_a-\mu_b}{\bar{\mu}}\right]^2\times\frac{4\alpha\beta\tau D_\tau}{1+\tau^2\omega^2}d\tau\,d^3x\,, \tag{3.11}$$

or, more compactly

$$S_{V\Delta}=\int\frac{(\vec{J}_0\cdot\vec{\nabla}Z)^2}{\bar{n}}\frac{\langle S_\mu\rangle}{\bar{\mu}^2}d^3x\,, \tag{3.12}$$

where

$$\vec{J}_0=q\bar{\mu}\bar{n}\vec{\mathscr{E}} \tag{3.13}$$

is the steady-state current density due to a bias direct current I injected in a driver point N and extracted from another M of the sample, M and N being, in general, distinct from the voltage probe points N' and M'.

Of course, if (3.9) holds true, the total spectrum $S_{V\Delta}$ given by (3.12) is also of $1/f^\gamma$ type. The conditions that may lead to (3.9) are examined in the following section.

IV. ISLAND MODEL

A. Physical model

Now the problem to be solved is one of finding the physical phenomenon and mechanism that give conditions a and b, characterized by (i) different mobility μ_a and μ_b, respectively, and, chiefly, (ii) a wide dispersion of the logarithm $\ln(\tau/\tau_0)=\theta_G$ of the decay time τ between them, which is necessary to account for the flicker noise existence over many frequency decades and down to however low a frequency.

Such a phenomenon may not be found in an intervalley process, which gives a as the fast valley and b as the slow valley,[13] because it cannot give a sufficient dispersion of τ. Instead, the mechanism that satisfies both requirements (i) and (ii) very well is the carrier trapping and releasing phenomenon occurring in conducting media containing localized states or, as previously called,[9–11] islands. In fact, for them (i) the two conditions may be a as the free carrier and b as the trapped carrier in an island, and (ii) the decay time τ, owing to its dependence on the island relaxation times, may have a sufficient dispersion.[9–11]

In this case, since it is

$$\mu_b=0\,, \tag{4.1}$$

from (3.3) and (3.10) the constant

$$\beta_P = \frac{\beta}{\alpha} D_G(-\theta_0) f_0^\delta \tag{4.2}$$

also becomes independent from the microscopic mobility μ_a of the free carrier. Moreover, we have

$$\beta = D_I \langle \bar{N}_I \rangle / \bar{n} = F \ , \tag{4.3}$$

$$\alpha \simeq 1 \ , \tag{4.4}$$

where D_I is the island density and $\langle \bar{N}_I \rangle$ is the time- and ensemble-average value of the carrier number of each island; since $\alpha+\beta=1$, (4.4) holds true with the assumption that $\beta \ll 1$.

B. Model comparison

More generally, from (3.3), (3.11), (4.1), (4.3), and (4.4), the spectrum $S_{V\Delta}$ becomes

$$S_{V\Delta} = \frac{\langle \Delta V^2 \rangle_t D_G(-\theta_\omega)}{f} \simeq \frac{f_0^\delta \langle \Delta V^2 \rangle_t D_G(-\theta_0)}{f^\gamma} \ , \tag{4.5}$$

where the variance $\langle \Delta V^2 \rangle_t$ of the voltage fluctuations is given by

$$\langle \Delta V^2 \rangle_t = \int_0^\infty S_{V\Delta} df = \int (\vec{J}_0 \cdot \vec{\nabla} Z)^2 \frac{F}{\bar{n}} d^3x \ . \tag{4.6}$$

Such relationships (4.5) and (4.6) are equal to Eqs. (6.4) and (6.11), respectively, of Ref. 11 obtained by means of the island model and IFM in the case of island sizes smaller than Debye's length.

Therefore, the island approach, as well as being able to given a unitary model of the flicker, burst, and generation-recombination noises,[11] accounts for and contains those theoretical results that may be achieved for the same physical phenomena by means of the diffusion-noise theory and IFM, or by means of the equivalent model, deriving from itself, of the hemimicroscopic-mobility fluctuation. The island model, however, is more general and chiefly it does not have the intrinsic constraints and limits of the diffusion theory that in practice make it inapplicable for the excess low-frequency noises.

In fact, for the validity of such a theory, firstly the smallest volume elements $\Delta\Omega$, into which the sample can be divided for analyzing the noise, must be many free paths in size (hemimicroscopic paths among the islands in our case); that is, each $\Delta\Omega$ must contain very many islands $\nu_I = D_I \Delta\Omega$ in order that the random drift velocity $\vec{U}_\Delta(t)$ of a carrier in a given volume $\Delta\Omega$ is not significantly correlated with that in another neighboring volume $\Delta\Omega$.[5,13]

Such ν_I islands, moreover, must have the same relaxation time $\tau_i = \tau$, because only in this case does the equation

$$\frac{d}{dt}\left[\sum_{i=1}^{\nu_I} \Delta n_{Ii}\right] = -\sum_{i=1}^{\nu_I} \frac{\Delta n_{Ii}}{\tau_i} \tag{4.7}$$

(obtained by summing the relaxation equations of the ν_I islands,[11] Δn_{Ii} being the carrier-number fluctuation of the ith island) reduce itself to the decay equation $d(\Delta n_b)/dt = -\Delta n_b/\tau$ for the number variation

$$\Delta n_b = -\Delta n_a = \sum_{i=1}^{\nu_I} \Delta n_{Ii}$$

of the carriers in the conditions b (trapped carriers) and a (free carrier), respectively, in the considered volume element $\Delta\Omega$, as required by the diffusion-noise theory.[13]

Finally, as said in the Sec. II C, in order to be able to proceed from (2.17) to (2.18), which is required to carry out the theory, the quantities that are distinct from τ must be practically constant throughout all the sample.

All these three very constrictive and limiting conditions, which make the approach unlikely, do not exist for the island model because it considers and determines separately the independent noise contribution of each single island by means of its stochastic relaxation equation, the Langevin method, the Schottky theorem, the vector dipole current induced by the island itself, and the IFM.

Moreover, the island method allows us to deal with the general case of islands of any nature and size and, chiefly, to determine their relaxation times, necessary to compute the noise spectral density, which the diffusion model can never yield.

Finally, the island-model method of computing the noise spectrum in the form $1/f^\gamma$ from the sum of the Lorentzian contributes of the islands is also significant. Such a procedure, however, as above shown, may also be applied to the diffusion and mobility approaches.

C. Reciprocal theorem and impedance field

The island model itself may be further developed by means of the reciprocal theorem[13] which, for the thermal equilibrium and no magnetic field, states that

$$Z = Z' \equiv \frac{\delta V'(M'\vec{r},N')}{\delta I'} , \tag{4.8}$$

where $\delta V'$ is the phasor of the ac voltage appearing between the points $\vec{r}$ and M' when a small ac current of phasor $\delta I'$ and frequency f is injected in N' and is extracted from M'.

In fact, on the assumption that (4.8) holds true also out of the thermal equilibrium and for any value of $\delta I'$ and in particular for $|\delta I'| = I$, where I is the direct bias current fed through the driver points M and N, and moreover if Z is real, which happens at the low frequencies associated with flicker noise, from (4.8) we have

$$\vec{\nabla} Z = -\vec{J}'_0/\sigma I , \tag{4.9}$$

where σ is the conductivity and $\vec{J}'_0$ is the current density due to the current I switched from the driver points M and N to the probe ones M' and N'. In particular (4.9) shows that $\vec{\nabla} z \propto \sigma^{-1}$.[11]

Therefore, from (4.6) and (4.9) we obtain the relation

$$\langle \Delta V^2 \rangle_t = \int \frac{(\vec{J}_0 \cdot \vec{J}'_0)^2 F}{\bar{n}\sigma^2 I^2} d^3x , \tag{4.10}$$

which reduces to the form

$$\langle \Delta V^2 \rangle_t = \int \frac{P^2 F}{I^2 \bar{n}} d^3x \tag{4.11}$$

when the probe and driver points coincide, i.e. $N \equiv N'$ and $M \equiv M'$, $P = J_0^2/\sigma$ being the direct power density dissipated at the point $\vec{r}$.

If, for a homogeneous device, one has $P\Omega = VI$, where V is the average voltage produced between M and N by the bias current I and Ω is the sample volume, from (4.11) we obtain directly the usual relationship[11]

$$\langle \Delta V^2 \rangle_t = \frac{FV^2}{\bar{n}\Omega} . \tag{4.12}$$

A relationship analogous to (4.10) has been found by Vandamme and Bokhoven[17] by their consideration on the sensitivity theorem in the electrical networks.

D. Notes

The island model also accounts for the experimental result[18] utilized to justify and to strengthen the mobility-fluctuation assumption,[2,5] whereby in the semiconductors the coefficient of Hooge's empirical formula[15] decreases when the impurity density increases. In fact, this result agrees with (4.3), (4.5), and (4.6), or (4.10)–(4.12), according to which F is inversely proportional to $\bar{n}$ when $\langle \bar{N}_I \rangle$ is a slowly varying function of $\bar{n}$ itself,[11] as may happen for both low and high doping.

Another consideration to be made concerns the flicker-noise dependence on the temperature. In fact, when the carrier trapping and releasing from the islands are due to the tunnel effect alone, which, according to the previous generalization of the McWhorter model,[11,12] may occur for islands both on the surface and in the bulk of the sample, the island relaxation times, their distribution D_G, and the noise spectrum, according to (4.3), (4.5), and (4.6), or (4.10)–(4.12), become independent of the temperature. This has been observed in semiconductor devices.

The opposite trends occur when the same phenomena are due to thermal activation processes.[3,11] Of course, a more complex situation occurs when both processes are present.

V. CONCLUSIONS

It has been shown that the flicker noise of the electronic devices—which, according to the previously discussed island model, is due to the localized states and to the great dispersion of their relaxation times—may be accounted for, although with approximations and limits, by means of the diffusion-noise theory applied also to systems containing islands. Through this approach it may be ascribed even to the fluctuations of the hemimicroscopic mobility, i.e., the mobility that takes into account the effects, on the drift velocity of the carrier, of its trapping and releasing processes from the islands themselves.

The hypothesis, postulated by several authors on the basis of Hooge's empirical formula, that the average spectrum of the hemimicroscopic mobility is of the flicker type $\beta_P \bar{\mu}^2/f^\gamma$, has been thus proved theoretically and the expression of the constant β_P has been achieved. In this way, the distinction between two model classes, based on the carrier-number and -mobility fluctuation, has also been removed, and the validity and generality of the island model are further strengthened.

In fact, comparison shows that such a model, since it starts directly from the island-fluctuation phenomena, does not suffer the constraints, approximations, and limits of the diffusion and mobility approaches; however, it accounts for and contains their results, when they may be considered as correct.

Therefore, in conclusion, for such a capability to justify, explain, and unify the methods and the results, although often partial and/or postulated, of other significant approaches such as just the diffusion-noise and mobility-fluctuation models, and for the previously shown ability to yield an unitary theory of the flicker, burst, and generation-recombination noises, the island model appears once again to be a general and correct analysis tool of the excess noises and, in particular, of the $1/f^{\gamma}$ noise.

APPENDIX

The microscopic mobility $\mu_c(j\omega)$ at ω of the carrier, associated to its Brownian motion in the c condition, is given by [19]

$$\mu_c(j\omega)=q\tau_c/m_c(1+j\omega\tau_c)\ , \qquad \text{(A1)}$$

and in consequence, since $\alpha\bar{n}$ and $\beta\bar{n}$ carriers are, on the average, subject to the conditions a and b for volume unit, respectively, the sample conductivity $\sigma(j\omega)$ at ω becomes

$$\sigma(j\omega)=q\bar{n}[\alpha\mu_a(j\omega)+\beta\mu_b(j\omega)]\ . \qquad \text{(A2)}$$

Therefore, from (2.16), (A1), and (A2), we have

$$S_{VT}=4kT\int |\nabla Z|^2\,\mathrm{Re}[\sigma(j\omega)]d^3x\ . \qquad \text{(A3)}$$

On the other hand, the reciprocal theorem (4.8) leads to the distributed power theorem[13]

$$R(\omega)=\int |\nabla Z|^2\,\mathrm{Re}[\sigma(j\omega)]d^3x\ , \qquad \text{(A4)}$$

where $R(\omega)$ is the real part of the sample impedance between the probe points M' and N'.

In conclusion, from (A3) and (A4) we have that

$$S_{VT}=4kTR(\omega)\ , \qquad \text{(A5)}$$

reduces to the Johnson-Nyquist thermal noise.[13]

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