Direct generation of ultrasound by electromagnetic radiation in metals in a magnetic field

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The theory of the anomalous skin effect in metals is generalized to obtain the direct generation of ultrasound by electromagnetic radiation incident on a metal surface, in the presence of a magnetic field. The nonmonotonic behavior of the acoustic flux as a function of the magnetic field, observed experimentally, can be explained within the framework of the free-electron model assuming that the electrons are scattered diffusely from the metal surface. The reason for this behavior is traced to the variation of the relative phases of the collision drag force on the bulk ions and that on the surface with increasing magnetic field. Quantitative studies indicate that the decrease in acoustic flux occurs when the product of the cyclotron resonance frequency and the collision time is approximately $3^{1/2}$. The relations between existing theories of direct generation of ultrasound have been investigated within both the free-electron approximation and the effective-mass theory. We conclude that, besides the collision drag force, the Lorentz force, and the Bragg reaction force, an additional "deformation" force acts upon the positive ions. The latter can, to a considerable extent, cancel the effect of the Bragg reaction force.

I. INTRODUCTION

The phenomenon of ultrasonic generation in metals by direct coupling of an electromagnetic wave to the phonon field has been studied by Gantmakher and Dolgopolov,[1] by Larsen and Saermark,[2] by Houck *et al.*,[3] and by Abeles.[4] Larsen and Saermark[2] investigated the excitation of shear acoustic waves in aluminum using radiation in the radio frequency range incident normally on the surface of the sample; a uniform magnetic field oriented perpendicular to the surface of the metal and of sufficient strength to allow the propagation of helicons was present. Houck *et al.*[3] performed similar experiments on silver, aluminum, and lead telluride using electromagnetic radiation in the same frequency range. They showed that the acoustic generation does not depend on the excitation of helicon waves in the normal sense but is always present. In the experiments carried out by Abeles[4] short microwave pulses were made to impinge normally on an indium film in a microwave cavity located over a germanium slab as substrate. The generation of acoustic waves was demonstrated by showing that the echoes of the pulses are delayed by exactly the transit times of shear acoustic waves within the germanium slab.

The theory of this phenomenon was investigated by Kaganov and Fiks,[5] Quinn,[6] Southgate,[7] and Alig.[8] These studies are based on the theory of the anomalous skin effect[9] because, as we shall see, it is only when the mean free path l of the electrons is long compared to the penetration depth δ of the electromagnetic wave that the effect is appreciable. For this reason the experiments are carried out at liquid-helium temperatures.

The mechanism of ultrasonic generation can be described as follows. In the presence of the electric field $\vec{E}$ of the electromagnetic wave the positive ions in the metal experience a force $\gamma e\vec{E}$ where

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γe is their charge. They also experience a second force arising from the transfer, through collisions, of the excess momentum of the electrons; this is called the collision-drag force. When $l \lesssim \delta$ these two forces cancel one another. However, when $l \gg \delta$ they are separated in space and produce a shear stress on the metal, thereby exciting a transverse-acoustic wave. The spatial separation of the electric and collision-drag forces depends, of course, on the way in which the electrons are scattered by the surface of the metal. Following Reuter and Sondheimer[9] we use, initially, the free-electron model of a metal assuming that a fraction p of the electrons is reflected specularly from the surface and the remaining fraction $1-p$ is scattered diffusely. In specular reflection, the momentum component perpendicular to the surface is reversed while the tangential components are conserved. In diffuse scattering we suppose that the velocity of an electron after scattering is directed in a random direction toward the interior of the metal.

The theory of direct ultrasonic generation for $p=0$ was given by Ram Mohan *et al.*[10] They showed that the efficiency of conversion of electromagnetic radiation to acoustic flux is of the same order of magnitude in the two limits $p=1$ and $p=0$. For parameters appropriate to potassium in a field of frequency of 9 MHz the conversion efficiency for diffuse scattering is only 40% larger than for specular scattering.[11]

The phenomenon described above is weak because of the electrical neutrality of the metal. However, in the presence of a large uniform magnetic field $\vec{B}_0$, the electrical current density in the metal acquires a component perpendicular to both $\vec{B}_0$ and $\vec{E}$. The resulting collision-drag force on the positive ions is parallel to $\vec{E}\times\vec{B}_0$ and proportional to $|\vec{B}_0|$ giving rise to an acoustic wave polarized in this direction with amplitude proportional to $|\vec{B}_0|$.

Experiments on potassium and aluminum in a magnetic field normal to the surface of the sample and in the direction of propagation of the incident radiation have been carried out by several investigators.[12–15] These experiments show that the acoustic amplitude increases linearly with the magnetic field at large fields in agreement with the theoretical prediction. However, at relatively low magnetic fields a nonmonotonic variation of the acoustic amplitude as a function of B_0 is observed.[13–15]

In the experiments reported in Ref. 14, the acoustic wave is produced on a surface of a slab normal to $\vec{B}_0$ and detected by electromagnetic induction at the opposite end. To establish the notation used in the present paper, we denote the amplitude of the shear wave by the real part of

$$\vec{\xi}=\vec{\xi}(z)e^{-i\omega t}=(\xi_x(z),\xi_y(z),0)e^{-i\omega t}, \qquad (1.1)$$

where we have taken the origin of the Cartesian coordinate system x,y,z at a point on the surface of the metal and let the material occupy the region $z>0$. The x axis is selected parallel to the direction of polarization of the incident radiation. The data in experiments of the type reported in Refs. 13–15 are displayed giving the values of $\vec{\xi}$ at the far end (with respect to the face of incidence) of the slab but are corrected to their values at $z=0$ by taking into account the attenuation of the acoustic wave. In our treatment we neglect the attenuation of the acoustic wave so that we designate the measured quantity by $\vec{\xi}(\infty)$. In general, even in the absence of ultrasonic attenuation, $\vec{\xi}(0)$ and $\vec{\xi}(\infty)$ are different. This was first demonstrated by Babkin and Kravchenko.[16]

To analyze their experimental results Chimenti *et al.*[14] made use of the free-electron model and the assumption of specular reflection[6] ($p=1$). While the theory is in agreement with experiment for large magnetic fields, it fails to account for the low-field behavior.[17]

Kaner and Fal'ko[18] made a calculation of $\vec{\xi}(0)$ which exhibits a behavior similar to that shown in the experimental results.[13] However, as we have pointed out above, what is actually measured is $\vec{\xi}(\infty)$; this quantity does not exhibit the nonmonotonic behavior observed experimentally. Banik and Overhauser[19] calculated $\vec{\xi}(0)$ and $\vec{\xi}(\infty)$ in the free-electron model. In the same publication they also took into account the reaction on the positive ions of the force responsible for the Bragg reflection of the electrons. Within the framework of the free-electron model and taking $p=1$ they find that neither $\vec{\xi}(0)$ nor $\vec{\xi}(\infty)$ exhibit the experimentally observed low B_0 behavior. In the course of the present investigation we performed similar calculations leading to identical conclusions. Since, as we shall prove, the treatments of Kaner and Fal'ko[18] and of Quinn[6] and Chimenti *et al.*[14] are completely equivalent within the free-electron model, we must conclude that the nonmonotonic variation of $\xi_y(0)$ as a function of B_0 given in Ref. 18 results from the mathematical approximations made.

One of the purposes of this paper is to discuss the theory of this effect when the electrons are scattered diffusely at the surface of the metal. We conclude that a nonmonotonic variation of $\xi_y(\infty)$ occurs in this case. Since this behavior appears in the free-electron model, it must be a universal phenomenon applicable to all metals under extreme anomalous skin-effect conditions ($l \gg \delta$). The physical origin of the effect is the following. The collision-drag force is essentially proportional to, but 180° out of phase with, the electron current density. Thus, its phase varies with the depth in the metal and with the magnitude of the applied magnetic field. In addition to the direct force $\gamma e \vec{E}$ on the positive ions we have also a surface stress because the electrons, upon being scattered by the metal surface, transfer to it, on the average, their tangential momentum. While the phase of the surface force does not change as the magnetic field increases, the y component of the collision-drag force changes phase when B_0 reaches the value for which $\omega_c\tau$ approaches unity. Here ω_c is the cyclotron frequency of the electrons and τ the average time between successive electron collisions. A quantitative study shows that this change in phase occurs when $(\omega_c-\omega)\tau=3^{1/2}$ under anomalous skin-effect conditions. Here ω is the angular frequency of the incident radiation and, hence, of the phonon generated. Thus, as this condition is reached we expect that $\xi_y(\infty)$ will decrease. Banik and Overhauser[19] took into account the surface force but calculated it using the electric field profiles obtained with the assumption of specular scattering at $z=0$. In the present work the electric field has been calculated exactly using the Wiener-Hopf method.[20]

Section II contains a description of the purely electrodynamic calculations used in the present work. Section III is devoted to the calculation of the acoustic amplitude generated by the incident radiation. Section IV gives an alternative formulation of the problem having the advantage of displaying the Lorentz force on the solid directly. In Sec. III this force is separated into a force on the positive ions while the force on the electrons is partially concealed in the collision-drag force. In Sec. IV A we demonstrate the equivalence of the approaches of Kaner and Fal'ko[18] and of Quinn[6]; we have followed the latter in Sec. III. Sec. IV B takes up the discussion of ultrasonic generation when the electronic states are described by Bloch wave functions rather than by free electrons. In this part we make use of Holstein's[21] microscopic theory of the collision-drag effect. The results of the theory yield the forces on the positive ions in the metal, including the Bragg reaction force in agreement with the work of Fiks.[22] We show, however, that failure to take into account the effect of deformation on the electron spectrum may partially or totally invalidate theories making use of the Bragg reflection force. This subject will be discussed also in Sec. IV B.

II. ANOMALOUS SKIN EFFECT IN A MAGNETIC FIELD

In this section we review the theory of the anomalous skin effect in the presence of a magnetic field normal to the surface of the metal in a manner suitable for our further developments.

We consider a plane-polarized monochromatic wave of angular frequency ω incident on the metal surface at $z=0$. The presence of the uniform magnetic field $\vec{B}_0=(0,0,B_0)$ causes a rotation of the plane of polarization inside the material. The electric and magnetic fields of the wave are taken as the real parts of $\vec{E}(z)e^{-i\omega t}$ and $\vec{B}(z)e^{-i\omega t}$, respectively. The electron current density is denoted by $\vec{j}(z)e^{-i\omega t}$. Instead of $E_x(z)$ and $E_y(z)$ we use the quantities

$$E_\pm(z)=E_x(z)\pm iE_y(z)$$

describing the circularly polarized components of the field. Similar combinations are defined for the other vector quantities involved. Associated with each circular polarization there is a surface impedance defined by

$$Z_\pm=\pm\frac{4\pi i}{c}\frac{E_\pm(0)}{B_\pm(0)}=\frac{4\pi i\omega}{c^2}\frac{E_\pm(0)}{E'_\pm(+0)}\ , \qquad (2.1)$$

where the second equality is obtained using the Faraday law of induction.

In our numerical calculations we used parameters appropriate for potassium and the experiments of Chimenti *et al.*[14] These are collected for ready reference in Table I. We expect

$$|Z_\pm|\approx Z_0(\omega\delta/c)\ , \qquad (2.2)$$

where

$$\delta=(4v_F c^2/3\pi\omega\omega_p^2)^{1/3} \qquad (2.3)$$

is of the order of the skin depth, v_F is the Fermi velocity, ω_p is the electron plasma frequency and $Z_0=(4\pi/c)=376.73\ \Omega$ the impedance of the vacuum. The parameter $(\omega\delta/c)=(2\pi\delta/\lambda)$ is of the order of the ratio of the skin depth to the wavelength in empty space of the incident radiation. This is about 10^{-7} so that $|Z_{\pm}| \ll Z_0$.

The electromagnetic wave in the empty region $z<0$ is of the form

$$E_{\pm}(z)=E_0 e^{i\omega z/c}+R_{\pm}e^{-i\omega z/c}\ , \tag{2.4}$$

where $R_{\pm}$ is the amplitude of the reflected wave. The boundary conditions at $z=0$ require

$$E_{\pm}(0)=2E_0Z_{\pm}/(Z_{\pm}+Z_0)\approx 2E_0Z_{\pm}/Z_0 \tag{2.5}$$

and

$$B_{\pm}(0)=\pm 2iE_0Z_0/(Z_{\pm}+Z_0)\approx \pm 2iE_0\ . \tag{2.6}$$

Since the mean free path l and the penetration depth δ are large compared to the de Broglie wavelength of the electrons at the Fermi surface, we use the distribution function $f(\vec{k},\vec{r},t)$ to provide a description of the dynamics of the electron gas. In equilibrium f equals the Fermi distribution $f_0(\epsilon_k)$ where $\epsilon=\epsilon_k$ is the kinetic energy of an electron of wave vector $\vec{k}$. Making the assumption of the existence of a constant relaxation time τ, f satisfies the Boltzmann equation

$$\frac{\partial f}{\partial t}+\dot{\vec{k}}\cdot\frac{\partial f}{\partial \vec{k}}+\vec{v}\cdot\frac{\partial f}{\partial \vec{r}}+\frac{f-f_0}{\tau}=0\ , \tag{2.7}$$

where $\partial f/\partial\vec{k}$ and $\partial f/\partial\vec{r}$ denote the gradients of f in $\vec{k}$ and $\vec{r}$ space, respectively. The velocity $\vec{v}=\hbar\vec{k}/m$ of an electron with wave vector $\vec{k}$ can be expressed in spherical coordinates (v,θ,ϕ) with $\hat{z}$ as the polar axis. We express f in the form

$$\begin{aligned} f&=f_0+f^{(1)}e^{-i\omega t}\\ &=f_0+(f_+^{(1)}e^{-i\phi}+f_-^{(1)}e^{i\phi})e^{-i\omega t}\ , \end{aligned} \tag{2.8}$$

where the deviation of f from f_0 has been expanded in a Fourier series in the azimuthal angle ϕ. The Fourier coefficients $f_{\pm}^{(1)}$ are, therefore, functions of z, v, and θ. In the free-electron model only two Fourier components are different from zero as indicated in Eq. (2.8). Keeping only that part of $f^{(1)}$ which is linear in the electric field and taking

$$\hbar\dot{\vec{k}}=-e(\vec{E}+c^{-1}\vec{v}\times\vec{B}_0)$$

we find the differential equations

$$\frac{\partial f_{\pm}^{(1)}}{\partial z}+\frac{1-i\alpha_{\pm}}{v\tau\cos\theta}f_{\pm}^{(1)}=\tfrac{1}{2}e\tan\theta E_{\pm}\frac{\partial f_0}{\partial\epsilon}\ , \tag{2.9}$$

where

$$\alpha_{\pm}=(\omega\pm\omega_c)\tau\ , \tag{2.10}$$

with $\omega_c=(eB_0/mc)$ the cyclotron frequency of the electrons. The general solutions of Eqs. (2.9) are

$$f_{\pm}^{(1)}=\left[F_{\pm}+\tfrac{1}{2}e\tan\theta\frac{\partial f_0}{\partial\epsilon}\times\int_{\infty}^{z}d\zeta E_{\pm}(\zeta)\exp(u_{\pm}\zeta)\right]\times\exp(-u_{\pm}z)\ , \tag{2.11}$$

where

$$u_{\pm}=(1-i\alpha_{\pm})/v\tau\cos\theta \tag{2.12}$$

and $F_{\pm}$ are arbitrary functions of v and θ to be determined by the boundary conditions. Since when z tends to infinity $f_{\pm}^{(1)}$ must approach zero, $F_{\pm}=0$ for $v_z=v\cos\theta<0$. If $v_z=v\cos\theta>0$ we determine $F_{\pm}$ by the condition

$$\begin{aligned} f_0+f_{\pm}^{(1)}(v_x,v_y,v_z;z=0)e^{-i\omega t} &=p[f_0+f_{\pm}^{(1)}(v_x,v_y,-v_z;z=0)e^{-i\omega t}]\\ &\quad+(1-p)f_0 \end{aligned} \tag{2.13}$$

TABLE I. List of material parameters for potassium used in this work.

Physical parameter	Symbol	Value
Experimental frequency	ω	$5.636\times10^{7}\ \mathrm{s}^{-1}$
Electron concentration	n	$1.402\times10^{22}\ \mathrm{cm}^{-3}$
Density of potassium (5 K)	ρ	$0.91\ \mathrm{g\,cm}^{-3}$
Fermi velocity	v_F	$8.633\times10^{7}\ \mathrm{cm\,s}^{-1}$
Relaxation time	τ	$1.646\times10^{-10}\ \mathrm{s}$
Mean free path	l	$1.421\times10^{-2}\ \mathrm{cm}$
Skin depth	δ	$2.357\times10^{-4}\ \mathrm{cm}$
Sound velocity	s	$1.780\times10^{5}\ \mathrm{cm\,s}^{-1}$
Plasma frequency	ω_p	$6.670\times10^{15}\ \mathrm{s}^{-1}$
dc conductivity	σ_0	$5.831\times10^{20}\ \mathrm{s}^{-1}$

describing the scattering of the electrons at $z=0$. This yields

$$F_\pm=\frac{e}{2}\tan\theta\frac{\partial f_0}{\partial\epsilon}\int_0^\infty d\zeta E_\pm(\zeta)[p\exp(-u_\pm\zeta)+\exp(u_\pm\zeta)] \tag{2.14}$$

for $v_z>0$. These equations provide a complete solution of f and hence give us a means to calculate the components of the electron current density. The simplest expression for $j_\pm(z)$ is obtained extending the definition of $E_\pm(z)$ to negative values of z by the convention $E_\pm(z)=E_\pm(-z)$. Physically this is equivalent to extending the metal to both sides of the plane $z=0$ and replacing the incident electromagnetic wave by rf current sheets localized at $z=0$. The result is

$$j_\pm(z)=p\sigma_0\int_{-\infty}^{\infty}G_\pm(z-\zeta)E_\pm(\zeta)d\zeta+(1-p)\sigma_0\int_0^\infty G_\pm(z-\zeta)E_\pm(\zeta)d\zeta\ , \tag{2.15}$$

where

$$G_\pm(z)=(3/4l)\int_0^{\pi/2}\sin^2\theta\tan\theta\times\exp(u_\pm|z|)d\theta\ . \tag{2.16}$$

Here l is the mean free path $v_F\tau$, $\sigma_0=ne^2\tau/m$, the dc electrical conductivity of the electron gas of concentration n, and now $u_\pm$ takes the value in Eq. (2.12) with v replaced by v_F. The result (2.16) was obtained assuming that the Fermi-degeneracy temperature is large compared to the temperature of the specimen. Thus, all calculated transport coefficients have, in effect, their zero temperature values. The Fourier transforms of $G_\pm(z)$ are

$$\begin{aligned}G_\pm(q)&=\int_{-\infty}^{\infty}G_\pm(z)\exp(-iqz)dz=(\tfrac{3}{4})\int_0^\pi(1-i\alpha_\pm+i\beta\cos\theta)^{-1}\sin^3\theta\,d\theta\\&=-\frac{3}{2\beta^2}(1-i\alpha_\pm)+\frac{3}{4\beta^3}[\beta^2+(1-i\alpha_\pm)^2]\\&\qquad\times\left[\arctan(\beta-\alpha_\pm)+\arctan(\beta+\alpha_\pm)-\frac{i}{2}\ln\left[\frac{1+(\beta-\alpha_\pm)^2}{1+(\beta+\alpha_\pm)^2}\right]\right]\ ,\end{aligned} \tag{2.17}$$

where

$$\beta=ql\ . \tag{2.18}$$

Since $(\omega\delta/c)\ll 1$ we can neglect the displacement current density in the Maxwell equations. We obtain

$$E''_\pm(z)=-(4\pi i\omega/c^2)j_\pm(z) \tag{2.19}$$

which allows us to solve for the components of $E_\pm(z)$ in conjunction with Eqs. (2.15). The solution is particularly simple for $p=1$. We find

$$E_\pm(z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}E_\pm(q)\exp(iqz)dq=-\frac{1}{\pi}E'_\pm(+0)\int_{-\infty}^{\infty}\left[q^2-\frac{4\pi i\omega\sigma_0}{c^2}G_\pm(q)\right]^{-1}\exp(iqz)dq\ . \tag{2.20}$$

We calculate the components of the electric field by performing a contour integration. For this purpose we introduce the complex variable

$$\zeta=\beta(i+\alpha_\pm)^{-1} \tag{2.21}$$

and write $G_\pm(q)$ in the form

$$G_\pm=\frac{3i}{2}(i+\alpha_\pm)^{-1}\left[\frac{1}{\zeta^2}+\frac{\zeta^2-1}{2\zeta^3}\ln\frac{1+\zeta}{1-\zeta}\right]\ . \tag{2.22}$$

The logarithmic function in (2.22) has essential singularities at $\zeta=\pm 1$. We cut the ζ plane along the segments $(-\infty,-1]$ and $[1,\infty)$ and select the Riemann sheet associated with this function such that $G_\pm$ is meromorphic in the finite ζ plane except along the cuts. We further require that, when β is real, $G_\pm$ coincides with the values in (2.17). This is accomplished by selecting the branch of $\ln[(1+\zeta)/(1-\zeta)]$ which vanishes at $\zeta=0$ and such that for $|\zeta|>1$,

$$\ln\left[\frac{1+\zeta}{1-\zeta}\right]=\ln\left[\frac{\zeta+1}{\zeta-1}\right]+i\pi(\operatorname{sgn}\operatorname{Im}\zeta)\ , \tag{2.23}$$

where $\operatorname{sgn}\operatorname{Im}\zeta=\pm 1$ if $\operatorname{Im}\zeta\gtrless 0$, respectively. Some details are given in the Appendix. The components of the electric field for $p=1$ are

$$E_\pm(z) = -(4i\omega\delta E_0/3c) \times \left[e_\pm + \frac{3}{2}\frac{l}{\delta(1-i\alpha_\pm)} I_\pm \right], \tag{2.24}$$

with

$$e_+ = \exp(-z/\delta) + \Delta_+ \exp\left[\frac{i\pi}{3} - \frac{z(1-i\sqrt{3})}{2\delta}\right] \tag{2.25}$$

and

$$e_- = \exp\left[\frac{i\pi}{3} - \frac{z(1-i\sqrt{3})}{2\delta}\right] + \Delta_- \exp\left[-\frac{i\pi}{3} - \frac{z(1+i\sqrt{3})}{2\delta}\right]. \tag{2.26}$$

Here $\Delta_+ = 1$ if $(\omega_c+\omega)\tau < \sqrt{3}$, and zero otherwise, while $\Delta_- = 1$ if $(\omega_c-\omega)\tau > \sqrt{3}$, and zero otherwise. The quantities $I_\pm$ are

$$I_\pm = \int_0^1 D_\pm^{-1} (t\kappa_\pm)^3 (1-t^2) \times \exp[(iz/\delta t\kappa_\pm)(2/\pi)^{1/3}]dt, \tag{2.27}$$

with

$$\kappa_\pm = (2/\pi)^{1/3}[l/\delta(i+\alpha_\pm)] \tag{2.28}$$

and

$$D_\pm = \left\{ 1 + (t\kappa_\pm)^3 \left[t + \frac{1-t^2}{2} \ln\left[\frac{1+t}{1-t}\right] \right] \right\}^2 + \frac{\pi^2}{4}(t\kappa_\pm)^6(1-t^2)^2. \tag{2.29}$$

These expressions are approximate in the sense that the zeros of the denominator in Eq. (2.20) have been calculated in the limit of large l/δ. The expression for E_+ is approximate but valid for all ω_c. However, that for $E_-(z)$ is only valid if $\omega_c > \omega$. From the expression for $E_+(z)$ we find

$$Z_+ = Z_0 \frac{\omega\delta}{c\sqrt{3}} \left[1 - i\sqrt{3} + \sqrt{3}\frac{l}{\delta(i+\alpha_+)} \times \int_0^1 D_+^{-1}(t\kappa_+)^3(1-t^2)dt \right]. \tag{2.30}$$

To verify the accuracy of the approximations used we calculated Z_+ exactly using Eq. (2.20) and then using the approximate expression (2.30). The exact calculation for $B_0 = 500$ G gave

$$Z_+ = 1.709\times 10^{-7}(1-i1.78)Z_0, \tag{2.31}$$

while Eq. (2.30) gave

$$Z_+ = 1.706\times 10^{-7}(1-i1.705)Z_0. \tag{2.32}$$

It is interesting to compare these results with the surface impedance obtained by Reuter and Sondheimer[9] for $p=1$ in the limit $l \gg \delta$, namely

$$Z = Z_0 \frac{2\delta\omega}{3\sqrt{3}c}(1-i\sqrt{3}) = 1.706\times 10^{-7}(1-i\sqrt{3})Z_0. \tag{2.33}$$

The results (2.31) and (2.32) are obtained for $l/\delta = 60.25$ while (2.33) holds in the limit l/δ approaching infinity.

Figure 1 gives the profiles of $E_x(z)$ and $E_y(z)$ for several values of B_0 taken at a given instant of time. In Fig. 2 we display similar results for $j_x(z)$ and $j_y(z)$.

In the case of diffuse scattering ($p=0$) from the surface we require the solutions of the equations

$$E''_\pm(z) = -(4\pi i\omega\sigma_0/c^2) \times \int_0^\infty G_\pm(z-\zeta)E_\pm(\zeta)d\zeta. \tag{2.34}$$

We used the Wiener-Hopf method to solve Eqs. (2.34) for $E_+(z)$ and $E_-(z)$. The method has been given in detail in Ref. 10. The only difference is that now we have two equations in which we formally replace ω by $\omega \pm \omega_c$ (except in the factor $4\pi i\omega\sigma_0/c^2$ and in ω^2/c^2; the latter arises from the displacement current and is neglected anyway).

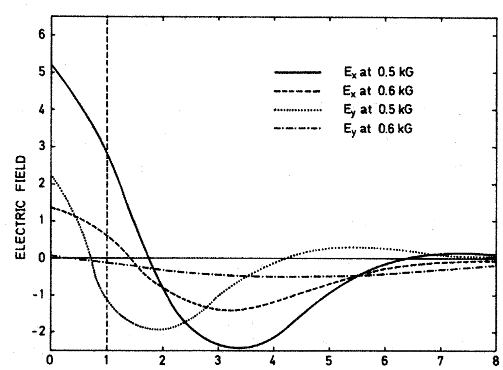

FIG. 1. Electric field at a fixed instant of time as a function of z/δ. z is the distance from the surface and δ the penetration depth. The vertical dashed line indicates the penetration depth δ. Electrical field is in arbitrary units.

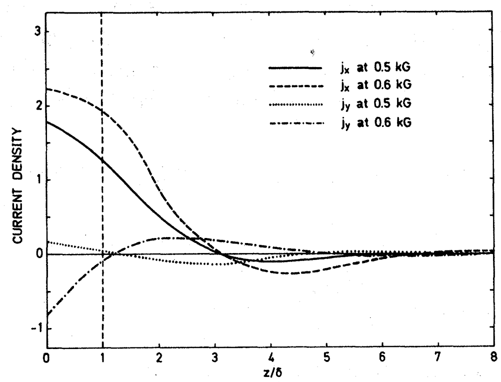

FIG. 2. Current density as a function of z/δ, in arbitrary units.

Since the results parallel exactly those in Ref. 10 we do not reproduce them here. The fields $E_x(z)$ and $E_y(z)$ were calculated numerically for use in the calculations of Sec. III but are not explicitly displayed.

III. GENERATION OF SHEAR ACOUSTIC WAVES

In the absence of external fields, the equation of motion of the displacement field $\vec{\xi}(\vec{r},t)$ of the lattice is given by

$$\frac{\partial^2\vec{\xi}}{\partial t^2}=s^2\frac{\partial^2\vec{\xi}}{\partial z^2},$$

where s is the velocity of shear acoustic waves. In the presence of the rf field $\vec{E}$ and the uniform magnetic field $\vec{B}_0$ the equation of motion of the positive ions is

$$M\ddot{\vec{\xi}}=Ms^2\frac{\partial^2\vec{\xi}}{\partial z^2}+\gamma e\vec{E}+\frac{\gamma e}{c}\dot{\vec{\xi}}\times\vec{B}_0+\vec{F}_c\ . \quad (3.1)$$

Here M is the mass of the positive ions. As with the case of the electron we neglect the Lorentz force arising from the rf magnetic field. The collision-drag force $\vec{F}_c$ arises from the transfer of momentum from the electrons to the positive ions through collisions. We suppose that the electrons have average velocity $\dot{\vec{\xi}}$ after each collision so that

$$\begin{aligned}\vec{F}_c&=(\gamma m/\tau)(\langle\vec{v}\rangle-\dot{\vec{\xi}})\\&=-(\gamma m/ne\tau)(\vec{j}+ne\dot{\vec{\xi}})\\&=-(\gamma m/ne\tau)\vec{J}\ ,\end{aligned} \quad (3.2)$$

where the last equality defines $\vec{J}$, the total current density, the sum of the electron current density $\vec{j}$, and the ionic contribution $ne\dot{\vec{\xi}}$. The latter is negligible in our study of ultrasonic generation and is disregarded in the calculation of the rf electric field inside the material. We shall, however, write our equations in more generality than is necessary for the problem at hand so that the theory can be utilized in studies in which there is no incident electromagnetic radiation. An example of such situations appears in the theory of ultrasonic attenuation. Equations (3.1) and (3.2) yield immediately

$$\frac{\partial^2\xi_\pm}{\partial z^2}+\frac{\omega^2}{s^2}\xi_\pm\mp\frac{\gamma eB_0\omega}{Mcs^2}\xi_\pm+\frac{i\gamma m\omega}{Ms^2\tau}\xi_\pm=-\frac{\gamma e}{Ms^2}\mathscr{E}_\pm(z)\ , \quad (3.3)$$

where

$$\mathscr{E}_\pm=E_\pm-\sigma_0^{-1}j_\pm\ . \quad (3.4)$$

The third term in the left-hand side of Eq. (3.3) can be neglected in comparison with the second. In fact, for potassium at the frequency $\omega=5.6\times10^7\ \mathrm{sec}^{-1}$ the two quantities become comparable for a field $B_0=2.3\times10^5$ G. If this term is not neglected there is a slight difference in the velocities of the acoustic waves of right and left circular polarizations; this gives rise to an acoustic rotatory power.

The fourth term in the left-hand side of Eq. (3.3) can also be neglected since it yields a correction to the ultrasonic amplitude of higher order in the parameter $\gamma m/M$. In studies of ultrasonic attenuation by electrons it cannot, however, be neglected. Dropping these two terms we solve Eq. (3.3) by the method employed by Kartheuser and Rodriguez in their similar study of acoustic generation in superconductors.[23] We obtain

$$\begin{aligned}\xi_\pm(z)=&A_\pm\exp(i\omega z/s)+B_\pm\exp(-i\omega z/s)\\&-(\gamma e/Ms\omega)\int_0^z\mathscr{E}_\pm(\zeta)\\&\times\sin[\omega(z-\zeta)/s]d\zeta\ .\end{aligned} \quad (3.5)$$

The requirement that for large z,

$$\xi_\pm(z)=\xi_\pm(\infty)\exp(i\omega z/s)\ , \quad (3.6)$$

i.e., that there be no reflected wave, gives

$$B_\pm=(i\gamma e/2Ms\omega)\int_0^\infty\mathscr{E}_\pm(\zeta)\exp(i\omega\zeta/s)d\zeta\ . \quad (3.7)$$

We obtain $A_\pm$ through the knowledge of the strain $\xi'_\pm(+0)$ at the surface, i.e., from

$$\xi'_\pm(+0)=(i\omega/s)(A_\pm-B_\pm)\ . \quad (3.8)$$

We find

$$\xi_\pm(\infty) = -(is/\omega)\xi'_\pm(+0) + (i\gamma e/2Ms\omega)\mathscr{E}_\pm(\omega/s)\ , \tag{3.9}$$

where $\mathscr{E}_\pm(\omega/s)$ is the Fourier transform $\mathscr{E}_\pm(q)$ of $\mathscr{E}_\pm(z)$ evaluated at the wave vector, $q=\omega/s$, of the shear acoustic wave of angular frequency ω. The components of the lattice displacement at $z=0$ are

$$\xi_\pm(0) = \xi_\pm(\infty) - (\gamma e/Ms\omega)\int_0^\infty \mathscr{E}_\pm(\zeta)\sin(\omega\zeta/s)d\zeta = \xi_\pm(\infty) - (\gamma e/2\pi Ms\omega)\mathscr{P}\int_{-\infty}^{\infty}(q-q_a)^{-1}\mathscr{E}_\pm(q)dq\ , \tag{3.10}$$

where $\mathscr{P}$ indicates that the principal value of the integral is to be taken. For $p=1$, $\xi'_\pm(+0)=0$ as we shall presently see, and

$$\xi_\pm(\infty) = \frac{2\gamma eE_0}{Msc}\frac{1-G_\pm(q_a)}{q_a^2-(4\pi i\omega\sigma_0/c^2)G_\pm(q_a)} \approx \frac{i\gamma ecE_0}{2\pi\omega\sigma_0 Ms}[G_\pm^{-1}(q_a)-1]\ . \tag{3.11}$$

In Eqs. (3.10) and (3.11) we have set

$$q_a = \omega/s\ . \tag{3.12}$$

When $\omega \ll \omega_c$, $G_+(q_a) \cong G_-^*(q_a)$, and

$$(\xi_x(\infty),\xi_y(\infty)) = \frac{i\gamma ecE_0}{2\pi\omega\sigma_0 Ms}(X,Y)\ , \tag{3.13}$$

where X and Y are the real and imaginary parts of $G_+^{-1}(q_a)-1$, respectively. In Figs. 3 and 4 we give graphs of the amplitudes of $\vec{\xi}(\infty)$ and $\vec{\xi}(0)$ as functions of B_0 for the parameters given in Table I. Notice that $\xi_y(0)$ does not exhibit a nonmonotonic behavior even for $\omega l/s = q_a l \approx 10$.

For future reference we give here the limiting behavior when $\omega_c\tau \gg 1$. Then, $G_\pm \approx (1-i\alpha_\pm)^{-1}$ and, neglecting $\omega\tau$ we find $\xi_x(\infty)\approx 0$ and

$$\xi_y(\infty) = -\frac{iE_0B_0}{2\pi\rho s\omega}\ , \tag{3.14}$$

where ρ is the mass density of the metal.

If the electrons are scattered diffusely at the surface ($p=0$) we obtain the surface stress from the components of the shear stress tensor

$$T_{ij}^{(1)} = (4\pi^3)^{-1}\int d\vec{k}\, mv_iv_jf^{(1)}\ . \tag{3.15}$$

The ij component of this tensor gives the i component of electron momentum transported per unit time across the unit surface perpendicular to the j direction. Thus, the components of the tensor $\underline{T}^{(1)}$ give the forces acting across a surface because of the deviation from the state of equilibrium. The complete stress tensor is

$$T_{ij} = (4\pi^3)^{-1}m\int d\vec{k}\, v_iv_jf = \tfrac{2}{5}n\epsilon_F\delta_{ij} + T_{ij}^{(1)}\ . \tag{3.16}$$

Here ϵ_F is the Fermi energy. The first term in Eq. (3.16) gives the uniform pressure exerted by the electron gas in equilibrium. The tensors T_{ij} and $T_{ij}^{(1)}$ are symmetric. In the present problem the only nonvanishing components of $T_{ij}^{(1)}$ are $T_{xz}^{(1)}$ and $T_{yz}^{(1)}$. The combinations $T_{xz}^{(1)} \pm iT_{yz}^{(1)}$ are

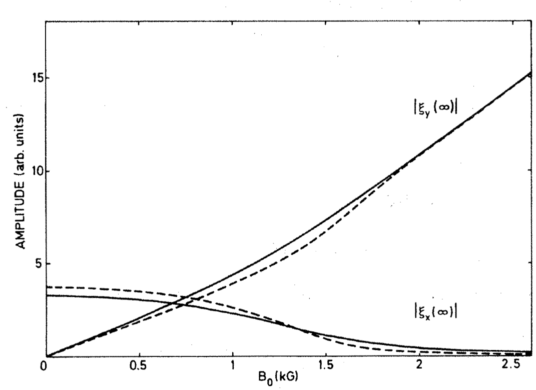

FIG. 3. Magnetic field dependence of the acoustic amplitudes at $z=\infty$ for specular scattering ($p=1$): a solid line for $q_al=4.5$ and a dashed line for $q_al=10$.

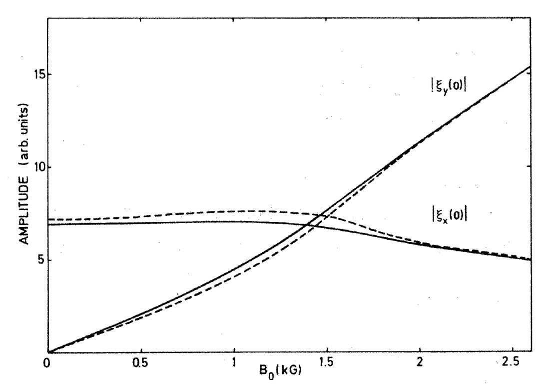

FIG. 4. Magnetic field dependence of the acoustic amplitudes at $z=0$ and for specular scattering ($p=1$): a solid line for $q_al=4.5$ and a dashed line for $q_al=10$.

$$T^{(1)}_{\pm,z}=T^{(1)}_{xz}\pm iT^{(1)}_{yz}$$

$$=-(3ne/4)\int_0^{\pi/2}\sin^3\theta\, d\theta\left[\int_0^z d\zeta\, E_\pm(\zeta)\exp(-u_\pm|\zeta-z|)-\int_z^\infty d\zeta\, E_\pm(\zeta)\exp(-u_\pm|\zeta-z|)+p\int_{-\infty}^0 d\zeta\, E_\pm(\zeta)\exp(-u_\pm|\zeta-z|)\right], \quad (3.17)$$

where $u_\pm=(1-i\alpha_\pm)/l\cos\theta$ as before. The components of surface strain are immediately obtained by

$$\xi'_\pm(+0)=(\rho s^2)^{-1}T^{(1)}_{\pm,z}(z=0)$$
$$=(3ne/4\rho s^2)(1-p)\int_0^{\pi/2}\sin^3\theta\, d\theta\int_0^\infty E_\pm(\zeta)\exp(-u_\pm\zeta)d\zeta\,. \quad (3.18)$$

Thus, as stated above, for $p=1$, $\xi'_\pm(+0)=0$. These results, together with those of the previous section, allow us to calculate $\xi_\pm(0)$ and $\xi_\pm(\infty)$ [see Eqs. (3.9), (3.10), and (3.18)]. In Fig. 5 we show the amplitude

$$|\vec{\xi}(\infty)|=[\,|\xi_x(\infty)|^2+|\xi_y(\infty)|^2]^{1/2}$$

as a function of B_0. The quantity $|\vec{\xi}(0)|$ is also of interest because it is accessible experimentally through Mössbauer studies such as those carried out in copper foils doped with ^{57}Co by Perlow *et al.*[24] For this reason we also show $|\vec{\xi}(0)|$ as a function of B_0 in Fig. 5. These numerical calculations, which are rather complex, were made using the parameters in Table I and $q_a l=4.5$. We could, of course, adjust the collision time τ in such a way as to obtain a better fit to the experimental data of Ref. 14. However, this seems hardly worthwhile since it is difficult to simultaneously adjust the values of $|\vec{\xi}(\infty)|$ at $B_0=0$ and at its minimum with a single parameter. The main conclusion of this investigation is that it demonstrates the possibility of obtaining a nonmonotonic behavior of $|\vec{\xi}(\infty)|$ as a function of an applied magnetic field within the free-electron model.

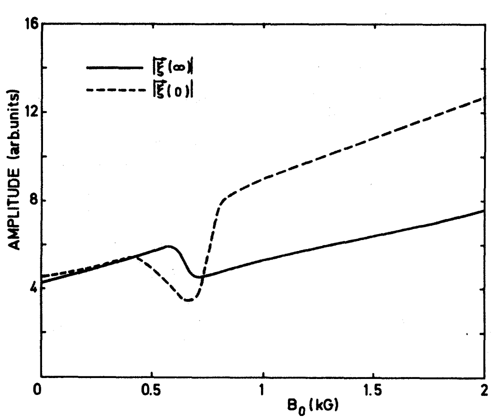

FIG. 5. Total acoustic amplitude as a function of the static magnetic field in the case of diffuse scattering ($p=0$).

The physical reason for the drop of $|\vec{\xi}|$ at $B_0\sim 0.6$ kG in Fig. 5 can be understood as arising from the change in the phase of ξ_y near this value of B_0. In fact, while the phase of the surface force remains fairly constant, the collision force changes sign as a function of B_0 at positions within the skin depth. This can be seen by inspecting Fig. 2 and remembering that the collision force is proportional to the current density [Eq. (3.2)]. The behavior of j_y as a function of B_0, shown in Fig. 2 for $p=1$, also occurs for $p=0$.

IV. ALTERNATIVE THEORY OF ULTRASONIC GENERATION

A. Free-electron model

The approach given in Sec. III has the advantage of simplicity. However, from the macroscopic point of view we recognize that the most important forces acting on the metal are the elastic stresses and the Lorentz force per unit volume $c^{-1}\vec{J}\times\vec{B}_0$. Inspection of Eq. (3.1) does not immediately exhibit this force. We expect that the force per unit volume $ne\vec{E}+(n/\gamma)\vec{F}_c$ can be written as the sum of $c^{-1}\vec{j}\times\vec{B}_0$ and additional smaller forces responsible for the amplitude of the generated ultrasonic wave present even when $B_0=0$. Separation of these forces is accomplished by the method used by Kaner and Fal'ko.[18] The purpose of this section is to demonstrate the equivalence of their procedure to that adopted[6] in Sec. III.

We start with the Boltzmann equation

$$\frac{\partial f}{\partial t}+\dot{\vec{k}}\cdot\frac{\partial f}{\partial \vec{k}}+\vec{v}\cdot\frac{\partial f}{\partial \vec{r}}+\frac{1}{\tau}(f-\bar{f}_0)=0\,, \quad (4.1)$$

where the collision term in Eq. (2.7) has been replaced by one in which f does not relax to the equilibrium condition but to a "local" distribution. The local distribution is one in which the electrons have average zero velocity in the frame of reference in which the lattice is instantaneously at rest. Thus we write

$$\bar{f}_0 = f_0(\epsilon_k - m\vec{v}\cdot\dot{\vec{\xi}}) , \tag{4.2}$$

where $\vec{v} = \hbar\vec{k}/m$ is the velocity and ϵ_k is the kinetic energy of an electron of wave vector $\vec{k}$ in the laboratory frame of reference. In the general case in which compressional waves are present, there are local fluctuations of the electron density. Relaxation, in this case, gives rise to diffusion currents. We need not, however, be concerned with this question since the waves are transverse and thus the density fluctuations are absent. We now define $f^{(1)}$ by

$$f = \bar{f}_0 + f^{(1)} . \tag{4.3}$$

We note that this is identical to the quantity $f^{(1)}$ used in Sec. II if $\dot{\vec{\xi}} = 0$, but is not equal to the deviation of f from the equilibrium distribution. The effects arising from the tendency of the electrons to screen the motion of the ions are, to a considerable extent, expressed in the form of $\bar{f}_0$. As before we neglect the Lorentz force on the electron from the rf magnetic field so that

$$\hbar\dot{\vec{k}} = -e\vec{E} - (e/c)\vec{v}\times\vec{B}_0 . \tag{4.4}$$

Substituting Eqs. (4.2)–(4.4) into Eq. (4.1) and making use of

$$\frac{\partial\bar{f}_0}{\partial t} = -m\ddot{\vec{\xi}}\cdot\vec{v}\frac{\partial f_0}{\partial\epsilon} , \tag{4.5}$$

$$\frac{\partial\bar{f}_0}{\partial\vec{k}} = \hbar(\vec{v} - \dot{\vec{\xi}})\frac{\partial f_0}{\partial\epsilon} , \tag{4.6}$$

and

$$\frac{\partial\bar{f}_0}{\partial x_i} = -m\frac{\partial f_0}{\partial\epsilon}\sum_j v_j\frac{\partial\dot{\xi}_j}{\partial x_i} = -m\frac{\partial f_0}{\partial\epsilon}\sum_j \dot{\epsilon}_{ij}v_j , \tag{4.7}$$

where

$$\epsilon_{ij} = \frac{1}{2}\left[\frac{\partial\xi_i}{\partial x_j} + \frac{\partial\xi_j}{\partial x_i}\right] , \tag{4.8}$$

is the strain tensor, we obtain

$$\frac{\partial f^{(1)}}{\partial t} - \frac{e}{\hbar c}(\vec{v}\times\vec{B}_0)\cdot\frac{\partial f^{(1)}}{\partial\vec{k}} + \vec{v}\cdot\frac{\partial f^{(1)}}{\partial\vec{r}} + \frac{1}{\tau}f^{(1)}$$
$$= \frac{\partial f_0}{\partial\epsilon}[e\vec{E}\cdot\vec{v} + \frac{e}{c}(\dot{\vec{\xi}}\times\vec{B}_0)\cdot\vec{v}$$
$$+ m\ddot{\vec{\xi}}\cdot\vec{v} + m\vec{v}\cdot\dot{\underline{\epsilon}}\cdot\vec{v}] . \tag{4.9}$$

In deriving Eq. (4.9) we have retained only linear terms in $\vec{E}$ and $\vec{\xi}$. Equation (4.7) follows from Eq. (4.2) and the fact that the crystal is not subject to a rigid rotation, i.e., that

$$\frac{\partial\xi_i}{\partial x_j} = \frac{\partial\xi_j}{\partial x_i} .$$

The electron current density $\vec{j}$ is given by

$$\vec{j} = -e(4\pi^3)^{-1}\int f\vec{v}\,d\vec{k}$$
$$= -e(4\pi^3)^{-1}\int \bar{f}_0\vec{v}\,d\vec{k}$$
$$-e(4\pi^3)^{-1}\int f^{(1)}\vec{v}\,d\vec{k} .$$

The first term, containing the distribution of electron velocities centered at $\dot{\vec{\xi}}$, gives the contribution to the electron current resulting from the screening of the ionic motion alone, i.e., $-ne\dot{\vec{\xi}}$. Hence,

$$-e(4\pi^3)^{-1}\int f^{(1)}\vec{v}\,d\vec{k} = \vec{j} + ne\dot{\vec{\xi}} = \vec{J} . \tag{4.10}$$

We now multiply both sides of Eq. (4.9) by $-e\vec{v}V^{-1}$, where V is the volume of the specimen and sum over all electron states. The first term in the left-hand side of Eq. (4.9) gives

$$\frac{\partial\vec{J}}{\partial t} ,$$

and the second gives

$$\frac{e}{mc}\vec{J}\times\vec{B}_0$$

after two integrations by parts; the i component of the vector resulting from the third term yields

$$-\frac{e}{V}\sum_k v_i\vec{v}\cdot\frac{\partial f^{(1)}}{\partial\vec{r}} = -e\sum_{j=1}^{3}\frac{\partial}{\partial x_j}\frac{1}{V}\sum_k v_iv_jf^{(1)}$$
$$= -\frac{e}{m}\sum_{j=1}^{3}\frac{\partial T_{ij}^{(1)}}{\partial x_j} ,$$

and finally, the fourth term simply results in

$$\frac{1}{\tau}\vec{J} .$$

All contributions originating from the first three terms on the right-hand side of Eq. (4.9) have the same structure and are obtained using

$$-\frac{e}{V}\sum_k \frac{\partial f_0}{\partial \epsilon} v_i v_j = \delta_{ij}\frac{ne}{m} .$$

The last term in Eq. (4.9) does not make a contribution to the average because it results in an integral of an odd function of $\vec{v}$. Combining these results we obtain

$$\frac{\partial \vec{J}}{\partial t}+\frac{e}{mc}\vec{J}\times\vec{B}_0-\frac{e}{m}\vec{\nabla}\cdot\underline{T}^{(1)}+\frac{1}{\tau}\vec{J} = \frac{ne^2}{m}\left[\vec{E}+\frac{1}{c}\dot{\vec{\xi}}\times\vec{B}_0+\frac{m}{e}\ddot{\vec{\xi}}\right] . \quad (4.11)$$

Combining Eqs. (3.1), (3.2), and (4.11) we obtain the desired equation of motion

$$\rho\frac{\partial^2\vec{\xi}}{\partial t^2}=\rho s^2\frac{\partial^2\vec{\xi}}{\partial z^2}+\frac{m}{e}\frac{\partial \vec{J}}{\partial t}+\frac{1}{c}\vec{J}\times\vec{B}_0-\vec{\nabla}\cdot\underline{T}^{(1)} , \quad (4.12)$$

which is identical to that of Ref. 18 for the free-electron model. It can also be demonstrated directly from Eq. (3.17) that

$$\frac{\partial}{\partial z}T^{(1)}_{\pm,z}=-neE_\pm(z)+\frac{m}{e\tau}(1-i\alpha_\pm)J_\pm , \quad (4.13)$$

which combined with Eq. (4.12) gives (3.3).

Before leaving this subject it is necessary to verify that, to first order in the small quantities $\vec{\xi}$ and $\vec{E}$, the expressions for $T^{(1)}_{ij}$ used here and in Sec. III are the same. This follows from

$$T_{ij}=\frac{m}{V}\sum_k v_i v_j(\bar{f}_0+f^{(1)})=T^{(1)}_{ij}+\frac{m}{V}\sum_k \bar{f}_0[(v_i-\dot{\xi}_i)(v_j-\dot{\xi}_j)+(v_i-\dot{\xi}_i)\dot{\xi}_j+(v_j-\dot{\xi}_j)\dot{\xi}_i+\dot{\xi}_i\dot{\xi}_j] .$$

Clearly this equals

$$T_{ij}=\tfrac{2}{5}n\epsilon_F\delta_{ij}+T^{(1)}_{ij}+nm\dot{\xi}_i\dot{\xi}_j \quad (4.14)$$

and the two quantities denoted by the symbol $T^{(1)}_{ij}$ are equal to first order in $\vec{\xi}$.

B. Effective-mass theory

The wave vector $\vec{k}$ of a Bloch electron in a crystal in the presence of a rf field $\vec{E}$ and a constant magnetic field $\vec{B}_0$ obeys Eq. (4.4) (we neglect the rf $\vec{B}$ field). The quantity $\hbar\vec{k}$ is not, however, the momentum. Thus, the force has an additional component due to the action of the periodic crystal potential. This is called the Bragg force,[25] and equals

$$m\frac{d\vec{v}}{dt}-\hbar\frac{d\vec{k}}{dt} ,$$

where $m\vec{v}$ is the expectation value of the momentum of the particle. Now, we know that $\vec{v}=\hbar^{-1}\partial\epsilon/\partial\vec{k}$ so that in the effective-mass theory

$$m\frac{d\vec{v}}{dt}-\hbar\frac{d\vec{k}}{dt}=(\underline{\alpha}-\underline{I})\cdot(\hbar\dot{\vec{k}}) = -e(\underline{\alpha}-\underline{I})\cdot(\vec{E}+c^{-1}\vec{v}\times\vec{B}_0) . \quad (4.15)$$

Here $\underline{\alpha}$ is the effective-mass tensor defined by

$$\alpha_{ij}=\left[\frac{m}{m^*}\right]_{ij}=\frac{m}{\hbar^2}\frac{\partial^2\epsilon}{\partial k_i\partial k_j} , \quad (4.16)$$

and $\underline{I}$ the unit tensor. Because of the law of action and reaction the positive ions experience the additional average force

$$\gamma e(\underline{\alpha}-\underline{I})\cdot\left[\vec{E}-\frac{\vec{J}\times\vec{B}_0}{nec}\right] .$$

Thus, the force on a positive ion is, after taking into account the collision force,

$$\vec{F}=\gamma e[\underline{\alpha}\cdot\vec{E}-(nec)^{-1}(\underline{\alpha}-\underline{I})\cdot(\vec{J}\times\vec{B}_0) -(m/ne^2\tau)\vec{J}]+(\gamma e/c)\dot{\vec{\xi}}\times\vec{B}_0 -(\gamma m/\tau)\dot{\vec{\xi}} . \quad (4.17)$$

The last two terms can be neglected in a study of ultrasonic generation.

To gain a better understanding of these questions we cut out a simply connected volume V inside the metal. Let S be the surface bounding V. The rate of change of the linear momentum of all particles inside V is equal to the force acting on the volume V minus the momentum flux through the surface S. This can be expressed by the following equation:

$$\frac{d}{dt}\int_{(V)}d\vec{r}\left[-\frac{m}{e}\vec{J}+\frac{nM}{\gamma}\dot{\vec{\xi}}\right] = \int_{(V)}d\vec{r}\,c^{-1}\vec{J}\times\vec{B}_0-\int_{(S)}\underline{\sigma}\cdot d\vec{S} -\int_{(S)}\underline{T}\cdot d\vec{S} . \quad (4.18)$$

Here $\underline{\sigma}$ is the elastic stress tensor arising from the matter inside V and $\underline{T}$ is the quantity defined in Eq. (3.16), except that now we are dealing with

Bloch electrons. The sign of the second term in the right-hand side of V is due to the fact that this is the force on the matter within V exerted by its surroundings. The form of $\underline{T}$ is however the same as before. Transforming the surface integrals into volume integrals by means of the divergence theorem and noting that the result is valid for any volume, we obtain

$$\frac{nM}{\gamma}\ddot{\vec{\xi}}=-\vec{\nabla}\cdot\underline{\sigma}+\frac{1}{c}\vec{J}\times\vec{B}_0+\frac{m}{e}\frac{\partial\vec{J}}{\partial t}-\vec{\nabla}\cdot\underline{T}^{(1)} . \tag{4.19}$$

We have replaced $\underline{T}$ by $\underline{T}^{(1)}$ since the equilibrium part of $\underline{T}$ has zero divergence. Strictly speaking, the momentum balance equation (4.18) should contain the contributions due to the electromagnetic field. However, the electromagnetic momentum tensor is quadratic in the field variables and, thus, of higher order than the terms kept in Eq. (4.18).

There must, of course, be a relation between this result and the total force obtained in Eq. (4.17). To study this question we describe the states in a single band as solutions of a one-particle Schrödinger equation in the periodic potential $U(\vec{r})$. These states are characterized by a wave vector $\vec{k}$ and have energy which we denote, as before, by ϵ_k. In the presence of the phonon field with strain tensor ϵ_{ij}, the one-electron energy ϵ_k experiences a (time-dependent) change

$$\Delta\epsilon(\vec{k})=\sum_{ij}\epsilon_{ij}L_{ij}(\vec{k}) , \tag{4.20}$$

where the quantities $L_{ij}(\vec{k})$ will be investigated later on in this section. We employ next the results obtained by Holstein[21] in his study of the collision-drag effect. He showed that the electron distribution function relaxes to the "local" equilibrium value

$$\bar{f}_0=f_0(\epsilon_k-m\dot{\vec{\xi}}\cdot\vec{v}_k) . \tag{4.21}$$

However, in Eq. (4.21) $\vec{v}_k$ is not the expectation value of the velocity in the Bloch state $\vec{k}$, namely $\hbar^{-1}\partial\epsilon/\partial\vec{k}$, but rather the expectation value of the velocity for states in which the acoustic wave produces virtual transitions to other bands. Holstein proved that $\vec{v}_k$ is

$$\vec{v}_k=\frac{1}{\hbar}\frac{\partial\epsilon}{\partial\vec{k}}+\dot{\vec{\xi}}-\frac{m}{\hbar}\dot{\vec{\xi}}\cdot\frac{\partial}{\partial\vec{k}}\vec{v}_k . \tag{4.22}$$

In a linear theory such as that to which this paper is devoted, we do not make a distinction between $\vec{v}_k$ and $\hbar^{-1}\partial\epsilon/\partial\vec{k}$ when either is multiplied by $\dot{\vec{\xi}}$ or $\vec{E}$. As shown by Holstein, this result leads to the conclusion that $\bar{f}_0$ carries a current density $-ne\dot{\vec{\xi}}$, i.e., the opposite of the ionic current density. This follows immediately from

$$-\frac{e}{V}\sum_k f_0(\epsilon_k-m\dot{\vec{\xi}}\cdot\vec{v})\vec{v}_k=-\frac{e}{V}\sum_k f_0(\epsilon_k-m\dot{\vec{\xi}}\cdot\vec{v})\left[\dot{\vec{\xi}}+\frac{1}{\hbar}\frac{\partial\epsilon_k}{\partial\vec{k}}-\frac{m}{\hbar}\dot{\vec{\xi}}\cdot\frac{\partial}{\partial\vec{k}}\vec{v}\right]=-ne\dot{\vec{\xi}} . \tag{4.23}$$

To set up a transport equation we describe the electrons in the states with wave vector $\vec{k}$ taking into account the virtual transitions caused by the acoustic wave. The appropriate transport equation assuming, as before, the existence of a constant relaxation time is

$$\frac{\partial f}{\partial t}-\frac{e}{\hbar}\left[E+\frac{1}{c}\vec{v}_k\times\vec{B}_0\right]\cdot\frac{\partial f}{\partial\vec{k}}+\vec{v}\cdot\frac{\partial f}{\partial\vec{r}}+\frac{1}{\tau}(f-\bar{f}_0)=0 . \tag{4.24}$$

We proceed in the same way as in Sec. IV A by letting $f=\bar{f}_0+f^{(1)}$. By virtue of Eq. (4.23), the average of $\vec{v}_k$ over the distribution $f^{(1)}$ is proportional to the total current density $\vec{J}$. The second term in Eq. (4.24) requires the consideration of the result stated in Eq. (4.22). In fact,

$$\frac{\partial\bar{f}_0}{\partial\vec{k}}=(\vec{v}_k-\dot{\vec{\xi}})\frac{\partial f_0}{\partial\epsilon}$$

and, to first order in small quantities,

$$-\frac{e}{\hbar}\left[\vec{E}+\frac{1}{c}\vec{v}_k\times\vec{B}_0\right]\cdot\frac{\partial\bar{f}_0}{\partial\vec{k}}=-\left[e\vec{E}\cdot\vec{v}+\frac{e}{c}\dot{\vec{\xi}}\times\vec{B}_0\cdot\vec{v}\right]\frac{\partial f_0}{\partial\epsilon} ,$$

where we write $\vec{v}$ whenever it is not necessary to distinguish between $\vec{v}_k$ and $\hbar^{-1}\partial\epsilon/\partial\vec{k}$. We find, after some simple transformations,

$$\frac{\partial f^{(1)}}{\partial t}+\frac{1}{\tau}f^{(1)}-\frac{e}{\hbar c}(\vec{v}\times\vec{B}_0)\cdot\frac{\partial f^{(1)}}{\partial\vec{k}}+\vec{v}\cdot\frac{\partial f^{(1)}}{\partial\vec{r}}=\left[e\vec{E}\cdot\vec{v}+\frac{e}{c}(\dot{\vec{\xi}}\times\vec{B}_0)\cdot\vec{v}+m\ddot{\vec{\xi}}\cdot\vec{v}+m\,\vec{v}\cdot\dot{\underline{\epsilon}}\cdot\vec{v}\right]\frac{\partial f_0}{\partial\epsilon} . \tag{4.25}$$

Multiplying both sides of Eq. (4.25) by $-(e/V)\vec{v}_k$ and adding over all electron states we obtain

$$\frac{\partial \vec{J}}{\partial t}+\frac{1}{\tau}\vec{J}+\frac{e}{mc}\underline{\alpha}\cdot(\vec{J}\times\vec{B}_0)-\frac{e}{m}\vec{\nabla}\cdot\underline{T}^{(1)}=\frac{ne^2}{m}\underline{\alpha}\cdot\left[\vec{E}+\frac{1}{c}\dot{\vec{\xi}}\times\vec{B}_0+\frac{m}{e}\ddot{\vec{\xi}}\right]. \quad (4.26)$$

Equations (4.19) and (4.26) yield

$$\rho\ddot{\vec{\xi}}=-\vec{\nabla}\cdot\underline{\sigma}+ne\left[\underline{\alpha}\cdot\vec{E}-\frac{\vec{J}\times\vec{B}_0}{nec}\cdot(\underline{\alpha}-\underline{I})+\frac{m}{e}\underline{\alpha}\cdot\ddot{\vec{\xi}}\right]-\frac{m}{e\tau}\vec{J}+\frac{ne}{c}\dot{\vec{\xi}}\times\vec{B}_0-\frac{mn}{\tau}\dot{\vec{\xi}}. \quad (4.27)$$

Equation (4.27) is identical to Eq. (4.17) except for the additional contribution $(m/e)\underline{\alpha}\cdot\ddot{\vec{\xi}}$. This arises because the tendency of the electrons to screen the motion of the ions subjects them to the additional force $m\ddot{\vec{\xi}}$. The reactions on the ions is equivalent to an additional effective electric field $(m/e)\ddot{\vec{\xi}}$. However, this contribution together with the last two terms in Eq. (4.27) is negligible when dealing with ultrasonic generation in metals.

An additional question concerns the dependence of the stress tensor $\underline{\sigma}$ on the fields $\vec{E}$ and $\vec{\xi}$. Certainly the most important contribution to $\underline{\sigma}$ is that occurring when the strain ϵ_{ij} is applied statically. The components of the stress tensor are obtained from

$$\sigma_{ij}=\frac{\partial F}{\partial \epsilon_{ij}}, \quad (4.28)$$

where F is the free energy of the system per unit volume. Now, the free energy in the dynamic situation can be regarded as time dependent. It can be separated into a contribution $F^{(0)}$ equal to the equilibrium value and the quantity $F^{(1)}$, the additional free energy due to the effect of the acoustic and electromagnetic waves on the electron energies. Thus we have

$$F=F^{(0)}+F^{(1)}=F^{(0)}+\frac{1}{V}\sum_k \Delta\epsilon(\vec{k})[f(\vec{k})-f_0(\vec{k})] \quad (4.29)$$

and

$$\sigma_{ij}=\sigma_{ij}^{(0)}+\frac{1}{V}\sum_k \frac{\partial}{\partial\epsilon_{ij}}\Delta\epsilon(\vec{k})(f-f_0). \quad (4.30)$$

Here $\sigma_{ij}^{(0)}$ is related to ϵ_{ij} by the ordinary elastic constants. The second term, which vanishes in the free-electron model, has been investigated by Fiks.[22] We follow the method of Bir and Pikus.[26] When a uniform strain $\underline{\epsilon}$ is applied, the lattice positions $\vec{n}$ of the crystal are transformed into $(\underline{I}+\underline{\epsilon})\cdot\vec{n}$. In general, the crystal symmetry is altered. Difficulties in the use of ordinary perturbation theory are circumvented by making the coordinate transformation

$$\vec{r}'=(\underline{I}+\underline{\epsilon})^{-1}\cdot\vec{r}=(\underline{I}-\underline{\epsilon})\cdot\vec{r} \quad (4.31)$$

such that the lattice positions of the strained crystal have the same coordinates in the new frame of reference as those of the unstrained crystal had in the old. The Hamiltonian of an electron in the deformed crystal is

$$H(\underline{\epsilon})=\frac{p^2}{2m}+U(\underline{\epsilon},\vec{r}), \quad (4.32)$$

where $U(\underline{\epsilon},\vec{r})$ is the potential energy of an electron at $\vec{r}$ when the deformation has been applied. When expressed in the new coordinate system $H(\underline{\epsilon})$ has the same symmetry as the unperturbed crystal $(\underline{\epsilon}=0)$. In this reference frame, since $\vec{p}'=(\underline{I}+\underline{\epsilon})\cdot\vec{p}$,

$$H(\underline{\epsilon})=\frac{p'^2}{2m}-\frac{1}{m}\sum_{ij}\epsilon_{ij}p'_ip'_j+U(\underline{\epsilon},(\underline{I}+\underline{\epsilon})\cdot\vec{r}'). \quad (4.33)$$

In general,

$$U(\underline{\epsilon},(\underline{I}+\underline{\epsilon})\cdot\vec{r}')=U(\vec{r}')+\sum_{ij}\epsilon_{ij}U_{ij}(\vec{r}')+\cdots. \quad (4.34)$$

The electron energy spectrum of the strained crystal is obtained by solving the Schrödinger equation for the Hamiltonian

$$H(\epsilon)=\frac{p'^2}{2m}+U(\vec{r}')+\sum_{ij}\epsilon_{ij}\left[U_{ij}-\frac{1}{m}p'_ip'_j\right]=\frac{p'^2}{2m}+U(\vec{r}')+\sum_{ij}\epsilon_{ij}D_{ij}, \quad (4.35)$$

which, we must remember, possesses the symmetry of the original crystal in the primed reference frame. The second equality in (4.35) defines D_{ij}. To first order in $\underline{\epsilon}$, the energy eigenvalue of $H(\epsilon)$ associated with the solution of

$$\left[\frac{p'^2}{2m}+U(\vec{r}')\right]\psi_{k'}=\epsilon_{k'}\psi_{k'} \quad (4.36)$$

is

$$\epsilon_{k'}+\sum_{ij}\epsilon_{ij}\langle \vec{k}'\,|\,D_{ij}\,|\,\vec{k}'\rangle \ . \qquad (4.37)$$

The eigenvector $\psi_{k'}$ is of the Bloch form $u_{k'}(\vec{r}')\exp(i\vec{k}'\cdot\vec{r}')$, and, when written in terms of the original coordinate system is

$$\psi_{k'}=u_{k'}[(\underline{I}+\underline{\epsilon})^{-1}\cdot\vec{r}]$$
$$\times\exp[i\vec{k}'\cdot(\underline{I}+\underline{\epsilon})^{-1}\cdot\vec{r}] \ . \qquad (4.38)$$

Therefore, $\psi_{k'}$ belongs to the wave vector

$$\vec{k}=(\underline{I}+\underline{\epsilon})^{-1}\cdot\vec{k}' \qquad (4.39)$$

and to energy ϵ_k. The difference in energy between the expression (4.37) and ϵ_k gives the shift of the energy levels of the electron upon application of the stress. This change equals

$$\Delta\epsilon=\epsilon_{k'}-\epsilon_k+\sum_{ij}\epsilon_{ij}\langle\vec{k}'\,|\,D_{ij}\,|\,\vec{k}'\rangle$$
$$=\sum_{ij}\epsilon_{ij}L_{ij}(\vec{k}) \ , \qquad (4.40)$$

with

$$L_{ij}(\vec{k})=\frac{1}{2}\left[k_i\frac{\partial\epsilon_k}{\partial k_j}+k_j\frac{\partial\epsilon_k}{\partial k_i}\right]+\langle\vec{k}\,|\,D_{ij}\,|\,\vec{k}\rangle \ . \qquad (4.41)$$

This equation gives the general expression for $L_{ij}(\vec{k})$. In the deformable ion model, we suppose that the potential at a point in the deformed crystal equals that at the original position, i.e.,

$$U(\underline{\epsilon},(\underline{I}+\underline{\epsilon})\cdot\vec{r}')=U(\vec{r}') \qquad (4.42)$$

and, hence, $U_{ij}=0$. Then, in the effective-mass approximation, with an isotropic effective mass m^*,

$$L_{ij}(\vec{k})=(m^*-m)v_iv_j \ . \qquad (4.43)$$

Combining this result with Eq. (4.30),

$$\sigma_{ij}=\sigma_{ij}^{(0)}+\frac{1}{V}\sum_k(m^*-m)v_iv_j(f-f_0) \ , \qquad (4.44)$$

which, when substituted into Eq. (4.19) yields

$$\rho\ddot{\vec{\xi}}=-\vec{\nabla}\cdot\underline{\sigma}^{(0)}+\frac{1}{c}\vec{j}\times\vec{B}_0+\frac{m}{e}\frac{\partial\vec{j}}{\partial t}$$
$$-\vec{\nabla}\cdot\underline{T}^{(1)}\frac{m^*}{m} \ . \qquad (4.45)$$

This result agrees with that obtained by Kontorovich[27] and, more recently, Fiks.[22] In this approximation all effects arising from the Bragg force cancel. In fact, from Eqs. (4.45) and (4.27), using an isotropic effective mass m^*, we obtain

$$\rho\ddot{\vec{\xi}}=-\vec{\nabla}\cdot\underline{\sigma}^{(0)}+ne\left[\vec{E}-\frac{1}{\sigma_0}\vec{j}\right]+\frac{ne}{c}\dot{\vec{\xi}}\times\vec{B}_0$$
$$-\frac{nm^*}{\tau}\dot{\vec{\xi}}-\frac{m^*-m}{e}\frac{\partial\vec{j}}{\partial t} \ , \qquad (4.46)$$

where now $\sigma_0=ne^2\tau/m^*$ is the dc electrical conductivity. Thus, it appears to be superfluous to attempt an explanation of the experimental results based on the effect of the Bragg reaction force in the absence of a more accurate knowledge of the deformation force.

However, we have made attempts to fit the data of Ref. 13 by neglecting the deformation force. From Eq. (4.27), taking $\underline{\sigma}=\underline{\sigma}^{(0)}$ and $\underline{\alpha}=(m/m^*)\underline{I}$, we obtain

$$\xi_x(\infty)=\frac{i\gamma ecE_0m}{2\pi\omega\sigma_0m^*sM}\mathrm{Re}\left[\frac{1}{G_+(q_a)}-1\right] \qquad (4.47)$$

and

$$\xi_y(\infty)=\frac{i\gamma ecE_0m}{2\pi\omega\sigma_0m^*sM}\left[\mathrm{Im}\left[\frac{1}{G_+(q_a)}-1\right]+\omega_c\tau\left[1-\frac{m^*}{m}\right]\right] \ . \qquad (4.48)$$

The cyclotron resonance frequency is $\omega_c=eB_0/$

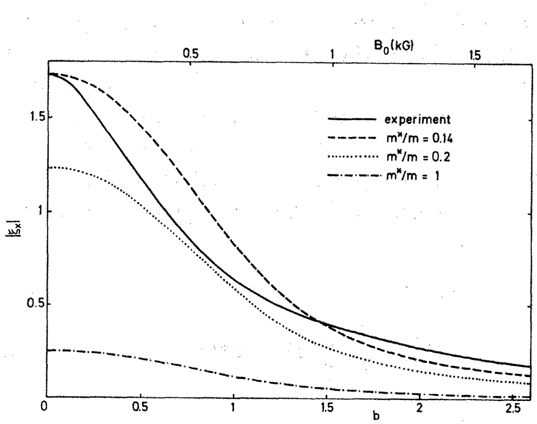

FIG. 6. Magnetic field dependence of the x component $|\xi_x(\infty)|$ of the acoustic amplitude for different values of the ratio m^*/m. The lower scale shows the parameter $b=\omega_c\tau/q_al$ and the solid line represents the experimental data of Ref. 13.

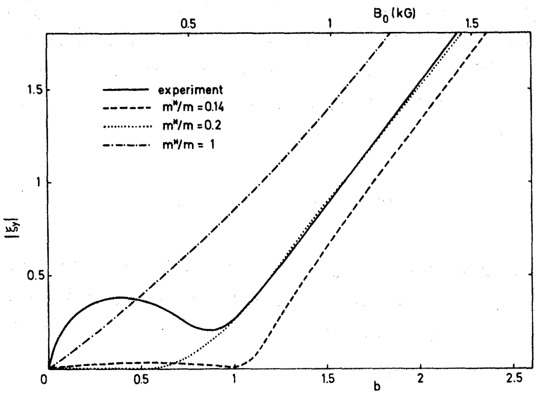

FIG. 7. Magnetic field dependence of the y component $|\xi_y(\infty)|$ of the acoustic amplitude for different values of the ratio m^*/m. The lower scale shows the parameter $b=\omega_c\tau/q_a l$ and the solid line represents the experimental data of Ref. 13.

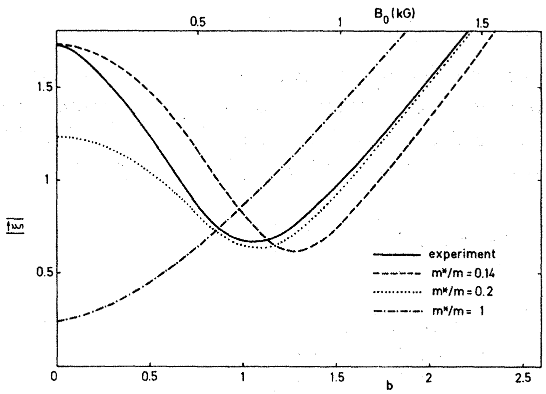

FIG. 8. Magnetic field dependence of the total sound amplitude for different values of the ratio m^*/m. The lower scale shows the parameter $b=\omega_c\tau/q_a l$ and the solid line represents the experimental data of Ref. 13.

m^*c and the electrodynamic part of the problem has been solved assuming specular scattering at $z=0$. The result of this analysis is shown in Figs. 6–8. In these figures the experimental data of Wallace *et al.*[13] are shown in solid lines. The data have been adjusted to coincide with the theoretical prediction for $(\omega_c\tau/q_a l)\gg 1$ given in Eq. (3.14) which is the same as that obtained from Eq. (4.48) even when $m^*\neq m$. Figure 6 shows $\xi_x(\infty)$ as a function of B_0. We notice that to fit $\xi_x(\infty)$ for $B_0=0$ we require an unrealistic effective mass $m^*=m/7$ and that, as seen in Fig. 7, the corresponding value of $\xi_y(\infty)$ is too small compared to the experimental value. The dip at $(\omega_c\tau/q_a l)=1$ arises because of a change in phase of $\xi_y(\infty)$ by 180° at that value of the magnetic field. Figure 8 shows $|\vec{\xi}(\infty)|$ for the values of $m^*/m=0.14$ and 0.2 as in the previous figures together with the experimental results. In Figs. 6–8 we have also shown, for convenience of reference the results using $m^*=m$. Similar conclusions were reached by Banik and Overhauser.[28]

V. CONCLUSIONS

Chimenti *et al.*[14] showed that the low magnetic field behavior of acoustic generation by electromagnetic radiation cannot be explained by the free-electron model assuming the electrons are scattered specularly at the surface of incidence. We have shown here, however, that the nonmonotonic variation observed is expected if the scattering is diffuse. The complexity of the numerical calculations has prevented us from making a detailed fit of the theory and experiment with limited resources. The nonmonotonic behavior is clearly exhibited in Fig. 5 using the parameters of Table I.

We proved, in agreement with Banik and Overhauser[28] that, in going beyond the free-electron model invoking the Bragg reflection force it is not possible to successfully account for the experimental data in potassium. However, a more important theoretical limitation arises from neglecting the reaction forces due to the electronic deformation potentials. These forces, which are difficult to evaluate with certainty, cancel the Bragg reflection force if we employ the deformable ion model. Thus, the nonmonotonic behavior obtained using the Bragg reflection force appears to be illusory. In a subsequent paper[29] the authors of Ref. 28 proposed a dynamic two-carrier model to explain the data obtained in aluminum films.[15] They included the effect of the Bragg reflection force but omitted the deformation forces.

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APPENDIX

The results in Eqs. (2.24)–(2.29) are obtained by transforming the variable of integration from q to ζ, using definition (2.21) in the integral (2.20). The path of integration is a straight line L from

$$-\infty \exp\left[\frac{i\pi}{2}+i\psi_{\pm}\right]$$

to

$$\infty \exp\left[\frac{i\pi}{2}+i\psi_{\pm}\right],$$

where

$$1-i\alpha_{\pm}=(1+\alpha_{\pm}^2)^{1/2}\exp(-i\psi_{\pm})\ . \tag{A1}$$

The integral is evaluated by completing the contour by arcs of circle at infinity lying to the right of L, two segments just above and below the cut $(1,\infty)$ and a small circle around the essential singularity $\zeta=1$, described in an anticlockwise fashion between arguments $+0$ and $2\pi-0$. The integrals along the arcs tend to zero at the appropriate limits. The zeros of the denominator of the integrand in Eq. (2.20) are calculated approximately as the roots of

$$\zeta^3=(\mathrm{sgn}\ \mathrm{Im}\zeta)(l/\delta)^3(1-i\alpha_{\pm})^{-3}\ . \tag{A2}$$

In the case of diffuse scattering ($p=0$), the calculations were rather complex so that the details are not reproduced here. It was also necessary to find the zeros of

$$\zeta^2+\frac{2}{\pi}(i+\alpha_{\pm})^{-3}(l/\delta)^3\left[\frac{1}{\zeta^2}+\frac{\zeta^2-1}{2\zeta^3}\ln\frac{1+\zeta}{1-\zeta}\right].$$

This function appeared, however, under a natural logarithm (see Ref. 10). For this situation, the approximation (A2) was not made but the zeros were obtained numerically by iteration.

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