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Critical behavior of impure superconductors in $4-\epsilon$ dimensions

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We show that, within the ϵ expansion, the impure superconductor coupled to a gauge field exhibits a new stable fixed point, indicating that the system has a second-order transition for sufficiently large impurity concentration. The critical exponents at this transition are computed to $O(\epsilon)$.

Some time ago it was argued by Halperin, Lubensky, and Ma[1] that the phase transition in superconductors should be weakly first order due to the long-range effects of the electromagnetic field. For type-II superconductors, where fluctuations in the order parameter are expected to be important, this conclusion was based on the runaway behavior of the renormalization-group (RG) trajectories, calculated within a $4-\epsilon$ expansion. This result is also consistent with calculations by Coleman and Weinberg[2] in the same model, and the first-order nature has been later confirmed by evaluating the free energy and the equation of state.[3]

In this Communication we consider the effect of quenched paramagnetic impurities in the superconductor, and show that, within the $4-\epsilon$ expansion, they act to restore the second-order nature of the transition with, however, very different critical exponents from those of the pure XY transition.

We take as a model the gauge-invariant Landau-Ginzburg action

$$F\{\phi_\alpha, A_\mu\} = \int d^dx \left[|D_\mu\phi_\alpha|^2 + [r+\delta r(x)]|\phi_\alpha|^2 + \frac{u_0}{4}(|\phi^\alpha|^2)^2 + \tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} \right] , \quad (1)$$

where $D_\mu = \partial_\mu + ie_0A_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. $\{\phi_\alpha\}$ is a set of $\frac{1}{2}n$ complex fields, $n=2$ corresponding to the superconductor. The quenched impurities are represented by the field $\delta r(x)$, whose quenched average satisfies $\langle\langle \delta r(x)\rangle\rangle = 0$, $\langle\langle \delta r(x)\delta r(x')\rangle\rangle = \Delta_0 f(x-x')$, where f is a short-ranged function which decays on a length scale of order ξ_0.[1] To investigate the critical behavior it is sufficient to replace f by a δ function. In the Lorentz gauge $\partial_\mu A_\mu = 0$, the photon propagator is $(g_{\mu\nu} - k_\mu k_\nu/k^2)/k^2$, and there are no new contributions of order $e^2\Delta$ to the wave-function or coupling-constant renromalization constants. To one loop, the Callan-Symanzik β functions in $4-\epsilon$ dimensions are

$$\beta_u = -\epsilon u + \tfrac{1}{6}(n+8)u^2 - 6e^2u + 18e^4 - 24\Delta u \ , \quad (2)$$

$$\beta_{e^2} = -\epsilon e^2 + \tfrac{1}{6}ne^4 \ , \quad (3)$$

$$\beta_\Delta = -\epsilon\Delta - 16\Delta^2 + \tfrac{1}{3}(n+2)\Delta u - 6e^2\Delta \ . \quad (4)$$

[A factor $2\pi^{d/2}/(2\pi)^d\Gamma(\frac{1}{2}d)$ has been absorbed in the coupling constants.] These reduce to well-known cases when either $\Delta = 0$ (Refs. 1–3) or $e^2 = 0$.[4–6] Besides the fixed points known in those cases there is now a new stable fixed point for $n > 1$:

$$u^* = \frac{3\epsilon}{4(n-1)} \left\{ \left(1+\frac{36}{n}\right) + \left[\left(1+\frac{36}{n}\right)^2 + \frac{3456(n-1)}{n^2} \right]^{1/2} \right\} , \quad (5)$$

$$\Delta^* = \frac{1}{16}\left[\left(\frac{(n+2)}{3}\right)u^* - \left(1+\frac{36}{n}\right)\epsilon \right] , \quad (6)$$

$$e^{*2} = 6\epsilon/n \ , \quad (7)$$

which, for the case $n=2$, become $u^* = 40.5\epsilon$, $\Delta^* = 2.19\epsilon$, and $e^{*2} = 3\epsilon$. At this fixed point there are three irrelevant eigenvalues $-\epsilon$ and $-(14.25 \pm 20.24i)\epsilon$. Thes complex eigenvalues (which arise in other random systems[5,6]) imply that the RG trajectories spiral into the fixed point, and give oscillatory corrections to scaling. The RG flows in the (u, Δ) plane, for which $e^2 = e^{*2}$ are shown in Fig. 1. There is a separatrix above which all flows terminate in the fixed point. Trajectories below the separatrix exhibit runaway behavior, ultimately crossing into the unstable region $u < 0$. We then argue that systems with sufficiently large Δ_0 (concentration of impurities) will exhibit a second-order transition, and those with a smaller value will have a weak first-order transition, although this latter conclusion strictly requires a calculation of the free energy.

Within the one-loop approximation the separatrix may be computed as a power series in u:

$$\Delta = \frac{4.05\epsilon^2}{u} - 0.68\epsilon + 0.06u + \cdots \quad . \tag{8}$$

The critical exponents computed at the new fixed point are

$$\eta = -9\epsilon + O(\epsilon^2) \quad ,$$

$$\nu^{-1} = 2 - 9.25\epsilon + O(\epsilon^2) \quad .$$

The above analysis applies only in $4 - \epsilon$ dimensions. In three dimensions, Dasgupta and Halperin[7] have suggested, on the basis of duality arguments and Monte Carlo simulations, that a fixed-length spin-lattice version of the pure superconductor has an inverted XY transition, rather than a first-order transition, although the RG interpretation of this is unclear. Since $\alpha \approx 0$ for this model, we would expect quenched impurities to be marginally relevant, so that our fixed point may still describe the impure superconductor. In this case, however, higher-order calculations are necessary to obtain reasonable values for the critical exponents.

The fact that inhomogeneities can make the transitions in gauge theories second order may also be of importance for theories of the early universe, where first-order transitions of the Coleman-Weinberg type play an important role.[8]

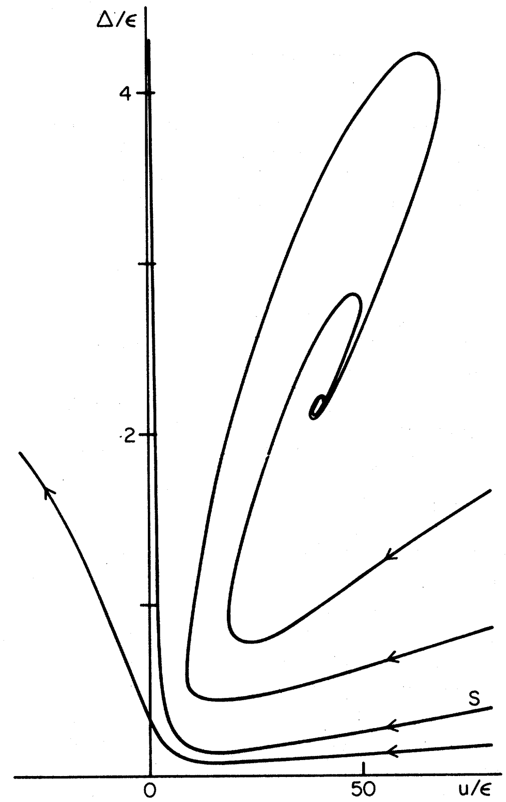

FIG. 1. The RG flows in the (u, Δ) plane where $e = e^*$. S labels the separatrix.

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[1] B. I. Halperin, T. C. Lubensky, and S. Ma, Phys. Rev. Lett. 32, 292 (1974).
[2] S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).
[3] I. D. Lawrie, Nucl. Phys. B200, [FS4], 1 (1982).
[4] T. C. Lubensky, Phys. Rev. B 11, 3573 (1975).
[5] S. N. Dorogovstev, Phys. Lett. 76A, 169 (1980).
[6] D. Boyanovsky and J. Cardy Phys. Rev. B (in press).
[7] C. Dasgupta and B. I. Halperin, Phys. Rev. Lett. 47, 1556 (1981).
[8] A. D. Linde, Rep. Prog. Phys. 42, 389 (1979).