Collective excitations in semiconductor superlattices

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Electronic collective modes of a system of large numbers of equally spaced, parallel two-dimensional electron layers are discussed within a self-consistent-field approach. Plasmon dispersion relations for the finite system as well as for the infinite periodic system are obtained. It is shown that the optical-plasmon frequency of the periodic system goes into the known two- or three-dimensional limit, respectively, depending on whether $qa \gg 1$ or $qa \ll 1$, where q is the wave number in the two-dimensional plane and a is the layer spacing. Effect of a uniform static external magnetic field oriented normal to the two-dimensional layers, on the collective-mode spectrum, is discussed with the use of the self-consistent-field and hydrodynamic approximations. It is shown that magnetoplasmons, helicon, and Alfvén waves can all exist in such a periodic system under suitable conditions. The theory is generalized to a system where the alternate layers are electrons and holes. The relevance of these results to semiconductor superlattice systems (both types I and II) is pointed out.

I. INTRODUCTION

The development of molecular beam epitaxy (MBE) in the last few years has made it possible to produce high-quality artificial superlattices made from two different semiconductor materials (e.g., InAs-GaSb, GaAs-AlAs, Ge-GaAs, etc.) with similar lattice structure and matching lattice parameters. In the direction of superlattice growth (taken to be the z direction), one has m atomic monolayers of material A deposited in an atomically sharp way on n atomic monolayers of material B to form a new superlattice unit cell of size $(m+n)$ lattice spacings "a" in the z direction. A macroscopic sample of such A-B superlattice is a new bulk material and has properties intermediate between materials A and B.

The physics of such semiconductor superlattices have been extensively studied[1] in the last few years. Two types of superlattices have so far been studied in great detail. One of these (referred to as type I) is the GaAs-$Al_xGa_{1-x}As$ system where the band gap of GaAs is contained entirely within the band gap of $Al_xGa_{1-x}As$ giving rise to band-gap discontinuities in both the conduction and valence bands of the resultant superlattice system.[2] This band-gap discontinuity at the interface of the two materials gives rise to potential wells in the conduction band of GaAs layers separated by potential barriers in the $Al_xGa_{1-x}As$ layers. Selective doping of the $Al_xGa_{1-x}As$ layers by the modulation-doping technique produces ionized donors in these layers. The electrons released by these donors drop into the potential wells on the GaAs side. The one-dimensional potential well quantizes the electronic motion along the superlattice direction, and the GaAs conduction band splits into a series of subbands, each of which represents "free" effective-mass-type electronic motion in the plane perpendicular to the superlattice direction. If the electron density is not too high so that only the lowest subband is occupied by electrons, then such a multilayer, superlattice system at low temperatures is a periodic system of two-dimensional electron gas (2DEG). Much attention has recently been focused on these modulation-doped GaAs-$Al_xGa_{1-x}As$ superlattice systems because of the possibility[2] of achieving very high mobility due to the spatial separation of the ionized donor impurities (in the $Al_xGa_{1-x}As$ layers) and the mobile carriers (in the GaAs layers).

The other type of superlattice system (referred to as type II) is typified by an InAs-GaSb system[3] where the band match-up is such that the conduction-band minimum of InAs is below the valence-band maximum of GaSb. Thus in the InAs-GaSb heterojunction, a spatial separation of electrons and holes takes place due to a transfer of charge ($\sim 10^{17}$ cm^{-3}) from GaSb to InAs due to the unusual lineup of bulk energy bands. Thin (layer thickness $\lesssim 170$ Å) type II superlattices of InAs-GaSb are found to be semiconductors exhibiting spatial separation and confinement of electrons (in InAs) and holes (in GaSb) which are thermally excited. However, superlattices of thickness greater then 170 Å behave as semimetals, containing spatially separated electrons (in InAs) and holes (in GaSb) even at zero temperature. For our purpose it is sufficient to consider the type-II superlattice as a one-dimensional periodic arrangement of two-dimensionally confined electrons and holes which are spatially spearated from each other.

Various aspects of the physics of these semiconductor superlattices have beem clarified in the last few years.[1-3] Thus, subband formation and cyclotron resonance of the two-dimensional carriers have been investigated by the far-infrared absorption spectroscopy[4-6] and resonant-light-scattering technique.[7-9] Much attention has also been focused[2,3] on the transport and magnetotransport behavior of these systems as well as on the possible laser operation[10] of the quantum wells. One aspect of the physics of these systems that is of particular interest to us is the electronic collective-excitation spectrum of a periodic array of equally spaced charge layers. The collective mode or the plasmon of a single charge layer is known[11] to have a frequency ω_p proportional to square root $q^{1/2}$ of the wave number q in the long wavelength limit. On the other hand, three-dimensional plasma frequency has the well-known[12] constant value independent of wave number in the long wavelength limit. The superlattice, being a periodic system of 2DEG, falls intermediate between two- and three-dimensional systems. When the separation between the adjacent layers is very large, we expect negligible coupling between them, giving rise to two-dimensional behavior. However, for vanishing separation between adjacent layers, the system is effectively three-dimensional. Since the layer thickness of a superlattice can be controlled by MBE technique, such a system provides a way of studying transition from two-dimensional to three-dimensional behavior in the collective-mode spectrum as the layer thickness is reduced. The situation under an external magnetic field along the direction of superlattice growth (i.e., perpendicular to the two-dimensional layers) is also interesting because the "three-dimensional" case of small separation should allow for extra modes that can propagate along the magnetic field direction in addition to the familiar magnetoplasmon modes.

In this paper we study the collective-excitation spectrum of such superlattices in the simplest possible model. Our model is that of a periodic arrangement in z direction of a large number of two-dimensional charge layers which occupy the xy plane. We consider the layers to be infinitely thin. To allow for both types-I and -II superlattices, we consider both the cases of the layers all having the same charge carriers (electrons) or alternate layers of different charge carriers (electrons and holes). We consider situations both in the presence and in the absence of external magnetic fields. Our basic formalism is that of self-consistent-field (SCF) technique. We obtain the dynamical-response function for the superlattice from which the collective-mode spectrum is calculated. In the case of nonzero magnetic field, we find it more convenient to obtain the collective-mode spectrum from a hydrodynamical formulation rather than working with the complete magnetoconductivity tensor calculated in the SCF formalism.

There have been a number of theoretical studies pertaining to the collective-excitation spectrum of 2DEG. Stern originally obtained[11] the dispersion relation of two-dimensional plasmons within a SCF approximation, showing that the plasma frequency vanishes in the long wavelength ($q \rightarrow 0$) limit as $q^{1/2}$, where q is the wave number. There have been a number of theoretical papers[13-16] following Stern's work where the plasma dispersion for the realistic two-dimensional system, as occurring, for example, in space-charge layers on semiconductor surfaces, have been derived taking into account physical effects arising from the fact that in any real system the 2DEG is submerged in a three-dimensional environment. There have also been papers[17,18] dealing with the collective modes of a single 2DEG in the presence of a perpendicular magnetic field. Experimentally, plasmons and magnetoplasmons have been observed[19] in 2DEG occurring in an n-type inversion layer on silicon surface as well as in electrons bound on helium surface. Theis has reviewed[20] the subject of collective modes in inversion layers (which serve as prototype 2DEG systems) in a recent paper.

Quite independent of these developments in actual two-dimensional systems, Fetter used[21] a hydrodynamical approach to treat the problem of electrodynamics of periodic array of electron layers. Part of our work could be considered as the SCF analog of Fetter's hydrodynamic treatment of electrodynamics in a periodic array. Results that we obtain are similar to those obtained by Fetter, since hydrodynamic theory gives results equivalent to SCF approach if the compressibility or the hydrodynamic pressure term is empirically chosen. Fetter, however, treated only the case of zero magnetic field. Our hydrodynamic treatment of collective modes in the presence of an external magnetic field is a generalization of his work.[21] In the context of transition-metal dichalcogenides, Visscher and Falicov treated[22] the problem of *static* response of a periodic array of electron layers within a SCF approach in the absence of any external magnetic field. They do not touch upon the subject of *dynamic* response and collective modes of such a layered system, which form the principal content of our work. Neither Fetter[21] nor Visscher and Falicov[22] considered the electron-hole case as we have done in this paper. After our work was completed, we learned that Bloss has done[23] a similar study of collective modes of a periodic array of 2DEG by generalizing earlier work[24] of Das Sarma and Madhukar on spatially separated two-component two-dimensional plasma in solids. Where the two works overlap his conclusions are similar to ours.

The plan of the present paper is the following: In Sec. II we describe the SCF treatment of a superlattice in the absence of any external magnetic field. In Sec. III we consider the SCF response for the nonzero magnetic field situation following the work of Greene *et al.*[25] for the 3DEG and of Chiu and Quinn[17] for the 2DEG. In Sec. IV we introduce a hydrodynamical model following Fetter[21] and explicitly obtain magnetoplasmons and the other possible modes in the presence of an external magnetic field. In Sec. V we consider the electron-hole system relevant for the type-II superlattices. We conclude in Sec. VI, pointing out some possible future improvements of our theory, discussing experimental significance of some of our findings, and connecting our work with prior work on the subject.

II. SELF-CONSISTENT RESPONSE FOR ZERO MAGNETIC FIELD

Our model for the superlattice system as discussed in the Introduction is the simplest one possible. We consider a periodic array of charged layers of two-dimensional electron density n_s per unit area. We consider the layers to occupy the xy plane and the separation between adjacent layers in the z direction is taken to be a length "a" which is the period of the superlattice. The motion of electrons in the xy plane is assumed to be completely free. For the actual superlattice this last approximation is quite valid within an effective-mass approximation.

The single-particle wave function for an electron in this model superlattice is given by (suppressing spin variables)

$$\phi_{\vec{k}l}(\vec{r},z)=\frac{1}{\sqrt{A}}e^{i\vec{k}\cdot\vec{r}}\xi(z-la)\ . \tag{1}$$

In Eq. (1), l is the layer index and $\vec{k}$ is a two-dimensional wave vector describing the planar motion in the (xy) plane. A is the two-dimensional area of each layer (needed only for normalization of the plane wave) and all vectors with lower case letters (e.g., $\vec{r}$, $\vec{k}$) denote two-dimensional vectors in the xy plane. The envelope function $\xi(z-la)$ denotes the confinement of the electrons in the lth layer which is positioned at $z=la$ with $l=0,\pm1,\pm2$, etc. In our approximation we choose

$$|\xi(z-la)|^2=\delta(z-la)\ . \tag{2}$$

Thus we are assuming each layer to be strictly two dimensional with no overlap between adjacent layers. This approximation is made mainly for the sake of convenience, allowing us to do most of our calculation analytically. It is expected to be reasonably good for low electron densities when only the lowest two-dimensional subband in each quantum well is occupied by electrons.

The noninteracting single-particle energy is given by (with m as the mass for planar motion)

$$E_{\vec{k},l}\equiv E(\vec{k})=\frac{\hbar^2k^2}{2m}+E_0\ , \tag{3}$$

where E_0 is the quantization energy for the subband motion which is the same for all the layers. With no loss of generality we put $E_0=0$ and neglect it from further considerations. The Hamiltonian for the superlattice can now be written as

$$H=H_0+H_{\rm int}\ , \tag{4}$$

where

$$H_0=\sum_{\vec{k},l}E_{\vec{k},l}C^{\dagger}_{\vec{k},l}C_{\vec{k},l}\ , \tag{5}$$

and

$$H_{\rm int}=\frac{1}{2}\sum_{\vec{k},\vec{k}',\vec{q}}\sum_{l_1l_2l_3l_4} v_{l_1l_2l_3l_4}(\vec{q}) \times C^{\dagger}_{\vec{k},l_1}C^{\dagger}_{\vec{k}',l_2}C_{\vec{k}'-\vec{q},l_4} \times C_{\vec{k}+\vec{q},l_3} . \quad (6)$$

The operator $C_{\vec{k},l}(C^{\dagger}_{\vec{k},l})$ annihilates (creates) an electron with 2D momentum $\hbar\vec{k}$ in layer l. The interaction $v_{l_1l_2l_3l_4}(\vec{q})$ is the matrix element of the Coulomb interaction $v(\vec{r},z)$ between wave functions of different layers given by Eq. (1). It is given by

$$v_{l_1l_2l_3l_4}(\vec{q})=\langle l_1l_2\,|\,v_q(z)\,|\,l_3l_4\rangle , \quad (7)$$

where $v_q(z)$ is the Fourier transform in the xy plane of the three-dimensional Coulomb interaction $v(\vec{r},z)$. Since the lattice dielectric constants of the two materials forming the superlattice are very similar, we neglect any image potential term in the interaction to write

$$v(\vec{r},z)=\frac{e^2}{\kappa(r^2+z^2)^{1/2}} , \quad (8)$$

where κ is the background dielectric constant of the system. The matrix element given by Eq. (7) is very simply obtained for our model wave functions of Eq. (2), giving

$$v_{l_1l_2l_3l_4}(\vec{q})=A^{-1}\delta_{l_1l_3}\delta_{l_2l_4}v_q(z)\,|_{z=(l_1-l_2)a} , \quad (9)$$

where $\delta_{l_1l_3}$, etc., are Kronecker delta functions. Using the known form of the Fourier transform of the two-dimensional Coulomb potential,

$$v_q(z)=2\pi e^2\exp(-q\,|z\,|)/q\kappa$$

in Eq. (9) gives

$$v_{l_1l_2l_3l_4}(\vec{q})=\delta_{l_1l_3}\delta_{l_2l_4}\frac{2\pi e^2}{\kappa qA}e^{-q\,|l_1-l_2|\,a} , \quad (10)$$

$$=\delta_{l_1l_3}\delta_{l_2l_4}V_l(q)\,|_{l=l_1-l_2} , \quad (11)$$

where

$$V_l(q)=\frac{2\pi e^2}{\kappa qA}e^{-q\,|l\,|\,a} . \quad (12)$$

Following the SCF prescription of Ehrenreich and Cohen,[26] we now introduce a single-particle density matrix defined as

$$\rho_0=\sum_{\alpha}f_\alpha\,|\,\alpha\rangle\langle\alpha\,| , \quad (13)$$

where $\alpha=(\vec{k},l)$ is a composite index defining the noninteracting single-particle state of Eq. (1), and f_α is the occupation factor which is unity or zero, depending on whether or not the state $|\,\alpha\rangle$ is occupied. We work strictly at zero temperature. In the presence of an external perturbing potential $v^{\rm ex}$ due to a point charge placed at the origin, the density matrix will be modified to

$$\rho=\rho_0+\rho' . \quad (14)$$

The perturbation ρ' is to be determined from the equation of motion[26] for the density matrix

$$\frac{\partial\rho}{\partial t}=\frac{i}{\hbar}[\rho,H+v] , \quad (15)$$

where

$$v=v^{\rm ex}+v^{\rm in} . \quad (16)$$

In Eq. (16) $v^{\rm in}$, the potential induced in the sytem due to the external perturbation, is to be obtained self-consistently. We make the Hartree SCF approximation to Eq. (15) (which has been implicit in our definition of ρ) in which the full Hamiltonian H of Eq. (4) is replaced by the noninteracting part H_0 given by Eq. (5). This would give us the so-called[12] random-phase-approximation (RPA) response function.

Replacing H by H_0 in Eq. (15) and introducing a frequency ω through the Fourier transform in time,

$$\rho(\omega)=\int_{-\infty}^{+\infty}dt\,\rho(t)e^{i\omega t} , \quad (17)$$

we can easily solve Eq. (15) in *linear approximation,* giving

$$\rho'_{\alpha\alpha'}=\frac{f_{\alpha'}-f_\alpha}{\hbar\omega+E_{\alpha'}-E_\alpha}v_{\alpha\alpha'} , \quad (18)$$

where $v_{\alpha\alpha'}=\langle\alpha\,|\,v\,|\,\alpha'\rangle$ and $E_\alpha\equiv\vec{E}_{k,l}$. To obtain $v_{\alpha\alpha'}=v^{\rm ex}_{\alpha\alpha'}+v^{\rm in}_{\alpha\alpha'}$, we follow the prescription leading to Eq. (11), remembering that $v^{\rm ex}$ is the potential due to a test charge of strength e placed at the origin. A straightforward calculation gives

$$v^{\rm ex}_{\alpha\alpha'}=\langle\alpha\,|\,v^{\rm ex}\,|\,\alpha'\rangle=\langle\vec{k},l\,|\,v^{\rm ex}\,|\,\vec{k}',l'\rangle =\delta_{ll'}V_l(q=|\,\vec{k}-\vec{k}'\,|) , \quad (19)$$

and

$$v^{\rm in}_{\alpha\alpha'}=\langle\vec{k},l\,|\,v^{\rm in}\,|\,\vec{k}',l'\rangle =\delta_{\vec{k},\vec{k}'+\vec{q}}\delta_{ll'}\sum_{l_1}An_{l_1}(q,\omega)V_{l-l_1}(q) . \quad (20)$$

$V_l(q)$ in Eqs. (19) and (20) has been defined in Eq.

(12). The function $n_l(q,\omega)$ in Eq. (20) is the Fourier transform of the induced electron density $n_l(\vec{r},t)$ on the lth plane. It is given by

$$n_l(q,\omega)=A^{-1}v_l(q,\omega)\sum_{\vec{k}}\frac{f_{\vec{k},l}-f_{\vec{k}-\vec{q},l}}{E_{\vec{k},l}-E_{\vec{k}-\vec{q},l}-\hbar\omega}\ , \tag{21}$$

where

$$v_l(q,\omega)=\langle l,\vec{k}+\vec{q}\,|\,v\,|\,l,\vec{k}\rangle\ . \tag{22}$$

Using Eqs. (19)–(22) in Eq. (16) we get the following:

$$v_l(q,\omega)=V_l(q)+\sum_{l_1}\left[v_{l_1}(q,\omega)\sum_{\vec{k}}\frac{f_{\vec{k},l_1}-f_{\vec{k}-\vec{q},l_1}}{E_{\vec{k},l_1}-E_{\vec{k}-\vec{q},l_1}-\hbar\omega}\right]V_{l-l_1}(q)\ . \tag{23}$$

Equation (23) is the basic dynamical self-consistent linear-response equation for the total potential of the system. Since our interest is to obtain the collective-mode spectrum it is more convenient to write down the self-consistent equation for the external potential using Eq. (19) in Eq. (23),

$$v_l^{\text{ex}}(q,\omega)=v_l(q,\omega)-\Pi(q,\omega)\times\sum_{l'}V_{l-l'}(q)v_{l'}(q,\omega)\ . \tag{24}$$

By writing $v_l(q,\omega)=v_l^{\text{ex}}(q,\omega)+v_l^{\text{in}}(q,\omega)$ where $v_l^{\text{in}}(q,\omega)$ is given by Eq. (20), we can obtain a true response equation in which the induced density fluctuation $n_l(q,\omega)$ is given in terms of the external potential $v_l^{\text{ex}}(q,\omega)$. In Eq. (24) $\Pi(q,\omega)$ is the polarizability[11] of a 2DEG and is defined to be

$$\Pi(q,\omega)=A\sum_{\vec{k}}\frac{f_{\vec{k},l}-f_{\vec{k}-\vec{q},l}}{E_{\vec{k},l}-E_{\vec{k}-\vec{q},l}-\hbar\omega}\ . \tag{25}$$

$\Pi(q,\omega)$ is independent of the layer index l for the obvious reason that the occupancies ($f_{\vec{k}}$'s) and the energies ($E_{\vec{k}}$'s) are independent of layer index l.

Collective modes of the system are given by the solutions of Eq. (24) for zero external potential. Putting $v_l^{\text{ex}}=0$ in Eq. (24) we get

$$v_l(q,\omega)-\Pi(q,\omega)\sum_{l'}V_{l-l'}(q)v_{l'}(q,\omega)=0\ . \tag{26}$$

In matrix notation Eq. (26) can be rewritten as

$$v-\Pi Vv=0\ . \tag{27}$$

For a self-sustaining solutation we must have

$$|\overleftrightarrow{1}-\Pi\overleftrightarrow{V}|=0\ . \tag{28}$$

Clearly $\overleftrightarrow{\epsilon}=\overleftrightarrow{1}-\Pi\overleftrightarrow{V}$ is the general dielectric matrix for the superlattice, and collective modes are given by the vanishing of the determinant of $\overleftrightarrow{\epsilon}$

$$|\delta_{ll'}-\Pi V_{l-l'}|=0\ . \tag{29}$$

In writing Eq. (29) we use the facts that $\Pi(q,\omega)$ is a scalar function independent of layer index l, and V depends on l,l' only through the difference $l-l'$ [cf. Eq. (26)].

For a system with a finite number N of layers (which is *not* a superlattice), Eq. (29) defines an $N\times N$ determinantal equation with N normal modes which are the collective excitations of an N component, spatially separated two-dimensional plasma. This is the N-component generalization of the earlier work[13,24,27] on the collective modes of a two-component, two-dimensional plasma.

Since we are interested in a true superlattice ($N\rightarrow\infty$), we would not pursue the N-component problem in any great detail here. However, we make two comments about the nature of the long wavelength, collective modes implied by Eq. (29) for an N-layer system in the strong coupling ($qa\ll 1$) limit: (i) The highest energy mode (the so-called optical plasmon or OP) of Eq. (29) would go as $O(q^{1/2})$ in the long wavelength ($q\rightarrow 0$) limit, with the coefficient of the $q^{1/2}$ term proportional to $(Nn_s)^{1/2}$. This is just like a regular (2D) plasmon of a single-layer 2DEG, but with an enhancement of the coefficient by a factor of $N^{1/2}$. (ii) All the other $(N-1)$ modes would be "acoustic" in nature in the long wavelength limit going linear as $O(q)$. These modes would, however, be damped in general.[24,27] In the opposite limit of weak coupling situation ($qa\gg 1$), one recovers N independent plasmons of single layer 2DEG each going as $O(n_s^{1/2}q^{1/2})$ in the long wavelength limit. The long wavelength behavior of Eq. (29) for a finite N layer system can be easily extracted explicitly by using the known[11] long wavelength form for the polarizability of a 2DEG:

$$\Pi(q,\omega)\simeq\left[\frac{n_s}{m}\right]\left[\frac{q^2}{\omega^2}\right]A\ . \tag{30}$$

The function V has been defined in Eq. (12) and is given by

$$V_{ll'}(q) \equiv V_{l-l'}(q) = \frac{2\pi e^2}{\kappa q A} e^{-q|l-l'|a} . \quad (31)$$

We now consider the periodic superlattice system and obtain the collective modes from Eq. (26). It is more convenient to work with the induced density fluctuation $n_l(q,\omega)$ which is given by Eq. (21). Using Eq. (21) in Eq. (26) and using the definition for $\Pi(q,\omega)$ from Eq. (25) gives

$$An_l(q,\omega)\Pi(q,\omega)^{-1} - A\sum_{l'} V_{l-l'}(q) n_{l'}(q,\omega) = 0 . \quad (32)$$

By replacing the zero on the right-hand side of Eq. (32) with $v_l^{\rm ex}(q,\omega)$, we get the induced density fluctuation on the lth layer in terms of the external potential.

Rewriting Eq. (32) gives the following condition for the self-sustaining collective mode in a superlattice:

$$n_l(q,\omega) - \Pi(q,\omega)\sum_{l'} V_{l-l'}(q) n_{l'}(q,\omega) = 0 . \quad (33)$$

However, a superlattice is completely periodic in z direction with periodicity a. Thus, we can use the following ansatz as a solution for Eq. (33):

$$n_l(q,\omega) = n_0(q,\omega) e^{ik_z la} , \quad (34)$$

where the solution is assumed to be of the form $e^{ik_z z}$ in z direction with $z = la$, where $l = 0, \pm 1, \pm 2$, etc., are the possible discrete z values allowed in the superlattice structure. The "wave number" k_z that labels the induced density fluctuation in the periodic system is restricted within the first Brillouin zone of the superlattice, i.e., $0 \le k_z \le 2\pi/a$.

With Eq. (34) in Eq. (33), we obtain

$$1 - \Pi(q,\omega)\sum_{l'} V_{l-l'}(q) e^{-ik_z(l-l')a} = 0 . \quad (35)$$

Substituting for $V_{l-l'}(q)$ from Eq. (31) in Eq. (35) gives

$$1 = \frac{2\pi e^2}{\kappa q a}\Pi(q,\omega)\sum_{l'} e^{-q|l-l'|a - ik_z(l-l')a} . \quad (36)$$

The sum over l' in Eq. (36) goes over all positive and negative integers including zero. Introducing the function $S(q,k_z)$ defined by

$$S = \sum_{l'} \exp[-q\,|l-l'|\,a - ik_z(l-l')a] , \quad (37)$$

allows us to write Eq. (36) as

$$1 = \frac{2\pi e^2}{\kappa q A}\Pi(q,\omega) S(q,k_z) . \quad (38)$$

We should point out that periodicity ensures that S is independent of layer index l. The sum S can in fact be evaluated exactly to give

$$S = \frac{\sinh qa}{\cosh qa - \cos k_z a} . \quad (39)$$

The collective-excitation spectrum is thus given by

$$1 = \frac{2\pi e^2}{\kappa q A}\Pi(q,\omega)\left[\frac{\sinh qa}{\cosh qa - \cos k_z a}\right] . \quad (40)$$

The solution of Eq. (40) for a given k_z can be obtained in a straightforward fashion since $\Pi(q,\omega)$ is exactly known.[11] We note that Eq. (40) defines a denumerably infinite set of collective modes defined by k_z which is a continuous variable in the first Brillouin zone ($0 \le k_z \le 2\pi/a$). Thus Eq. (40) is the $N \to \infty$ generalization of the matrix equation (29) defining the N modes of an N-layer system.

We consider the strong ($qa \ll 1$) and the weak ($qa \gg 1$) coupling limits of Eq. (40) explicitly in the following.

1. Strong coupling case ($qa \ll 1$)

We first consider the $k_z \ne 0$ situation. Taking $qa \to 0$ limit of Eq. (40) gives

$$1 = \frac{2\pi e^2}{\kappa q A}\Pi(q,\omega)\left[\frac{qa}{1 - \cos k_z a}\right] . \quad (41)$$

By using Eq. (30) for the long wavelength polarizability $\Pi(q,\omega)$, we obtain the following collective-excitation frequency in the long wavelength limit ($q \to 0$):

$$\omega \simeq \left[\frac{2\pi n_s e^2}{\kappa m}\right]^{1/2} (1 - \cos k_z a)^{-1/2}\sqrt{a}\, q . \quad (42)$$

Thus, all the modes for nonzero k_z in the strong coupling case are "acoustic" plasmons proportional to wave number q in the long wavelength limit.

For $k_z=0$, Eq. (40) has the following solution in the $qa \to 0$ limit:

$$\omega \simeq \left[\frac{4\pi n_s e^2}{\kappa m a}\right]^{1/2} + O(q^2) \ . \tag{43}$$

The coefficient of the $O(q^2)$ term in Eq. (43) can be evaluated by retaining[24] the next-order [of $O(q^4/\omega^4)$] term in the expansion of $\Pi(q,\omega)$ in Eq. (30). The coefficient is found to be $(\frac{3}{16})v_f^2/\omega_p$, where v_f is the Fermi velocity of the 2DEG and $\omega_p=(4\pi n_s e^2/\kappa m a)^{1/2}$ is the leading-order term in Eq. (43). Thus the $k_z=0$ mode in the strong coupling case is just like a three-dimensional plasmon with plasma frequency $\omega_p=(4\pi n_B e^2/\kappa m)^{1/2}$ of the same form as that of bulk plasmons where $n_B=n_S/a$ is the "effective" three-dimensional electron density of the superlattice. The dispersion of this mode as defined by Eq. (43) is also of the three-dimensional form, with the only difference that the coefficient of the $O(q^2)$ term is $(\frac{3}{16})v_f^2/\omega_p$ rather than $(\frac{3}{10})v_f^2/\omega_p$ as in the bulk case.[12] The mode defined by Eq. (43) is the "optical plasmon" or the highest frequency mode of the superlattice carrying the maximum spectral weight at long wavelengths.

2. *Weak coupling case* ($qa \gg 1$)

By taking the $qa \to \infty$ limit in Eq. (40) we obtain the following for all values of k_z:

$$1=\frac{2\pi e^2}{\kappa q A}\Pi(q,\omega) \ . \tag{44}$$

But Eq. (44) is just the condition for a regular two-dimensional plasmon going as $\omega \simeq (2\pi n_S e^2/\kappa m)^{1/2} q^{1/2}$ in the long wavelength limit. Thus in the weak coupling situation each layer independently supports its own two-dimensional plasmon as we expect on intuitive grounds.

Before concluding this section, we write down the general dynamical response equation for an arbitrary external potential within RPA. This is obtained by generalizing Eq. (32) to get:

$$n_l(q,\omega)-\Pi(q,\omega)\sum_{l'} V_{l-l'}(q)n_{l'}(q,\omega) = A^{-1}\Pi(q,\omega)v_l^{\text{ex}}(q,\omega) \ , \tag{45}$$

where $n_l(q,\omega)$ is the induced electron density fluctuation due to an external potential $v_l^{\text{ex}}(q,\omega)$. If we take the external potential to be arising from some charge density distribution $e\rho^{\text{ex}}(\vec{r},z,t)$, then we can write

$$v_l(q,\omega)\equiv v(q,\omega;z=la) = \frac{2\pi e^2}{\kappa q}\int_{-\infty}^{+\infty} dz' e^{-q|z'-la|} \times \rho^{\text{ex}}(q,\omega;z') \ . \tag{46}$$

In Eq. (46), $\rho^{\text{ex}}(q,\omega;z')$ is the Fourier transform of $\rho^{\text{ex}}(\vec{r},t;z')$ in the xy plane and in time, and we have used Poisson's equation to obtain Eq. (46). Equations (45) and (46) together with Eqs. (25) and (31) defining $\Pi(q,\omega)$ and $V_{l-l'}(q)$, respectively, give us the complete Hartree self-consistent-field response of the superlattice. We can solve for $n_l(q,\omega)$ by inverting Eq. (45) to get

$$n_l(q,\omega)=(M^{-1})_{ll}\frac{2\pi e^2}{\kappa q A}\Pi(q,\omega) \times \int_{-\infty}^{+\infty} dz' e^{-q|z'-la|}\rho^{\text{ex}}(q,\omega;z') \ . \tag{47}$$

The matrix M is given by

$$M_{ll'}=\delta_{ll'}-\frac{2\pi e^2}{qA}e^{-qa|l-l'|}\Pi(q,\omega) \ . \tag{48}$$

Equations (47) and (48) are appropriate for an N-layer system rather than a superlattice. For a completely periodic superlattice, one can go back to Eqs. (45) and (46) and solve for $n_l(q,\omega)$ using the technique employed for obtaining the collective modes.

III. SELF-CONSISTENT RESPONSE FOR NONZERO MAGNETIC FIELD

We consider a static external magnetic field B_0 oriented along z direction perpendicular to the electron layers. One-electron wave functions are no longer plane waves in the xy plane because the electronic motion is now quantized in Landau levels. By choosing a particular gauge (so-called Landau gauge) for the vector potential $\vec{A}_0\equiv(0,B_0x,0)$ corresponding to the external magnetic field B_0 (remembering that $\vec{B}_0=\vec{\nabla}\times\vec{A}_0$), we obtain for the one-electron wave functions of the superlattice

$$\phi_{nk_yl}(\vec{r},z)=\frac{1}{\sqrt{L_y}}e^{ik_y y}u_n(x+l_c^2k_y)\xi(z-la) \ , \tag{49}$$

where L_y is the normalization length in y direction. In Eq. (49), $u_n(x+l_c^2k_y)$ is the simple harmonic oscillator wave function with a displaced center, $l_c=(c\hbar/eB_0)^{1/2}$ is the Landau radius, and n is the Landau quantum number. We assume that the magnetic field does not affect the motion of the electrons in z direction. The one-electron energy levels corresponding to wave functions in Eq. (49) are given by

$$E_{nk_yl}\equiv E_n=(n+1/2)\hbar\omega_c+E_0\ , \tag{50}$$

with $n=0,1,2$, etc., denoting the quantized Landau levels in xy plane and $\omega_c=eB_0/mc$ is the cyclotron frequency.

A. Density response

We can now follow the self-consistent-field response formalism of Sec. II by introducing the single-particle density matrix with respect to the Landau basis set of Eqs. (49) and (50). Calculating the *density-density* response function, we get an equation which is formally the same as Eq. (45), but with a modified polarizability function $\Pi^{(B_0)}(q,\omega)$ in the presence of the external magnetic field, defined by

$$\Pi^{(B_0)}(q,\omega)=\left[\frac{A}{2\pi l_c^2}\right]\sum_{nn'}\left|\int_{-\infty}^{+\infty}dx\,u_{n'}(x+l_c^2q_y)\times e^{iq_xx}u_n(x)\right|^2\times\left[\frac{f_{n'}-f_n}{E_{n'}-E_n-\hbar\omega}\right]. \tag{51}$$

We can express[28,29] the matrix elements in Eq. (51) in terms of associated Laguerre polynomials. However, because our main interest in this paper is long wavelength collective behavior, we concentrate on the small q behavior of $\Pi^{(B_0)}(q,\omega)$:

$$\Pi^{(B_0)}(q\to 0,\omega)\simeq\frac{n_sq^2A}{m(\omega^2-\omega_c^2)}\ . \tag{52}$$

The condition for the existence of self-sustaining oscillation is obtained by applying periodicity condition in the z direction as in Sec. I. We obtain [cf. Eq. (38)] the following:

$$1=\frac{2\pi e^2}{\kappa qA}\Pi^{(B_0)}(q,\omega)S(q,k_z)\ , \tag{53}$$

where S is given by Eq. (39). Using Eqs. (39) and (52) in Eq. (53) gives the following magnetoplasma modes for the superlattice:

$$\omega^2=\omega_c^2+\left[\frac{2\pi n_se^2}{\kappa m}\right]q\left[\frac{\sinh qa}{\cosh qa-\cos k_za}\right]. \tag{54}$$

In the strong ($qa\ll 1$) and weak ($qa\gg 1$) coupling limits, we get the following long wavelength ($q\to 0$) magnetoplasma modes for the superlattice.

1. *Strong coupling ($qa\ll 1$) case*

$$\omega=\left[\omega_c^2+\frac{2\pi n_se^2}{\kappa m}\frac{aq^2}{1-\cos k_za}\right]^{1/2}\quad\text{for } k_z\neq 0\ , \tag{55}$$

$$=\left[\omega_c^2+\frac{4\pi n_se^2}{\kappa ma}\right]^{1/2}\quad\text{for } k_z=0\ . \tag{56}$$

2. *Weak coupling ($qa\gg 1$) case*

$$\omega=\left[\omega_c^2+\frac{2\pi n_se^2}{\kappa m}q\right]^{1/2}\quad\text{for all } k_z\ . \tag{57}$$

We note that in the strong coupling limit the $k_z=0$ mode is a bulk[30] magnetoplasmon given by Eq. (56), whereas in the weak coupling limit we recover two-dimensional magnetoplasmons[17] given by Eq. (57) for each layer. The $k_z\neq 0$ modes in the strong coupling case given by Eq. (55) are the acoustic magnetoplasmons corresponding to the acoustic plasmons given in Eq. (42).

B. Current response

We follow the self-consistent procedure of Greene *et al.*[25] and of Chiu and Quinn[17] to obtain the current response of a superlattice to a general external electromagnetic perturbation in the presence of a static magnetic field B_0 along the superlattice direction.

The total one-electron Hamiltonian neglecting electron-electron interaction effects can be written in the *first quantized* representation as

$$H = H_0 + H' , \tag{58}$$

where

$$H_0 = \frac{1}{2m}\left[\vec{p} + \frac{e}{c}\vec{A}_0\right]^2 , \tag{59}$$

and

$$H' = \frac{e}{2mc}\left[\left[\vec{p} + \frac{e}{c}\vec{A}_0\right]\cdot\vec{A} + \vec{A}\cdot\left[\vec{p} + \frac{e}{c}\vec{A}_0\right]\right] - e\Phi . \tag{60}$$

H_0, the unperturbed Hamiltonian in the presence of the static magnetic field, has the eigenstates and eigenenergies given by Eqs. (49) and (50), respectively. H' is the perturbation due to the total electromagnetic perturbation described by $\vec{A}$ and Φ which are, respectively, the *total* vector and scalar potentials of the system. The perturbing potential H' has been expressed in linear approximation neglecting terms of order A^2.

Introducing the single-particle density matrix ρ as before, we can express the current density $\vec{j}(\vec{r}_0, z_0, t)$ induced in the superlattice as

$$\vec{j}(\vec{r}_0, z_0, t) = \mathrm{Tr}\left\{-\frac{e}{2m}\left[\left[\vec{p} + \frac{e}{c}\vec{A}_0\right] + \frac{e}{c}\vec{A}(\vec{r}, z, t)\right]\delta(\vec{r} - \vec{r}_0)\delta(z - z_0)\rho + \mathrm{H.c.}\right\} . \tag{61}$$

In Eq. (61) Tr denotes trace and H.c. is the Hermitian conjugate. Since current and charge densities are related by the equation of continuity, we show only the current response.

Introducing the complete set of noninteracting states defined by Eq. (49), we can finally express the Fourier transform of the induced current density $\vec{j}(q, \omega; z_0 = la)$ on the lth layer of the superlattice as

$$\vec{j}(\vec{q}, \omega; l) = \overleftrightarrow{R}(\vec{q}, \omega; l)\vec{A}(\vec{q}, \omega; l) , \tag{62}$$

with

$$\overleftrightarrow{R}(\vec{q}, \omega; l) = \left[\frac{n_s e^2}{mc}\right](-\overleftrightarrow{1} - \overleftrightarrow{K}) . \tag{63}$$

Here $\overleftrightarrow{1}$ is the unit tensor, and $\overleftrightarrow{K}$ is given by

$$\overleftrightarrow{K} = \left[\frac{2m}{n_s}\right]\sum_{\alpha\alpha'}\langle\alpha|\vec{F}(\vec{q})|\alpha'\rangle\langle\alpha'|\vec{F}(\vec{q})|\alpha\rangle\rho_{\alpha'\alpha} , \tag{64}$$

with

$$\vec{F}(\vec{q}) = \frac{1}{2m}\left[\left[\vec{p} + \frac{e}{c}\vec{A}_0\right]\cdot e^{i\vec{q}\cdot\vec{r}} + e^{i\vec{q}\cdot\vec{r}}\cdot\left[\vec{p} + \frac{e}{c}\vec{A}_0\right]\right] . \tag{65}$$

The state $\alpha = (n, k_y)$ is the composite index for the wave function in Eq. (49) describing the Landau quantized motion in the xy plane.

In obtaining the *irreducible* response function $\overleftrightarrow{R}$ of Eq. (62) describing response to the total potential, we have chosen a particular gauge in which the scalar potential Φ has been set equal to zero. This is allowed since RPA is known[12] to be gauge invariant. To obtain the collective modes, we have to determine the *reducible* response function to the external potential. This can be done by writing $\vec{A} \equiv \vec{A}^{\mathrm{ex}} + \vec{A}^{\mathrm{in}}$ and expressing $\vec{A}^{\mathrm{in}}$ in terms of $\vec{j}$ through Maxwell's equations. Without giving any detail, we quote the result for the condition of general collective modes of a superlattice in the presence of an external static magnetic field:

$$\det|\delta_{ij} - L_{ij}| = 0 . \tag{66}$$

The matrix $\overleftrightarrow{L}$ is given by

$$\overleftrightarrow{L} = \frac{2\pi n_s e^2 S(k)}{\kappa m c^2 k}\left[\overleftrightarrow{R} + \left[\frac{c}{\omega}\right]\cdot\vec{I}\,\vec{q}\right] , \tag{67}$$

where $S(k)$ is determined in Eq. (39) with $k = (q^2 - \omega^2/c^2)^{1/2}$ replacing q, and $\vec{I}$ is given by

$$\vec{I} = \frac{2mc}{n_s}\sum_{\alpha\alpha'}\rho_{\alpha\alpha'}\langle\alpha'|\vec{F}(q)|\alpha\rangle\langle\alpha|e^{i\vec{q}\cdot\vec{r}}|\alpha'\rangle . \tag{68}$$

One can express[17,25] $\vec{I}$ and $\overleftrightarrow{R}$ in terms of Bessel functions and discuss the collective modes implied by Eq. (66) in various limits. We leave that for a future study, basing our discussion of various collective modes of a superlattice in the presence of an external magnetic field on a hydrodynamic model developed in the next section.

IV. HYDRODYNAMICAL THEORY OF LINEAR RESPONSE

Since Fetter has given[21] a detailed hydrodynamic theory for the linear response of a superlattice in the absence of any external magnetic field, we concentrate in this section on the situation where an external static magnetic field B_0 is present in z direction.

Hydrodynamical theory describes electronic motion in terms of two dynamical variables, namely the electron-density fluctuation, $n_l(\vec{r},t)$ on the lth layer, and $\vec{v}_l(\vec{r},t)\equiv(v_{lx},v_{ly})$, the electron velocity on the lth layer. The linearized equations[21] of motion are

$$\frac{\partial\vec{v}_l(\vec{r},t)}{\partial t}=-\frac{1}{mn_s}\vec{\nabla}P_l(\vec{r},t)-\frac{e}{m}\vec{E}_l(\vec{r},t)-\frac{e}{mc}\vec{v}_l\times\vec{B}\ , \tag{69}$$

$$\frac{\partial n_l(\vec{r},t)}{\partial t}+n_s\vec{\nabla}\cdot\vec{v}_l(\vec{r},t)=0\ . \tag{70}$$

In Eqs. (69) and (70), n_s is the average electron density per unit area introduced in Sec. I and P_l is the hydrodynamic pressure of the electron liquid in the lth layer. $\vec{E}_l$ and $\vec{B}$ are the total local self-consistent electric field and the external magnetic field, respectively. We choose[21] the simplest form of the pressure term:

$$P_l\equiv m\beta^2 n_l\ , \tag{71}$$

where β^2 is a constant coefficient giving the "sound" velocity of the liquid. Also, the magnetic field is given by $\vec{B}=(0,0,B_0)$ in our configuration.

The total self-consistent field $\vec{E}$ can be written as $\vec{E}=\vec{E}^{\text{ex}}+\vec{E}^{\text{in}}$. The induced field $\vec{E}^{\text{in}}$ can be expressed in terms of induced density fluctuation n. We introduce Fourier transforms in (xy) plane and in time as before. Then, one can combine Eqs. (69) and (71) to get a couple of algebraic equations for the velocity field v_{lx} and v_{ly}. However, applying the periodicity, we can write

$$\vec{v}_l(q,\omega)=\vec{v}_0(\vec{q},\omega)e^{iq_zla}\ . \tag{72}$$

We can now introduce current operator $\vec{J}(z)$ defined as

$$\vec{J}(q,\omega;z)=-n_se\sum_l\vec{v}_l\delta(z-z_l)=-n_se\sum_l\vec{v}_0e^{iq_zla}\delta(z-z_l)\ . \tag{73}$$

We can formally introduce a z-Fourier transform of $\vec{J}(q,\omega;z)$ through the definition,

$$\vec{J}(q,\omega;q_z)=\int_{-\infty}^{+\infty}dz\,\vec{J}(q,\omega;z)e^{-iq_zz}\ , \tag{74}$$

giving

$$\vec{J}(q,\omega;q_z)=-n_se\sum_l\vec{v}_l(q,\omega)e^{-iq_zla}\ . \tag{75}$$

By using Eqs. (69)–(75) and doing some algebra, we can write down the following equations for $J_{x,y}(q,\omega;q_z)$:

$$(\omega^2-\omega_c^2-\beta^2q^2)J_x=\frac{in_se^2\omega}{m}\sum_lE_{lx}e^{-iq_zla}+\frac{n_se^2\omega_c}{m}\sum_lE_{ly}e^{-iq_zla}\ , \tag{76}$$

and

$$(\omega^2-\omega_c^2)J_y=\frac{i\beta^2\omega_cq^2}{\omega}J_x-\frac{n_se^2\omega_c}{m}\sum_lE_{ly}e^{-iq_zla}\ . \tag{77}$$

In Eqs. (76) and (77) we have written $\vec{J}\equiv\vec{J}(q,\omega;q_z)$ and all other quantities (e.g., $\vec{v}_l,\vec{E}_l$) are $(\vec{q}-\omega)$ transforms. We have also taken $\vec{q}\equiv(q,0)$ by choosing the xy axes suitably. This particular choice of axes (with no loss of generality) facilitates further analysis considerably. If we now assume the electric field $\vec{E}(\vec{r},z,t)$ to be arising from some kind of charge-density distribution, we immediately obtain

$$E_{ly}(\vec{q},\omega)=0\ , \tag{78}$$

and

$$\sum_lE_{lx}e^{-iq_zla}=\sum_lE_{lx}(q,\omega)e^{-iq_zla}\ ,=\frac{S}{2}[qE_x(q,q_z;\omega)+q_zE_z(q,q_z;\omega)]\ , \tag{79}$$

where

$$E_x(q,q_z;\omega)=\int_{-\infty}^{+\infty} dz\, E_x(q,\omega;z)e^{-iq_z z} . \quad (80)$$

S is the sum defined in Eq. (39). Using Eqs. (78) and (79) in Eqs. (76) and (77), we finally obtain the following:

$$J_x=\sigma_{xx}E_x , \quad (81)$$

and

$$J_y=\sigma_{yx}E_x , \quad (82)$$

where

$$\sigma_{xx}(q,q_z;\omega)=\frac{in_s e^2\omega q}{2m(\omega^2-\omega_c^2-\beta^2 q^2)} \times\left[\frac{\sinh qa}{\cosh qa-\cos q_z a}\right], \quad (83)$$

and

$$\sigma_{yx}(q,q_z;\omega)=-\frac{n_s e^2\omega_c q}{2m(\omega^2-\omega_c^2-\beta^2 q^2)} \times\left[\frac{\sinh qa}{\cosh qa-\cos q_z a}\right] . \quad (84)$$

Notice that Eqs. (81) and (82) take these simple forms because of our choice of axes.

In the limit of zero magnetic field, $\omega_c=eB_0/mc$ vanishes and consequently $\sigma_{yx}=0$. Thus, in the absence of any magnetic field, we are left with only the diagonal, isotropic conductivity,

$$\sigma=\frac{in_s e^2\omega q}{2m(\omega^2-\beta^2 q^2)}\left[\frac{\sinh qa}{\cosh qa-\cos q_z a}\right] . \quad (85)$$

This is the same result that we obatin from Sec. I in the long wavelength limit.

We introduce conductivity function $\sigma_\pm$ defined by

$$\sigma_\pm=\sigma_{xx}\pm i\sigma_{xy} . \quad (86a)$$

Dielectric functions $\epsilon_\pm$ corresponding to $\sigma_\pm$ are defined[30] by $\epsilon_\pm=1+(4\pi i/\kappa\omega)\sigma_\pm$ and are given by

$$\epsilon_\pm=1-\frac{2\pi n_s e^2 q(\omega\pm\omega_c)}{\kappa m\omega(\omega^2-\omega_c^2-\beta^2 q^2)} \times\left[\frac{\sinh qa}{\cosh qa-\cos q_z a}\right] . \quad (86b)$$

We can now consider wave propagation[30,31] implied by the circularly polarized dielectric function ϵ_+. In particular, helicon waves propagate[30,31] under the condition

$$\epsilon_+(\omega)=\frac{c^2Q^2}{\omega^2} , \quad (87)$$

where $Q=(q^2+q_z^2)^{1/2}$ is a three-dimensional wave-number.

By using Eq. (86b) in Eq. (87), we conclude that standard three-dimensional[30,31] helicon waves propagate in a semiconductor superlattice in the strong coupling situation $qa \ll 1$. In the weak coupling situation ($qa \gg 1$), such modes cannot exist.

Using Eq. (86b) one can discuss dispersion of such helicon waves in the superlattice. Magnetoplasmons derived in Sec. III are also obtainable from the hydrodynamical model by looking for the zeros of the dielectric function $\epsilon_{xx}=1+(4\pi i/\kappa\omega)\sigma_{xx}$. This gives the following condition:

$$1-\frac{2\pi n_s e^2 q}{\kappa m(\omega^2-\omega_c^2-\beta^2 q^2)}\left[\frac{\sinh qa}{\cosh qa-\cos q_z a}\right]=0 . \quad (88)$$

This equation is the same as Eq. (54) of Sec. III if the hydrodynamic pressure term is neglected (i.e., $\beta^2=0$). By choosing $\beta^2=(\frac{3}{8})v_F^2$ we can make RPA and hydrodynamic theory agree to $O(q^2)$ in plasma dispersion. This long wavelength agreement between hydrodynamic and RPA theories is well known in the literature.[12,21]

V. GENERALIZATION TO ELECTRON-HOLE SYSTEMS

In this section we generalize our results of Secs. II–IV to type-II superlattice where alternate layers of electrons and holes form the periodic array, rather than carriers of one type only that we have been dealing with so far. We consider electrons of density n_e per unit area and mass m_e occupying all the even layers with $l=0,\pm2,\pm4$, etc., and holes of density n_h per unit area and mass m_h occupy all the odd layers. The separation between adjacent layers ($|z_l - z_{l-1}|$) is taken to be a as before.

The generalization of Secs. II–IV to the electron-hole system is quite straightforward following the formalism developed in the preceding sections, since the forms for the one-electron wave functions and energies remain unchanged except for the trivial mass differences in odd and even layers. We will therefore be quite succinct in this section, essentially quoting the important results without giving the calculational details.

A. Situation without any external magnetic field

Plasmons of the electron-hole superlattice are given by 2×2 determinantal equation (since there are two kinds of charge carriers) instead of the simple equation (38) for a type-I system. The collective-excitation spectrum for the type-II system is given by the equation,

$$\left[1-\frac{2\pi e^2}{\kappa q A}\Pi_e(q,\omega)S_e(q,k_z)\right]\left[1-\frac{2\pi e^2}{\kappa q A}\Pi_h(q,\omega)S_h(q,k_z)\right] - \left[\frac{2\pi e^2}{\kappa q A}\right]^2 \Pi_e(q,\omega)\Pi_h(q,\omega)S_e(q,k_z)S_h(q,k_z)=0\ , \tag{89}$$

where Π_e,Π_h are, respectively, the electron and the hole polarizabilities given by Eq. (25) with electron and hole single-particle energies appearing in the respective electron and hole functions. The sums S_e and S_h are given by

$$S_e = \sum_{l=0,\pm 2,\pm 4\ldots} \exp(-q\,|l|\,a + ik_z la)\ , \tag{90}$$

and

$$S_h = \sum_{l=\pm 1,\pm 3,\pm 5\ldots} \exp(-q\,|l|\,a + ik_z la)\ . \tag{91}$$

Introducing $2l'=l$ in Eq. (90) gives

$$S_e = \sum_{l'=0,\pm 1,\pm 2} \exp(-2q\,|l'|\,a + 2ik_z\,|l'|\,a)\ . \tag{92a}$$

The sum in Eq. (92a) has been evaluated before [Eq. (39)] in the context of obtaining S in Eq. (37). We obtain the following:

$$S_e = \frac{\sinh 2qa}{\cosh 2qa - \cos 2k_z a}\ . \tag{92b}$$

By comparing Eqs. (37), (90), and (91) we conclude that $S = S_e + S_h$, giving

$$S_h = S - S_e = \frac{\sinh qa}{\cosh qa - \cos k_z a} - \frac{\sinh 2qa}{\cosh 2qa - \cos 2k_z a}\ . \tag{93}$$

By using Eqs. (92b) and (93) in Eq. (89), we can obtain the collective spectrum for the electron-hole superlattice.

Equation (89) can be simplified (suppressing variables) to

$$1 = \frac{2\pi e^2}{\kappa q A}(\Pi_e S_e + \Pi_h S_h)\ . \tag{94}$$

We now discuss the long wavelength collective modes implied by Eq. (94) in the strong ($qa \ll 1$) and weak ($qa \gg 1$) coupling regimes. We use [see Eq. (30)] the following long wavelength form for the polarizability:

$$\Pi_{e,h}(q,\omega) \simeq \left[\frac{n_{e,h}}{m_{e,h}}\right]\left[\frac{q^2}{\omega^2}\right]A\ . \tag{95}$$

1. Strong coupling ($qa \ll 1$) case

Appropriate ($qa \ll 1$) expansions of Eqs. (92b) and (93) when inserted in Eq. (94) along with Eq. (95) yield the following:

$$\omega^2 = \frac{2\pi e^2}{a}\left[\frac{n_e}{m_e}+\frac{n_h}{m_h}\right] + O(q^2) \quad \text{for } k_z = 0\ , \tag{96}$$

$$\omega^2 = 2\pi e^2 a q^2 \left[\frac{n_e}{m_e}(1-\cos 2k_z a)^{-1} + \frac{n_h}{m_h}[(1-\cos k_z a)^{-1} - (1-\cos 2k_z a)^{-1}]\right] \quad \text{for } k_z \neq 0\ . \tag{97}$$

Equation (96) gives the OP for a "bulk" electron-hole system[30] with three-dimensional electron and hole densities given by $n_e/2a$ and $n_h/2a$, respectively. This is sensible because the basic period of the type-II superlattice is $2a$ since carriers of similar charge are on layers separated by a minimum distance of $2a$. The collective modes given by Eq. (97) for nonzero k_z are the acoustic plasmons for the type-II superlattice. Note that k_z is now restricted in a reduced Brillouin zone $0 \le k_z \le \pi/a$ since the period has doubled to $2a$.

2. *Weak coupling ($qa \gg 1$) case*

It is easier to take this limit by going back to Eq. (89). One finds the only collective modes to be either electron or hole plasmons corresponding to a pure 2DEG:

$$1 = \frac{2\pi e^2}{\kappa q A}\Pi_e(q,\omega)$$

or (98)

$$1 = \frac{2\pi e^2}{\kappa q A}\Pi_h(q,\omega)$$

These are the 2D plasmons going as $q^{1/2}$ for all k_z with a coefficient given by $(2\pi n_{e,h}e^2/m_{e,h})^{1/2}$ as the case may be.

B. Situation with an external magnetic field

We can follow the RPA treatment of Sec. III A to get the following equation for the magnetoplasmons of an electron-hole superlattice:

$$1 = \frac{2\pi e^2}{\kappa q A}[\,\Pi_e^{(B_0)}(q,\omega)S_e + \Pi_h^{(B_0)}(q,\omega)S_h\,] , \tag{99}$$

where $\Pi_{e,h}^{(B_0)}(q,\omega)$ is the electron or hole polarizability [given by Eq. (51)] in the presence of external magnetic field B_0 along the z direction.

In the strong coupling ($qa \ll 1$) situation we get the following condition for the $k_z=0$ mode which carries most of the spectral weight:

$$\frac{\omega_{pe}^2}{\omega^2-\omega_{ce}^2} + \frac{\omega_{ph}^2}{\omega^2-\omega_{ch}^2} = 1 , \tag{100}$$

where

$$\omega_{pe}^2 = \frac{2\pi n_e e^2}{\kappa a m_e} = \frac{4\pi n_e e^2}{\kappa m_e(2a)} ,$$
$$\omega_{ph}^2 = \frac{2\pi n_h e^2}{\kappa m_h a} = \frac{4\pi n_h e^2}{\kappa m_h(2a)} , \tag{101}$$

and

$$\omega_{ce,ch} = \frac{eB_0}{m_{e,h}c} . \tag{102}$$

Equation (101) gives the coupled magnetoplasmon modes of a "bulk" electron-hole system in the long wavelength limit. Other modes with $k_z \neq 0$ have more complicated structure that can be obtained by an appropriate expansion of Eq. (99). In the strong coupling ($qa \ll 1$) case these other modes ($k_z \neq 0$) carry small spectral weight and are coupled cyclotron-acoustic plasmon "bulk" modes.

In the weak coupling ($qa \gg 1$) situation we get either the electron magnetoplasmon or the hole magnetoplasmon modes for a 2DEG. Since these modes can all be obtained by an appropriate expansion of Eq. (99), we do not explicitly show those modes for the sake of brevity.

Finally, we can apply hydrodynamic theory to the electron-hole superlattice system in the presence of an external magnetic field by an appropriate generalization of Sec. IV. One can easily obtain the magnetoplasmons that we have obtained earlier within RPA. Instead we obtain the dielectic functions $\epsilon_\pm(\omega)$ defined in Eq. (86) of IV. For the electron-hole system, $\epsilon_\pm(\omega)$ are given by

$$\epsilon_\pm = 1 - \frac{\omega_{pe}^2 S_e a q(\omega \pm \omega_{ce})}{\omega(\omega^2-\omega_{ce}^2-\beta_e^2 q^2)} - \frac{\omega_{ph}^2 S_h a q(\omega \mp \omega_{ch})}{\omega(\omega^2-\omega_{ch}^2-\beta_h^2 q^2)} , \tag{103}$$

where β_e, β_h are the electron and hole "sound" velocities, respectively.

In the strong coupling ($qa \ll 1$) limit Eq. (103) gives the familiar helicon waves[30,31] of three-dimensional uncompensated electron-hole plasma ($n_e \neq n_h$). In the compensated situation ($n_e = n_h$) one gets the Alfvén waves.[30,31] These modes can all be calculated from Eq. (103) by demanding $\omega^2 = Q^2c^2/\epsilon_+$, where $Q = (q^2+k_z^2)^{1/2}$ is a three-dimensional wave number. The dispersion of bulk helicon waves and Alfvén waves are well known in the literature.[30,31]

VI. DISCUSSIONS AND CONCLUSION

In this paper we have studied theoretically the collective-excitation spectrum of semiconductor superlattices by modeling the system to be a periodic array of charged layers. Within the simple model of purely two-dimensional charge layers ("zero thickness"), we have considered both types-I and -II superlattices containing periodic array of electron layers only or of both electron and hole layers. We also consider the experimentally interesting situation where a static magnetic field is present along the superlattice growth direction. We find a rich variety of possible collective modes of these systems—optical plasmons, acoustic plasmons, magnetoplasmons, helicon, and Alfvén waves, to name some of these modes. A particularly interesting feature is the change in dimensionality of the systems from two- to three-dimensional behavior as the coupling between the layers is increased by decreasing the dimensionless quantity qa where q is the wave number and a is the superlattice period. In the strong coupling ($qa \ll 1$) situation, the plasmon of the system is essentially a three-dimensional plasmon, $\omega_p = (4\pi n_s e^2/ma)^{1/2}$, corresponding to a volume density n_s/a where n_s is the two-dimensional density. In the weak coupling ($aq \gg 1$) situtation, we get the two-dimensional plasmon with frequency proportional to $n_s^{1/2} q^{1/2}$ in the long wavelength. In the intermediate situation ($qa \sim 1$), plasmons of the superlattice have character intermediate between two and three dimensions. In the presence of an external static magnetic field, we obtain magnetoplasmons and other collective modes (helicons, etc.) which also change their dimensional character with coupling strength. In addition to the regular optical plasmons which carry the maximum spectral weight at long wavelength, we get a large number of "acoustic" plasmons which have frequencies proportional to q in the long wavelength limit. These modes carry negligible spectral weight in the $q \rightarrow 0$ and signify out-of-phase oscillations of electrons on different layers. In the highest frequency optical-plasmon mode, electrons of different layers oscillate in phase.

Most of the experimental information[20] on plasmons of 2DEG are on the single-layer system like inversion and accumulation layers on semiconductor surfaces or electrons bound on liquid-helium surface.[32] Far-infrared absorption[19,20] and emission[33] technique were employed to observe plasmons in single-layer 2DEG. Recently an electron-energy-loss scattering experiment has successfully been performed[34] on zinc oxide accumulation layer to observe two-dimensional plasmons. To our knowledge no experimental observation of plasmons in a superlattice has yet been reported. However, very recently experimental observations of magnetoplasmons in GaAs-$Al_xGa_{1-x}As$ superlattice[35] and of helicons in InAs-GaSb superlattice[36] have been reported. The first study uses resonant inelastic light scattering technique[35] whereas the second employs far-infrared transmission spectroscopy.[36] These experimental results are rather preliminary and have not been analyzed in complete detail yet. These data seem to be completely consistent with our theoretical results in this paper. In particular, experimental parameter values are such that one is in the strong coupling ($qa \ll 1$) situation in both the experiments. In the magnetoplasmon experiment the resonant frequency is found to scale with $(\omega_c^2 + \omega_p^2)^{1/2}$ where ω_p^2 is found to be proportional to electron volume density. This is exactly what we find in our analysis in Secs. III and IV. Details of the helicon experiment are not quite available yet. However, preliminary results[36] are consistent with our findings in Sec. V.

Experimental techniques that seem to be the best suited for studying the collective excitations discussed in this paper are resonant Raman scattering spectroscopy,[35] far-infrared absorption measurements,[36] and inelastic electron-energy-loss scattering experiments.[34] Typical values for the superlattice period (the parameter a) in the experimental systems are in the 50–200-Å range. This makes it very difficult to explore anything but the strong coupling limit ($qa \ll 1$) experimentally since typical wave numbers involved in light scattering and absorption measurements are small. Also, in the long wavelength limit, most of the spectral weight is carried by the optical plasmons, making it difficult to experimentally observe the "acoustic" plasmons. However, we may mention that very recently observation of acoustic plasmons has been reported[37] for the first time in a solid-state plasma in an inelastic light scattering experiment.

We have made a number of simplifying approximations in our theory. We have neglected, among other effects, effect of finite layer thickness, damping effect, coupling of electronic modes to LO phonons, effect of subband structure, etc. These effects are all expected to be important in actual systems. However, we believe that the basic physics of electronic collective excitations of superlattice system are all contained in our simple model. Inclusion of damping is straightforward within hydrodynamic formalism by introducing a phenomenological damping parameter $1/\tau$. Since mobility in these systems is rather high, we expect the collective modes to satisfy $\omega\tau > 1$, making them observable in these systems. The effect that we believe should be included in a future improvement of our theory is that of finite thickness of charge layers in the model. This means relaxing the $|\xi(z)|^2 = \delta(z)$ approximation of Eq. (2). This introduces two important modifications: (i) Overlap of electron wave functions from adjacent layers, and (ii) introduction of subbands in each layer. Subbands have their own resonant contribution to the dynamical response since electrons can make intersubband transitions. Much is known[7,8] about such intersubband transition from intersubband spectroscopy. These intersubband transitions and overlap of electronic wave functions in z direction give rise to an actual electron current in z direction not allowed within our model. This may produce collective effects not contained in our theory. Finally, one expects the electronic collective modes discussed in this paper to be coupled to LO phonons of the polar semiconductors (GaAs,AlAs, InAs,GaSb,etc.) forming the superlattice since the two energies are comparable. Such LO-phonon-plasmon coupling has been discussed[38] in the context of bulk and surface collective modes in the polar materials. Such coupling is also very important[7,8,39] in the intersubband transition in multilayer systems. We can include LO-phonon coupling in our formalism in a straightforward fashion. Such coupling effects are expected to be important in these weakly polar systems only in the vicinity of $\omega_{LO} \simeq \omega_{ex}$ and ω_{ex} is a typical collective-excitation frequency obtained in this paper.

We have restricted ourselves to RPA or a self-consistent-field Hartree approximation. The hydrodynamical model gives the same results as RPA in the $q \to 0$ limit. It is difficult to go beyond RPA for a complicated system such as a superlattice. We believe that it is more important to include some of the realistic effects discussed earlier (particularly the finite-layer thickness) than trying a more complicated many-body theory for the response. Part of our work (Sec. I) is a finte frequency generalization of the static self-consistent-field treatment[22] of Visscher and Falicov. The hydrodynamical calculation of Sec. IV is a generalization of Fetter's work[21] to finite magnetic fields. Our theory for the collective modes of the electron-hole systems is to our knowledge the first calculation of its kind for type-II superlattices.

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