PHYSICAL REVIEW B VOLUME 25, NUMBER 12 15 JUNE 1982

Quantum-limit cyclotron resonance linewidth due to an electron-phonon interaction

Akira Suzuki and Dennis Dunn
J. J. Thomson Physical Laboratory, University of Reading, Whiteknights, Reading, RG6 2AF, United Kingdom
(Received 26 May 1981)

The cyclotron resonance line shape is dependent upon the current relaxation rate $\Gamma(k)$. We have numerically determined $\Gamma(k)$ and hence the cyclotron resonance linewidth γ in the quantum limit for a nondegenerate semiconductor using a previously derived formula [J. Phys. Chem. Solids 41, 735 (1980)]. The electrons are assumed to interact with acoustic phonons via a deformation potential. For high temperatures we have found that the dependence of the linewidth γ and peak absorption $P^{\max}$ on the temperature and the resonance frequency ω_0 is given approximately by $\gamma \propto T^{1/2}\omega_0$, $P^{\max} \propto T^{-1/2}\omega_0^{-1}$, whereas for low temperatures $\gamma \propto T^{6.5}\omega_0^{-2}$, $P^{\max} \propto T^{-6.5}\omega_0^{2}$. The line shape is found to be almost Lorentzian. We have compared our numerical results for $\Gamma(k)$ with the various approximations which are commonly employed.

I. INTRODUCTION

In 1955 Dresselhaus *et al.*[1] observed cyclotron resonance (CR) in germanium. Since then many CR experiments have been carried out to study electronic band structures: The position of the absorption peak gives the value of the effective mass of conduction electrons, $m^* = eB/\omega_0$, where $-e$, B, and ω_0 are, respectively, the electronic charge, the applied magnetic field strength, and the resonance frequency. The linewidth or line shape of the power absorption has received less attention. However, in recent years the high-frequency CR experiments performed with far-infared lasers have opened up a new domain in semiconductor physics, and special attention has been paid to the study of the shape of a cyclotron resonance power absorption line: The shape of the line, and in particular the linewidth (γ) and its dependence on temperature (T) and magnetic field strength (B), depend critically on the detailed nature of the scattering mechanisms. Thus they provide a sensitive probe for these interactions.

Lodder and Fujita[2] developed a general theory to describe a CR line shape of the power absorption, starting from Kubo's formula and employing the proper connected diagram expansion. In their theory, the *gain* and *loss* terms occurring in the kinetic description of the relaxation process are built in automatically and the inelastic scattering mechanisms are included for an electron-phonon system unlike other theories.[3] Therefore their theory gives a clear picture of the *emission* and *absorption* of a phonon by electrons, especially in the low-temperature region where one-phonon inelastic processes play an important role. Recently one of the authors, Suzuki *et al.*[4] extended the general theory and obtained a formula valid in the vicinity of ω_0 in the quantum limit ($\hbar\omega_0 >> k_B T$), and studied the CR line shape near $\omega \simeq \omega_0$ at high temperature, neglecting the inelastic effect. Their results agree with recent experiments on Ge.[5]

In this paper we have carried out a quantum-mechanical calculation of the CR linewidth for a nondegenerate semiconductor in the quantum limit for an electron-acoustic-phonon interaction via the deformation potential, starting from the formulas obtained previously [see Eqs. (3.2) and (3.7) in Ref. 4]. The CR line shape and hence its linewidth γ depend critically on the current relaxation rate $\Gamma(k)$. To obtain correct information on the temperature and magnetic field dependence of γ, we need to evaluate the expression for $\Gamma(k)$. To do so, various approximations have been suggested. For the high-temperature region if we neglect the inelasticity of the scattering caused by the phonon energies $\pm\hbar\omega_{\vec{q}}$ and apply the high-temperature approximation for the phonon distribution function $N_{\vec{q}}$, we can obtain

$$\Gamma(k) = \frac{1}{\pi}\frac{D^2}{\rho_m s_0^2}\frac{m^{*2}}{\hbar^4}\omega_0 k_B T \,|\, k_z \,|^{-1} . \qquad (1.1)$$

This expression diverges as $k_z \to 0$ and as we shall

show greatly exaggerates the magnitude of $\Gamma(k)$ for the important region of the k_z values. Therefore the usual approximation

$$P(\omega)\propto\left\langle\frac{\Gamma}{(\omega-\omega_0)^2+\Gamma^2}\right\rangle \sim\frac{\langle\Gamma\rangle}{(\omega-\omega_0)^2+\langle\Gamma\rangle^2} \tag{1.2}$$

breaks down. To offset this divergence, the effect of inelasticity of the acoustic phonon is important.[6,7] For the low-temperature region, the equilibrium phonon occupation number is often approximated as[8]

$$N_{\vec{q}}\sim 0$$

In fact as seen in Figs. 1(a)–1(c), this approximation is invalid.

To obtain correct information on the temperature and magnetic field dependence of γ, which should be determined through its defining Eq. (2.6), we retain the phonon energies $\pm\hbar\omega_{\vec{q}}$ and include the distribution function $N_{\vec{q}}$ without the usual approximations and so we can correctly determine the effects of the inelasticity of the scattering processes and of the temperature dependence of the line shape. In Sec. II the basic formulas for the further calculations are presented. In Sec. III the current relaxation rate $\Gamma(k)$ is examined and the expressions for high- and low-temperature limits are given. In Sec. IV the numerical results are presented and discussed. In Sec. V concluding remarks are given.

II. THEORY

The power absorption for a circularly polarized electric field $\vec{E}(t)$ ($\perp\vec{B}$) of frequency ω is given by

$$P(\omega)=\tfrac{1}{2}E^2\mathrm{Re}[\sigma_{+-}(\omega)]\ , \tag{2.1}$$

where the symbol Re means the real part, and $\sigma_{+-}(\omega)$ is a combination of transverse components of the conductivity tensor. In an earlier paper, one of the present authors has extended the general theory to the electron-phonon system[2] and derived a formula for the conductivity valid in the vicinity of $\omega\simeq\omega_0$ in the quantum limit ($\hbar\omega_0>>k_BT$) (see Ref. 4 for details). The results are

$$\mathrm{Re}[\sigma_{+-}(\omega)]=\frac{2e^2}{m^*}\lim\frac{1}{V}\sum_k\frac{f_{0k}\Gamma(k)}{(\omega-\omega_0)^2+\Gamma(k)^2}\ , \tag{2.2}$$

$$\hbar\Gamma(k)=\pi\sum_{\vec{q}}(1+N_{\vec{q}})\,|\,C_q\,|^2t^2e^{-t}\delta(\epsilon(k_z-q_z)-\epsilon(\vec{k}_z)+\hbar\omega_{\vec{q}}) +\pi\sum_{\vec{q}}N_{\vec{q}}\,|\,C_q\,|^2t^2e^{-t}\delta(\epsilon(k_z-q_z)-\epsilon(k_z)-\hbar\omega_{\vec{q}})\ . \tag{2.3}$$

where lim means the bulk limit, f_{0k} is the distribution function for the lowest Landau states $N=0$, namely in the extreme quantum limit, all electrons occupy Landau states with the lowest oscillator quantum number, $\omega_0=eB/m^*$ the cyclotron frequency, and $\Gamma(k)$ is the current relaxation rate associated with transitions between states $N=0$ and $N=1$. $N_{\vec{q}}$ is the Planck distribution function for phonons:

$$N_{\vec{q}}=[\exp(\hbar\omega_{\vec{q}}/k_BT)-1]^{-1}\ , \tag{2.4}$$

and t is given by

$$t\equiv\frac{\hbar}{2m^*\omega_0}q_\perp^2\equiv\frac{\hbar}{2m^*\omega_0}(q_x^2+q_y^2)\ . \tag{2.5}$$

$\epsilon(k_z)$ is the energy associated with electron's motion along the magnetic field direction. C_q and $\hbar\omega_{\vec{q}}$ are, respectively, a coupling constant and a phonon energy, which depend on the type of phonon. It is noted that expression (2.3) is strictly valid for the vicinity of $\omega\simeq\omega_0$ and is obtained under the high quantum and weak coupling limit. In fact, for a strong coupling case we have to solve the self-energy (the memory function) self-consistently, from which we can obtain the width and the shift.[2,9] The expressions, (2.2) and (2.3), are the basic equations for a further study of cyclotron resonance width. In the following sections, we study the current relaxation rate $\Gamma(k)$, which contains both gain and loss contributions correctly,[2,3] and derive the expressions for the resonance width γ of the power absorption line. The cyclotron resonance width γ is herein determined through the following defining equations for a width:

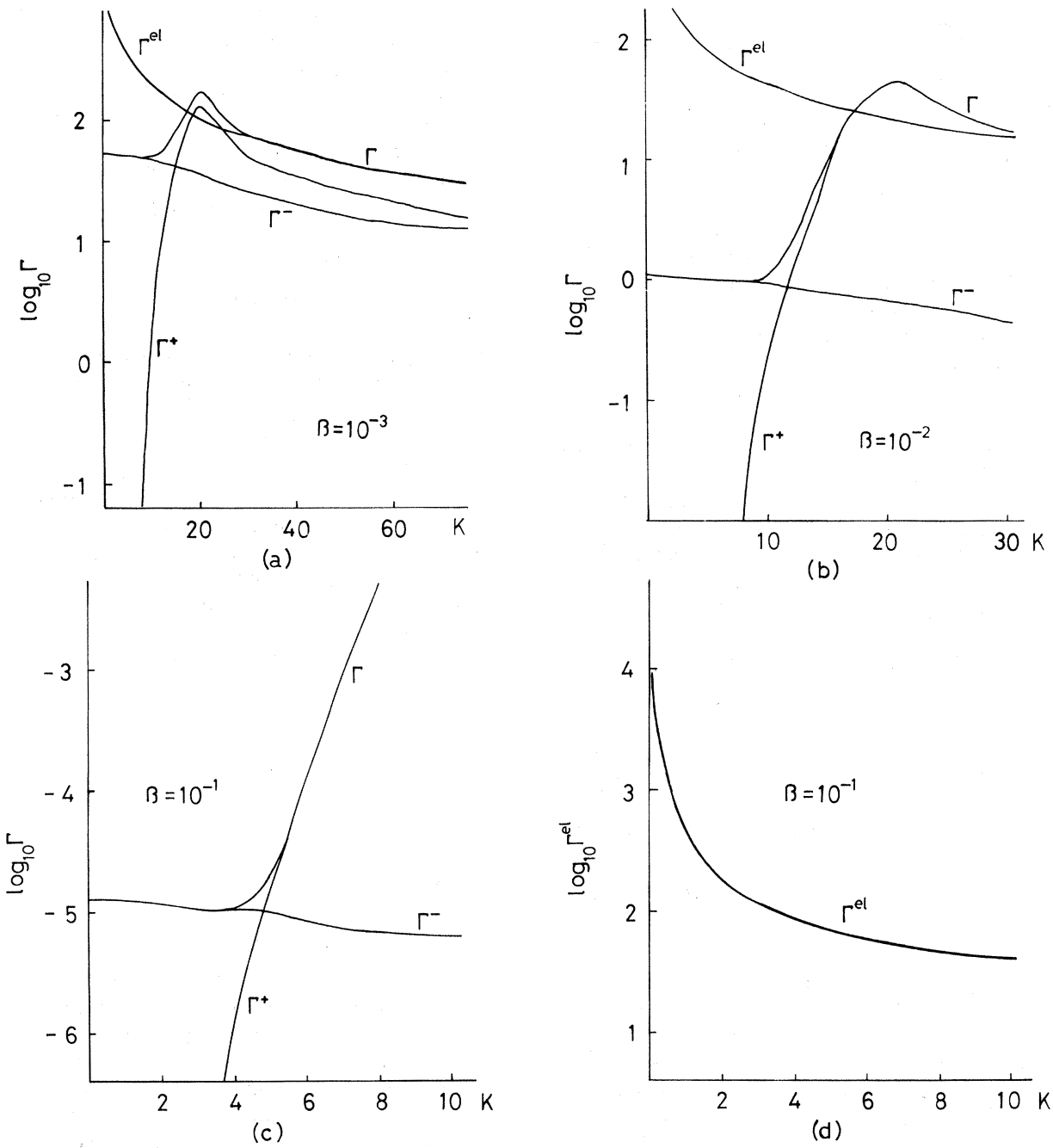

FIG. 1. Current relaxation rate. Resonance frequency is taken to be $\Omega_0=1.0\times10^4$. Γ is the total current relaxation rate, $\Gamma^{\pm}$ are the phonon emission (+) and absorption (−) parts, and Γ^{el} is the total current relaxation rate in the elastic scattering approximation. (Γ, $\Gamma^{\pm}$, and Γ^{el} are in the units of $m^*S_0^2/2\hbar$.)

$$P(\omega_{max}+\gamma_R)=\tfrac{1}{2}P(\omega_{max}),$$
$$P(\omega_{max}-\gamma_L)=\tfrac{1}{2}P(\omega_{max}), \qquad (2.6)$$
$$\gamma=\gamma_R+\gamma_L,$$

where ω_{max} corresponds to the maximum power absorption. For a symmetric line, of course, $\gamma_R=\gamma_L$.

II. CURRENT RELAXATION RATE $\Gamma(k)$ AND CR WIDTH γ

The current relaxation rate $\Gamma(k)$ can be evaluated from (2.3) by inserting the appropriate coupling constant C_q. We consider the case of acoustic phonon scattering via deformation potential cou-

pling, taking into account the inelasticity of the scattering process. Then C_q takes the form[6]

$$C_q = iD(\hbar q/2\rho_m S_0 V)^{1/2} , \tag{3.1}$$

where D is the deformation potential constant, ρ_m the mass density of the bulk, and S_0 the sound speed. The presence of $\hbar\omega_{\vec{q}}$ in the δ function makes it difficult for us to obtain the analytical expression. Some approximation methods have been introduced to study the effect of the inelasticity of the acoustic phonon scattering by making some assumptions on $\hbar\omega_{\vec{q}}$.[5,6] However to obtain the exact information about this effect, we evaluate the current relaxation rate $\Gamma(k)$ without making any approximation on $\hbar\omega_{\vec{q}}$.

Let us introduce dimensionless variables:

$$K^2 \equiv \left[\frac{\hbar^2 k_z^2}{2m^*}\right] \Big/ \left[\frac{m^* s_2^2}{2}\right], \quad Q_\perp^2 \equiv \left[\frac{\hbar^2 q_\perp^2}{2m^*}\right] \Big/ \left[\frac{m^* S_0^2}{2}\right], \quad Q_z^2 \equiv \left[\frac{\hbar q_z^2}{2m^*}\right] \Big/ \left[\frac{m^* S_2^2}{2}\right],$$

$$\Omega \equiv \hbar\omega \Big/ \left[\frac{m^* S_0^2}{2}\right], \quad \Omega_0 \equiv \hbar\omega_0 \Big/ \left[\frac{m^* S_0^2}{2}\right], \quad \beta \equiv \left[\frac{m^* S_0^2}{2}\right] \Big/ (k_B T) . \tag{3.2}$$

The current relaxation rate $\Gamma(k)$, Eq. (2.3), can then be expressed by

$$\Gamma(K) = \Gamma^+(K) + \Gamma^-(K) , \tag{3.3}$$

$$\Gamma^\pm(K) = \pm \frac{A_{ac}}{\Omega_0^2} \int_{-\infty}^{\infty} dQ_z \int_0^{\infty} dQ_\perp [\exp(-Q_\perp^2/\Omega_0)] \{1 - \exp[\mp 2\beta(Q_\perp^2 + Q_z^2)]^{1/2}\}^{-1}$$

$$\times Q_\perp^5 (Q_\perp^2 + Q_z^2)^{1/2} \delta(Q_z^2 - 2KQ_z \pm 2(Q_\perp^2 + Q_z^2)^{1/2}) , \tag{3.4}$$

$$A_{ac} \equiv D^2 m^{*3} S_0 / (4\pi\hbar^4 \rho_m) . \tag{3.5}$$

where $+$ $(-)$ sign corresponds to the phonon emission (absorption) part of the current relaxation rate. It is apparent that $\Gamma^\pm(K)$ are positive and that

$$\Gamma^\pm(K) = \Gamma^\pm(-K) , \tag{3.6}$$

so that we need only consider positive values of K.

Let us consider $\Gamma^+(K)$. To perform the $Q_\perp$ integral, we seek the solutions for $Q_\perp$ of the argument of the δ function:

$$2KQ_z - Q_z^2 = 2(Q_\perp^2 + Q_z^2)^{1/2} \tag{3.7}$$

The right-hand side of (3.7) is positive so solutions exist only if

$$0 \le Q_z \le 2K \quad \text{for} \quad K \ge 0 , \tag{3.8}$$

and the solutions of (3.7) for $Q_\perp$ are given by

$$Q_\perp = \frac{|Q_z|}{2} [(Q_z - 2K)^2 - 4]^{1/2} , \tag{3.9}$$

which contribute to the $Q_\perp$ integration only if

$$|Q_z - 2K| \ge 2 \tag{3.10}$$

is satisfied. From (3.8) and (3.10), Q_z must be in the range

$$0 \le Q_z \le 2(K-1) , \tag{3.11}$$

which imposes the restriction on K:

$$K \ge 1 . \tag{3.12}$$

Keeping in mind the above restriction and the integral region for Q_z, after the $Q_\perp$ integration, we obtain $\Gamma^+(K)$ as

$$\Gamma^+(K) = \begin{cases} 0 \quad \text{for} \quad 0 \le K \le 1 \\ \dfrac{A_{ac}}{128\Omega_0^2} \displaystyle\int_0^{2(K-1)} dQ_z \frac{\exp\{-Q_z^2[(Q_z-2K)^2-4]/4\Omega_0\}}{1-\exp[\beta Q_z(Q_z-2K)]} Q_z^6 (Q_z-2K)^2 [(Q_z-2K)^2-4]^2 \quad \text{for} \quad K \ge 1 . \end{cases} \tag{3.13}$$

From the similar argument given above, $\Gamma^-(K)$ is given by

$$\Gamma^{-}(K)=-\frac{A_{\rm ac}}{128\Omega_0^2}\left[\int_{\max[0,2(1-K)]}^{\infty}dQ_z\frac{\exp\{-Q_z^2[(Q_z+2K)^2-4]/4\Omega_0\}}{1-\exp[\beta Q_z(Q_z+2K)]}Q_z^6(Q_z+2K)^2[(Q_z+2K)^2-4]^2\right.$$
$$\left.+\int_{2(K+1)}^{\infty}dQ_z\frac{\exp\{-Q_z^2[(Q_z-2K)^2-4]/4\Omega_0\}}{1-\exp[\beta Q_z(Q_z-2K)]}Q_z^6(Q_z-2K)^2[(Q_z-2K)^2-4]^2\right]$$
$$\text{for } K\geq 0\,. \quad (3.14)$$

It should be noted that for $0\leq K<1$, i.e., $0\leq k_z<m^*S_0/\hbar$, the phonon emission process is prohibited and only the phonon absorption process takes place. In terms of these variables and assuming a nondegenerate semiconductor (i.e., a Boltzmann distribution for f_{0k}), the power absorption is, apart from a constant, given by

$$P(\omega)\propto \mathrm{Re}[\sigma_{+-}(\omega)]$$
$$\propto\beta^{1/2}n(\beta)\int_0^{\infty}dk\frac{e^{-\beta k^2}\Gamma(k)}{(\omega-\omega_0)^2+\Gamma(K)^2}\,. \quad (3.15)$$

$n(\beta)$ is the free-carrier density.

To perform the Q_z-integration analytically in (3.13) and (3.14) is difficult. However, in certain limiting cases we can determine the form of the temperature and magnetic field dependence of $\Gamma(K)$ and hence of γ. For numerical evaluations see Sec. IV.

A. High- and low-temperature limits

1. $\beta<<(\beta\Omega_0)^{-1}<<1$,
viz., $(\frac{1}{2}m^*S_0^2\hbar\omega_0)^{1/2}<<k_BT<<\hbar\omega_0$

In this high-temperature limit, we can extract the temperature and the magnetic field dependence of $\Gamma(K)$ as

$$\Gamma(K)=\beta^{-1}\Omega_0^{3/4}f(K/\Omega_0^{1/4})\,, \quad (3.16)$$

where f is a complicated temperature-independent function of $K/\Omega_0^{1/4}$. For large values of $K/\Omega_0^{1/4}$, we can approximate $f(K/\Omega_0^{1/4})$ as

$$F(K/\Omega_0^{1/4})=f_0\frac{\Omega_0^{1/4}}{K}\,, \quad (3.17)$$

and so we obtain $\Gamma(K)$ as

$$\Gamma(K)=f_0\frac{\Omega_0}{\beta K}\,, \quad (3.18)$$

where f_0 is a constant. From (2.6), (3.15), and (3.18), we obtain a CR linewidth γ with the following temperature and magnetic field dependence:

$$\gamma\propto T^{1/2}\omega_0\,. \quad (3.19)$$

Letting $\omega=\omega_0$ and using (3.18) in (3.15), we obtain a peak value $P^{\max}(\omega_0)$ with the dependence

$$P^{\max}(\omega_0)\propto T^{-1/2}\omega_0^{-1}n(T)\,. \quad (3.20)$$

Under the high-quantum and high-temperature limits, the temperature and the magnetic field dependence of our results, (3.19) and (3.20), appears to be in good agreement with the experimental evidence on germanium. See also Ref. 4.

2. $(\beta\Omega_0)^{-1}<<\beta<<1$,
viz., $\frac{1}{2}m^*S_0^2<<k_BT<<(\frac{1}{2}m^*S_0^2\hbar\omega_0)^{1/2}$

In this low-temperature limit, we obtain the temperature and the magnetic field dependence of $\Gamma(K)$ as

$$\Gamma(K)=\beta^{-13/2}\Omega_0^{-2}g(K\beta^{1/2})\,, \quad (3.21)$$

where g is a complicated function of $K\beta^{1/2}$. A very rough estimate of the function g may be given by

$$g(K\beta^{1/2})=g_0(K\beta^{1/2})^5[(K^2\beta+1)^{1/2}-K\beta^{1/2}]^6\,, \quad (3.22)$$

where g_0 is a constant. This gives the temperature and the magnetic field dependence of the CR linewidth γ as

$$\gamma\propto T^{6.5}\omega_0^{-2} \quad (3.23)$$

and of the peak value $P^{\max}(\omega_0)$ as

$$P^{\max}(\omega_0)\propto T^{-6.5}\omega_0^2n(T)\,. \quad (3.24)$$

Experiments in the high quantum and low-temperature limiting case are much more difficult than the high-temperature limiting case because they require a significantly higher magnetic field and lower temperatures. Unfortunately we are unaware of any experimental results in this limiting case. It is interesting to note that Meyer's

theory gives for γ a linear dependence on ω_0 and no temperature dependence.[10]

IV. NUMERICAL ANALYSIS AND DISCUSSIONS

A. Current relaxation rate $\Gamma(K)$

The temperature and magnetic field dependence of the cyclotron resonance line shape and linewidth are determined by formulas (2.6) and (3.15), making use of the expressions (3.13) and (3.14) for $\Gamma(K)$. The temperature dependence arises both from the temperature dependence of $\Gamma(K)$ and from the Boltzmann factor $\exp(-\beta K^2)$. In the approximation we are using, the power absorption as a function of ω has a symmetrical line shape (centered on ω_0) because $\Gamma(K)$ is independent of ω. The experimental results are, however, usually expressed as a function of ω_0, with fixed ω. In this case our theory gives a slightly asymmetric line,[4] because $\Gamma(K)$ does depend on ω_0.

We firstly consider the current relaxation rate $\Gamma(K)$, and its constitutents $\Gamma^{\pm}(K)$, which arise from the emission and absorption of phonons. The Boltzmann factor $\exp(-\beta K^2)$ in Eq. (3.15) implies that we need to know $\Gamma^{\pm}(K)$ only for K in the range

$$0 \leq K \lesssim 3/\sqrt{\beta}\,. \tag{4.1}$$

In the case of low temperatures the contribution from the phonon absorption process $\Gamma^{-}(K)$ is usually ignored by setting the phonon occupation number of zero.[8,10] It is therefore of interest to directly test the validity of this assumption over this range of K values. Figures 1(a)–1(c) show the results of the numerical evaluation of Eqs. (3.13) and (3.14) for $\Gamma^{\pm}(K)$ as a function of K in the range (4.1) for an interaction constant characteristic of germanium and three values of the inverse temperature $\beta=10^{-3}$, 10^{-2}, 10^{-1}. These correspond approximately to real temperatures of 100, 10, and 1 K.

We see from these results that the phonon absorption process is important in all these cases which cover the usual experimental temperature ranges. It is therefore certainly not valid to set the phonon occupation $N_{\vec{q}}$ to zero.

Another approximation which is commonly employed in cyclotron resonance theory is to assume $\Gamma(K)$ is a constant ($=\frac{1}{2}\gamma$) and hence that the line shape is Lorentzian. The results show that this also is invalid: $\Gamma(K)$ varies significantly over the required range of K values, particularly at low temperatures.

B. Cyclotron resonance linewidth γ

We consider the magnetic field B and hence ω_0 and Ω_0 to be fixed and investigate the power absorption as a function of ω (or Ω). In this case the line is symmetric (see Fig. 2) and the width γ is given by the equation:

$$\int_0^{\infty} dk \frac{e^{-\beta K^2}\Gamma(K)}{(\gamma/2)^2+\Gamma(K)^2} = \tfrac{1}{2}\int_0^{\infty} dk \frac{e^{-\beta K^2}}{\Gamma(K)}\,, \tag{4.2}$$

where $\Gamma(K)$ is given by Eqs. (3.13)–(3.14). For a very narrow line $\gamma << \omega_0$ the difference in plotting the line shape as a function of Ω_0 with fixed Ω is negligible. We solve Eq. (4.2) numerically for γ and hence obtain the temperature and magnetic field (i.e., resonance frequency) dependence.

1. Temperature dependence

Figure 3(a) shows the temperature dependence of the cyclotron resonance linewidth at two different

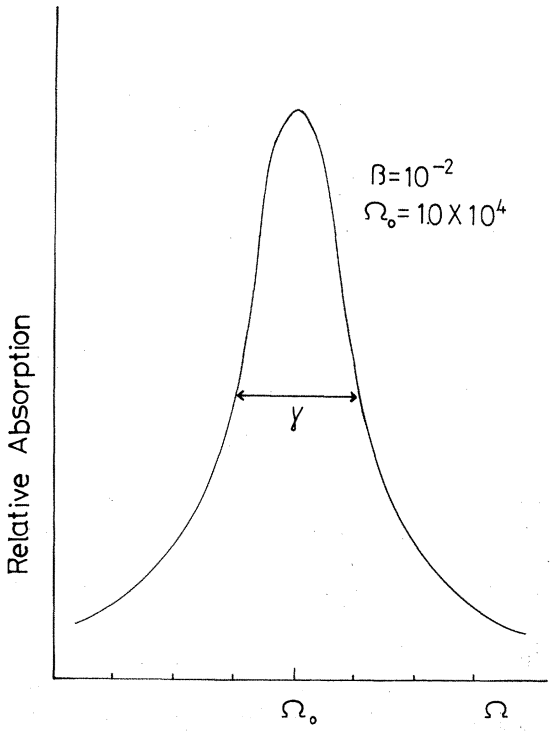

FIG. 2. Typical power absorption line shape.

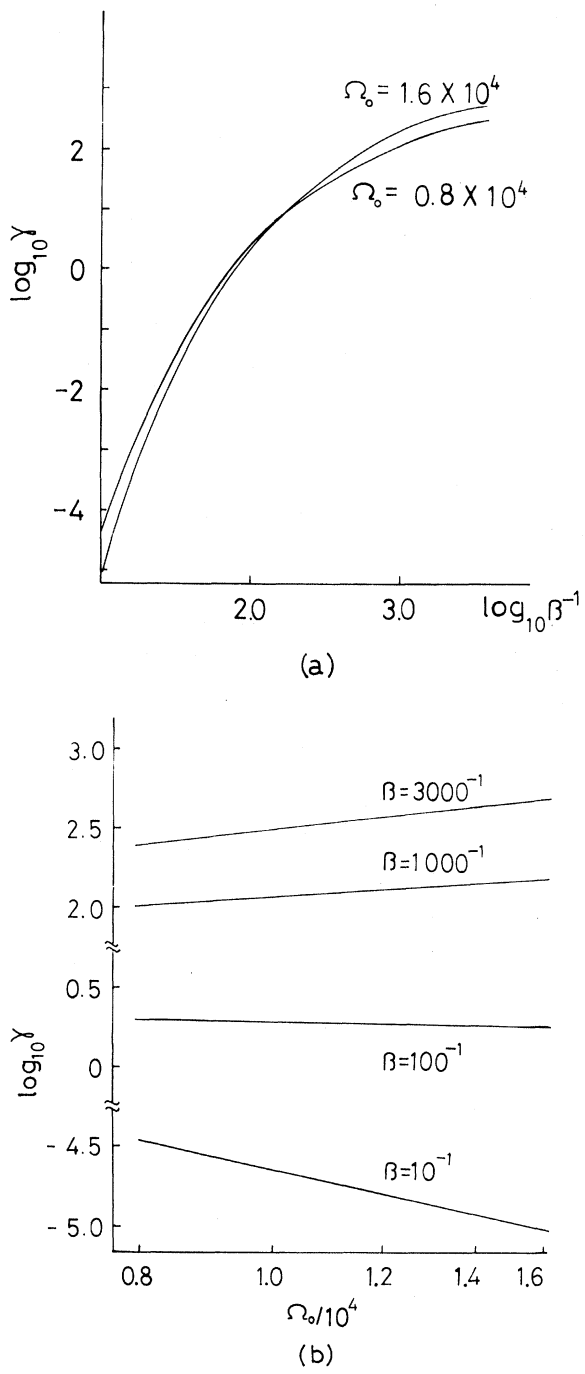

FIG. 3. (a) Temperature (β^{-1}) dependence of the linewidth (γ). (Linewidth γ is in the units of $m^*S_0^2/2\hbar$.) (b) Resonance frequency (Ω_0) dependence of the linewidth (γ). (Linewidth γ is in the units of $m^*S_0^2/2\hbar$.)

values of Ω_0 (0.8×10^4, 1.6×10^4) corresponding to ω_0 in an infrared region. (We have used the same data as in Sec. IV A.) It is seen from Fig. 3(a) that in the high-temperature region the width increases with temperature but at a much slower rate than the normally expected linear growth[10] and varies approximately as $T^{1/2}$. A $T^{1/2}$ dependence of the cyclotron resonance linewidth in a high-temperature region agrees with the recent experimental evidence.[9] In the low-temperature region, the width grows very rapidly and varies as $T^{6.5}$. To this date the authors were unaware of quantum limit cyclotron resonance experiments in a low-temperature region. We hope these theoretical results will stimulate experimental work in this temperature region ($T \lesssim 15$ K for Ge).

2. *Magnetic field (resonance frequency) dependence*

The dependence of the cyclotron resonance linewidth γ on the resonance frequency Ω_0 can be evaluated numerically as in Sec. IV A 1. For different values of a fixed inverse temperature β ($=10^{-1}, 25^{-1}, 50^{-1}, 100^{-1}, 500^{-1}, 1000^{-1}, 3000^{-1}$) which corresponds approximately to a real temperature T (≈ 1, 2.5, 5, 10, 50, 100, 300 K), we plot the frequency Ω_0 dependence of the cyclotron resonance width γ. It is seen from Fig. 3(b) that near an inverse temperature $\beta \sim \Omega_0^{-1/2}$, which corresponds to a real temperature given by

$$k_B T \sim (\tfrac{1}{2} m^* S_0^2 \hbar\omega_0)^{1/2} ,$$

the width is almost independent of magnetic field. In the case we are considering, Ge, this corresponds to a temperature of about 10 K. For $k_B T > (\frac{1}{2} m^* S_0^2 \hbar\omega_0)^{1/2}$, the width increases as Ω_0 whereas for $k_B T < (\frac{1}{2} m^* S_0^2 \hbar\omega_0)^{1/2}$, the width decreases like Ω_0^{-2}. From a least-squares linear regression method, we have found that the resonance frequency dependence of the cyclotron resonance width is given by $\Omega_0^{-1.8}$ for $\beta = 10^{-1}$, $\Omega_0^{-0.9}$ for $\beta = 50^{-1}$, $\Omega_0^{-0.2}$ for $\beta = 100^{-1}$, $\Omega_0^{0.9}$ for $\beta = 1000^{-1}$, $\Omega_0^{1.0}$ for $\beta = 3000^{-1}$.

C. Peak value of power absorption

The temperature and resonance frequency dependence of the peak value of the power absorption is obtained by setting $\omega = \omega_0$ in Eq. (3.15). The peak absorption is then given as a function of the resonance frequency Ω and the inverse temperature β. It is seen from Fig. 4(a) that in the high temperature region the dependence of the peak value of the power absorption on temperature behaves like $T^{-0.5}$ whereas in the low-temperature region like $T^{-6.5}$. It is interesting to note that, as seen in

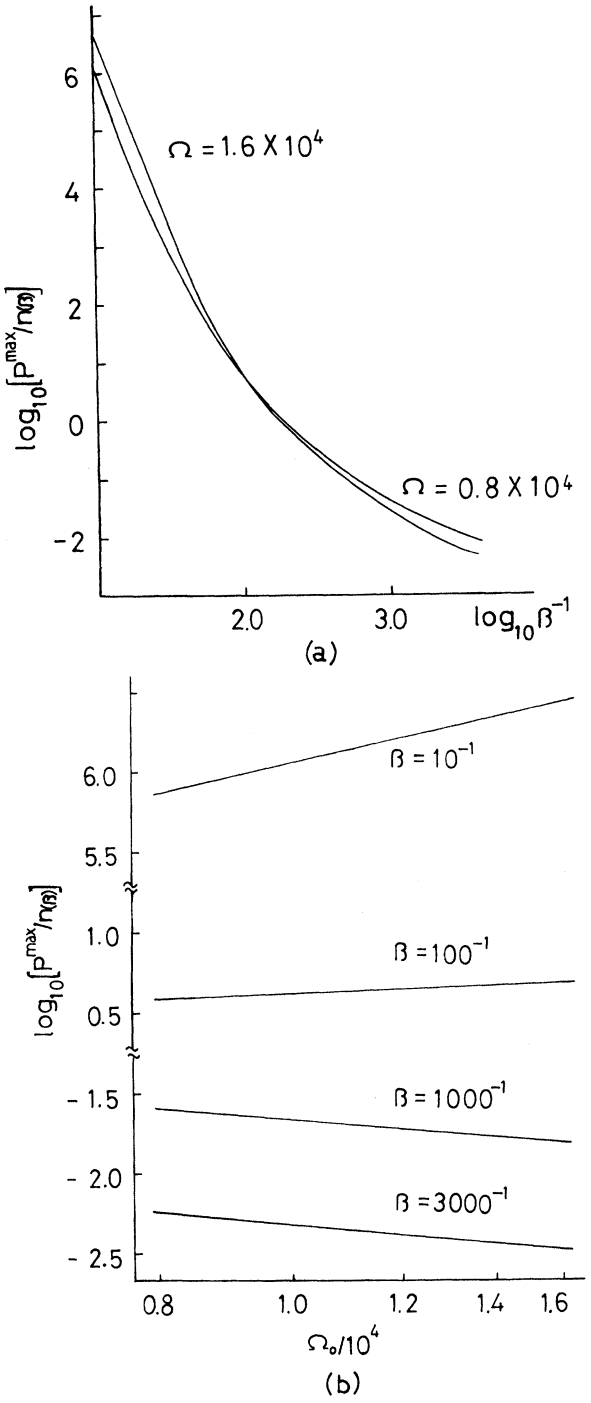

FIG. 4. (a) Temperature (β^{-1}) dependence of the peak value of the power absorption. (b) Resonance frequency (Ω_0) dependence of the peak value of the power absorption.

Figs. 3(a) and 4(a), the width gets narrower and the peak value grows significantly as the temperature decreases. This characteristic behavior for a nondegenerate semiconductor agrees with experiments by Fukai *et al.*[11] (although their experiment does not satisfy the quantum condition).

The resonance frequency dependence of the peak value of the power absorption [Fig. 4(b)] is found to be almost inversely proportional to the resonance frequency-dependent linewidth $\gamma(\Omega_0)$ as would be expected for a Lorentzian line shape. This dependence in the high-temperature region also agrees with experiment.[5]

V. CONCLUDING REMARKS

The basis of the cyclotron resonance absorption process is an electronic transition from the (N,k_z) Landau state to the $(N+1,k_z)$ state. In the *high quantum limit*, $\hbar\omega_0 >> k_B T$, the major contribution arises from the $N=0$ to $N=1$ transitions. These transitions by themselves would, of course, yield a line of zero width.

The actual width (in the absence of impurity or defect scattering) arises from the interaction of the electrons with lattice vibrations. An electron in state (N,k_z) can absorb or emit a phonon of wavevector $\vec{q}$ providing energy conservation can be satisfied, to yield a new state $(N',k_z \pm q_z)$. Only the intra-Landau-level scattering, $N'=N$, appreciably affects the linewidth. So the four processes which significantly affect the linewidth are the emission and absorption of a phonon in both the lowest and first excited Landau levels. These processes are shown graphically in Fig. 5.

The theory of Suzuki *et al.*[4] includes those four processes. We have, starting from the expression Suzuki *et al.*[4] which should be valid for narrow resonance lines, numerically determined the real part of the ac conductivity as a function of the phonon frequency ω, the magnetic frequency ω_0, and the temperature T. We have used an electron-phonon interaction applicable to the deformation potential interaction in Ge.

The main conclusions can be summarized as follows:

(i) In the *high-temperature region*,

$$(\tfrac{1}{2}m^* S_0^2 \hbar\omega_0)^{1/2} << k_B T << \hbar\omega_0 ,$$

the linewidth γ varies with temperature and magnetic frequency as

$$\gamma \propto T^{1/2}\omega_0 . \tag{5.1}$$

In the *low-temperature region*,

$$\tfrac{1}{2}m^* S_0^2 << k_B T << (\tfrac{1}{2}m^* S_0^2 \hbar\omega_0)^{1/2} ,$$

$$\gamma \propto T^{6.5}\omega_0^{-2} , \tag{5.2}$$

and in the *extreme low-temperature region*,

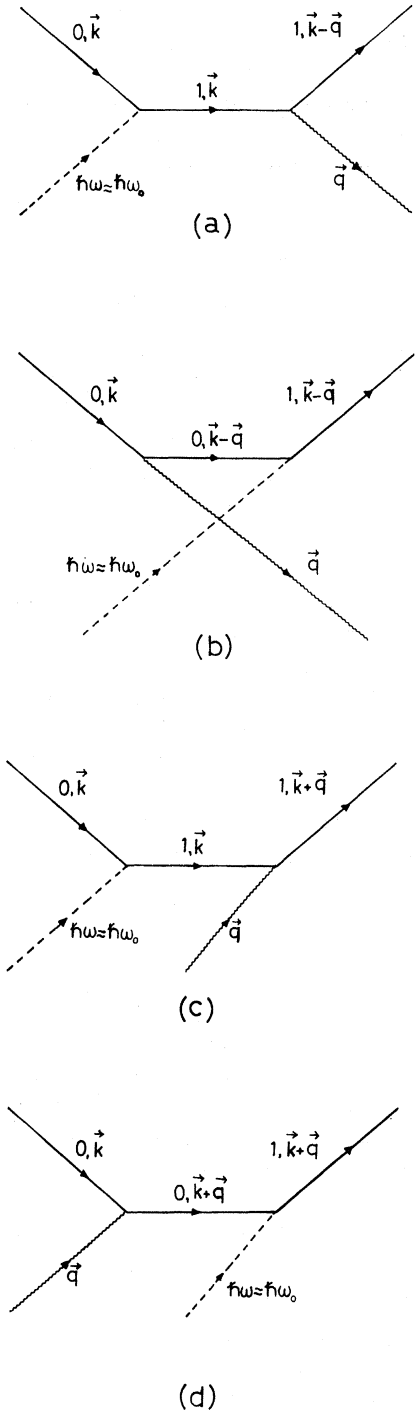

FIG. 5. Phonon emission (a) and (b) and absorption (c) and (d) in the lowest and the first excited Landau levels. The electron states are represented by—, the phonons by ⁓⁓, and the photons by - - -. Time increases from left to right.

$k_BT << \frac{1}{2}m^*S_0^2$, the linewidth dependence is

$$\gamma \propto T^3\omega_0^{-2}\exp\left[-\frac{2m^*S_0^2}{k_BT}\right]. \tag{5.3}$$

Numerical results for this extreme low-temperature region, $T << 0.1$ K for Ge, have not been included because in practice this region would be determined by impurity scattering.

(ii) Over the whole temperature range of interest the phonon absorption process, which is proportional to the phonon occupation number $N_{\vec{q}}$, produces a significant proportion of the linewidth. Thus the frequently used approximation of setting $N_{\vec{q}}$ to zero at low temperatures is *not* valid.

(iii) The current relaxation rate $\Gamma(k_z)$ varies so much with k_z that it is not practicable to replace it by a constant.

(iv) The inelastic nature of the electron-phonon interaction is important for all but the highest temperatures.

Although these conclusions follow directly from the numerical results, it is instructive to consider the scattering processes in more detail and to determine those values of the electron wave vector k_z and of the components of the phonon wave vector $q_z, q_\perp$ which significantly affect the linewidth. We call such values the "typical' values.

In the extreme low temperature region, $k_BT << \frac{1}{2}m^*S_0^2$, such a typical process is specified by

$$\frac{\hbar^2k_z}{2m^*} \approx k_BT, \quad \frac{\hbar^2q_z^2}{2m^*} \approx m^*S_0^2, \quad q_\perp \approx 0 \tag{5.4}$$

and only the phonon absorption produces the linewidth.

In the low-temperature region,

$$\tfrac{1}{2}m^*S_0^2 << k_BT << (\tfrac{1}{2}m^*S_0^2\hbar\omega_0)^{1/2},$$

the typical scattering process has

$$\frac{\hbar^2k_z}{2m^*} \approx k_BT, \quad \frac{\hbar^2q_z^2}{2m^*} \approx k_BT, \quad \frac{\hbar^2q_\perp^2}{2m^*} \approx \frac{(k_BT)^2}{\frac{1}{2}m^*S_0^2}. \tag{5.5}$$

For high temperatures,

$$(\tfrac{1}{2}m^*S_0^2\hbar\omega_0)^{1/2} << k_BT << \hbar\omega_0,$$

the typical values of the parameters k_z and $q_\perp$ are given by

$$\frac{\hbar^2k_z^2}{2m^*} \approx k_BT, \quad \frac{\hbar^2q_\perp^2}{2m^*} \approx \hbar\omega_0, \tag{5.6}$$

and there are two typical q_z regions:

$$\frac{\hbar^2 q_z^2}{2m^*} \approx \frac{\hbar\omega_0 \frac{1}{2} m^* S_0^2}{k_B T}, \qquad \frac{\hbar^2 (q_z - 2k_z)^2}{2m^*} \approx \frac{\hbar\omega_0 \frac{1}{2} m^* S_0^2}{k_B T} . \tag{5.7}$$

Now consider the elastic approximation to the scattering process. This involves the neglect of the phonon energy $\hbar\omega_{\vec{q}}$ in the energy conservation and is valid only if the "typical" phonon energy is small compared to the incident electron energy $\hbar^2 k_z^2/2m^*$. Using the above list of typical values, we see that the elastic approximation is valid only in the high temperature region where the typical phonon energy is $(\hbar\omega_0 \frac{1}{2} m^* S_0^2)^{1/2}$.

The approximation of neglecting the phonon occupation number $N_{\vec{q}}$ is valid only if, in the typical process, $\hbar\omega_{\vec{q}} >> k_B T$. In the above three temperature ranges the typical phonon eneriges are $\hbar\omega_{\vec{q}} \approx (\frac{1}{2} m^* S_0^2 \hbar\omega_0)^{1/2}$ (high temperatures), $\hbar\omega_{\vec{q}} \approx k_0 T$ (low temperatures), and $\hbar\omega_{\vec{q}} \approx \frac{1}{2} m^* S_0^2$ (extreme low temperatures). In the high- and low-temperature regions—this typically covers the range 0.5—300 K in high magnetic field cyclotron resonance in Ge—the criterion is certainly not valid and it is only in the extreme low-temperature region, less than say 0.05 K for Ge, that the typical phonon occupation number becomes very small. However in this region the phonon emission process is forbidden—energy conservation cannot be satisfied for $\hbar^2 k_z^2/2m^* < \frac{1}{2} m^* S_0^2$—and so the linewidth, although very small, is due almost entirely to the phonon absorption process.

The typical values listed above do not depend on the electron-phonon interaction constant C_q: They are equally valid for the piezoelectric interaction as for the deformation potential interaction. However the linewidth does depend on the specific form of the interaction. If the deformation potential, Eq. (3.1), is used together with the typical values (5.4)—(5.7) then the results (5.1)—(5.3) for the linewidth follow.

Consider, for example, the high-temperature region. In this region we have shown that the elastic approximation is valid and so the relaxation rate $\Gamma(k)$ is to a good approximation given by Eq. (1.1):

$$\Gamma(k) = \frac{1}{\pi} \frac{D^2}{\rho_m S_0^2} \frac{m^{*2}}{\hbar^4} \omega_0 k_B T \, | k_z |^{-1} .$$

This shows in particular that the relaxation process *for a given electron wave vector k* increases linearly with temperature. It should not however be concluded from this that the linewidth γ has this temperature dependence. As a temperature is increased so is the mean electron energy and hence so is the mean wave vector k_z. The linewidth is approximately given by $2\Gamma(k)$ with k_z equal to the mean electron wave vector $(m^* k_B T/\hbar^2)^{1/2}$. That is, γ is approximately given by

$$\gamma \approx \frac{1}{\pi} \frac{D^2}{\rho_m S_0^2} \frac{m^{*2}}{\hbar^4} \omega_0 \left[\frac{\hbar^2}{m^*} \right]^{1/2} (k_B T)^{1/2} \tag{5.8}$$

and so varies linearly with magnetic field but is proportional to the square root of the temperature.

This square root dependence on the temperature has been predicted previously by Suzuki *et al.*[4] and also by Arora and Spector[12] and seems to explain the experimental results.[8]

The same procedure can be used to derive the corresponding results for the coupling to piezoelectric phonons. If we use the isotropic model of Mahan[13] then we have

$$\gamma \propto \begin{cases} T \ \text{(high temperatures)} \\ T^{4.5} \omega_0^{-2} \ \text{(low temperatures)} \\ T^3 \omega_0^{-2} \exp\left[-\dfrac{2m^* S_0^2}{k_B T} \right] \\ \quad \text{(extreme low temperatures).} \end{cases}$$

[1] G. Dresselhaus, A. F. Kip, and C. Kittel, Phys. Rev. 98, 368 (1955); R. Ito, H. Kawamura, and M. Fukai, Phys. Lett. 13, 26 (1964); N. Miura, G. Kido, and S. Chikazumi, Solid State Commun. 18, 885 (1976).

[2] A Lodder and S. Fujita, J. Phys. Soc. Jpn. 25, 774 (1968).

[3] A. Kawabata, J. Phys. Soc. Jpn. 23, 999 (1967).

[4] A. Suzuki, S. D. Choi, and S. Fujita, J. Phys. Chem. Solids 41, 735 (1980).

[5] N. Miura, G. Kido, K. Suzuki, and S. Chikazumi, in *Proceedings of the International Conference on the Application of High Magnetic Fields in Semiconductor Physics,* edited by G. Landwehr (Würzburg University Press, Würzburg, 1974), p. 441; D. Fink and R. Braunstein, Solid State Commun. 15, 1627 (1974).

[6] R. Kubo, S. J. Miyake, and N. Hashitsume, in *Solid State Physics,* edited by F. Seitz and D. Turnbull (Academic, New York, 1965), Vol. 17, p. 269.

[7]V. K. Arora, Phys. Rev. B 13, 2532 (1976).
[8]C. K. Sarkar, Phys. Status Solidi B 102, K81 (1980); R. Ito, M. Fukai and I. Imai, J. Phys. Soc. Jpn. 21, Suppl. 357 (1966); S. D. Choi and S. Fujita, Solid State Commun. 17, 298 (1981).
[9]A. Suzuki and D. Dunn, Phys. Rev. A 25, 2247 (1982).
[10]H. J. G. Meyer, Phys. Lett. 2, 259 (1961).
[11]M. Fukai, H. Kawamura, K. Sekido, and I. Imai, J. Phys. Soc. Jpn. 19, 30 (1962).
[12]V. K. Arora and H. N. Spector, Phys. Status Solidi B 94, 701 (1979).
[13]G. D. Mahan, in *Polarons in Ionic Crystals and Polar Semiconductors,* edited by J. T. Devreese (North-Holland, Amsterdam, 1972), p. 553.