Rapid Communications

The Rapid Communications section is intended for the accelerated publication of important new results. Manuscripts submitted to this section are given priority in handling in the editorial office and in production. A Rapid Communication may be no longer than 3½ printed pages and must be accompanied by an abstract. Page proofs are sent to authors, but, because of the rapid publication schedule, publication is not delayed for receipt of corrections unless requested by the author.

Nonreciprocal surface-acoustic-wave propagation in aluminum

J. Heil and B. Lüthi
Physikalishes Institut der Universität, Robert-Mayer-Strasse 2-4, D-6000 Frankfurt am Main 1, Federal Republic of Germany

P. Thalmeier
Max-Planck-Institut für Festkörperforschung, Heisenbergstrasse 1, D-7000 Stuttgart 80, Federal Republic of Germany
(Received 23 March 1982)

Nonreciprocal effects in surface-acoustic-wave (SAW) velocities for single-crystal aluminum have been observed at low temperatures in the presence of a magnetic field. They are of the order of 0.1% in the relative velocity change. This effect is due to coupling of the SAW displacement field to the conduction electrons moving on cyclotron orbits. We give a quantitative explanation for this effect.

Magnetostatic Damon-Eshbach modes and spin waves on magnetic surfaces show nonreciprocal behavior,[1,2] i.e., $\omega(\vec{k}_{\parallel}) \neq \omega(-\vec{k}_{\parallel})$. Here ω is the mode frequency and $\vec{k}_{\parallel}$ its wave vector parallel to the surface. This behavior is due to the axial vector nature of the magnetic field. Analogous nonreciprocal effects have not been observed so far for surface acoustic waves (SAW). However, they should exist as shown by the following symmetry argument. Consider an external magnetic field $\vec{B} \perp \vec{k}_{\parallel}$ and $\vec{B} \perp \hat{n}$ (Fig. 1) ($\hat{n}$ the surface normal). For a semi-infinite medium symmetry requirements give $\omega(\vec{k}_{\parallel}, \vec{B}) = \omega(-\vec{k}_{\parallel}, -\vec{B})$. This is in contrast to bulk acoustic waves where the reciprocal conditions $\omega(\vec{k}, \vec{B}) = \omega(-\vec{k}, \vec{B})$ and $\omega(\vec{k}, \vec{B}) = \omega(\vec{k}, -\vec{B})$ have to be satisfied. One should therefore expect for SAW $\omega(\vec{k}, \vec{B}) \neq \omega(-\vec{k}, \vec{B})$ if a sufficiently strong mechanism exists to make this symmetry breaking observable.

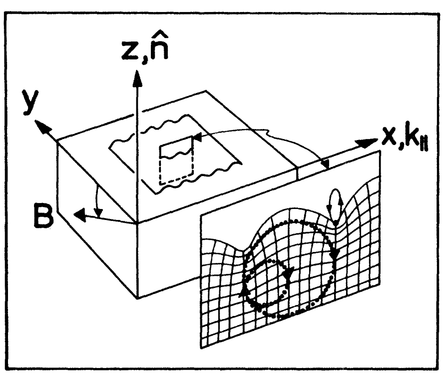

FIG. 1. Experimental arrangement: The vectors $\hat{n}$, $\vec{k}_{\parallel}$, $\vec{B}$ are shown. A picture of a SAW displacement field with the particle rotational sense is given (dimensions exaggerated for clarity). Electronic cyclotron orbits for B_y configuration are indicated as dotted lines.

In this Communication we give for the first time experimental results for nonreciprocal behavior of SAW in single-crystal aluminum. We also give a quantitative interpretation of these experiments. As shown pictorially in Fig. 1 a Rayleigh wave is elliptically polarized in the sagittal plane ($\vec{k}_{\parallel}$-$\hat{n}$ plane). The rotational sense of its displacement field with respect to the conduction electrons moving on cyclotron orbits (for $\vec{B} \perp \vec{k}_{\parallel}, \vec{B} \perp \hat{n}$) depends on the direction of $\vec{k}_{\parallel}$. The SAW velocity which is influenced by electron impurity scattering should therefore be nonreciprocal. In addition, this mechanism should lead to geometric resonances and in higher fields to de Haas–Shubnikov oscillations in the SAW velocity, effects well known for bulk acoustic waves.[3]

Two Al single crystals were investigated; either a (001) or a (011) surface with propagation in both cases along [100] direction. Both crystals gave similar results. The magnetic field $\vec{B}$ could be rotated in the plane perpendicular to $\vec{k}_{\parallel}$ (Fig. 1). SAW of 12.4 MHz are generated with a comb structure using CdS transducers. Details of the generation and the phase-sensitive detection are described elsewhere.[4]

In Fig. 2 we give typical examples of reciprocal and nonreciprocal behavior for SAW and bulk c_{44} shear waves. Figure 2(a) shows a clearcut example of a strong nonreciprocal effect (of the order of 0.1% for the relative velocity change) for SAW for $\vec{B} \perp \vec{k}_{\parallel}$ and $\vec{B} \perp \hat{n}$. Figure 2(b) gives an example of reciprocal behavior for $\vec{B} \perp \vec{k}_{\parallel}$ and $\vec{B}$ parallel or antiparallel to $\hat{n}$. Finally, in Fig. 2(c) reciprocal behavior is clearly evident for the bulk c_{44} mode and is shown for the $\vec{B}$ geometry given Fig. 2(a). In addition to the results given in Fig. 2 we found geometric resonances and de Haas–Shubnikov oscillations for both SAW and bulk waves in the field region 0–12 T. An indication of geometric resonances can be seen in Fig. 2(c) where they clearly show up for $B > 0$ and are slightly masked due to a small misorientation for $B < 0$. A full discussion of these effects will be given elsewhere.[5]

We give now a quantitative theoretical explanation of the observed SAW nonreciprocity in aluminum. The SAW dispersion is determined by the boundary condition of a stress-free surface. In addition to the elastic stress tensor s_{ik} $(i,k = x,z)$ electron scattering from impurities leads to a kinetic nonequilibrium stress tensor $t_{ik}(B)$ with the property $t_{ik}(-\vec{B}) \neq t_{ik}(\vec{B})$. Therefore the total stress tensor $\sigma_{ik} = s_{ik} + t_{ik}$ depends on the field direction and the boundary condition of a stress-free surface

$$\sigma_{iz} = s_{iz} + t_{iz}(\vec{B}) = 0 \quad (i = x,z) \tag{1}$$

leads to a nonreciprocal Rayleigh SAW dispersion. This approach is valid as long as the classical skin depth is much smaller than the acoustic penetration depth of SAW so that electromagnetic boundary conditions do not have to be included explicitly.

The kinetic stress tensor $\underline{t}$ has been calculated with

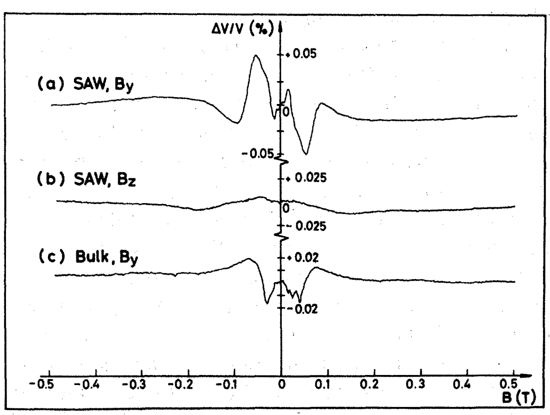

FIG. 2. Relative velocity change $\Delta v/v$ in percent as a function of B for $T = 4.2$ K and $0 \leqslant B \leqslant 0.5$ T. (a) Nonreciprocal effects for SAW and B_y geometry. (b) Reciprocal effect for SAW and B_z geometry. (c) Reciprocal effect for bulk c_{44} waves and B_y geometry.

the help of Boltzmann's equation to treat the electron scattering[6]:

$$\underline{t} = -m \int \vec{v}\,\vec{v} f_1(\vec{r}, \vec{v}, t)\, d^3v = \underline{\Lambda}(\vec{B})\, \vec{u}(r,t) \quad .$$

Here m, $\vec{v}$ are the band mass and the velocity of the conduction electron and f_1 is the nonequilibrium electron distribution function. $\underline{t}$ is linear in the displacement field $\vec{u}(\vec{r},t)$ and the third rank tensor $\underline{\Lambda}$ has been calculated using a free-electron model in the limit $|kR| \ll 1$. $R\,(= v_F/\omega_c)$ is the Larmor radius; $\omega_c(= eB/mc)$, the cyclotron frequency. Calculations for $|kR| \gtrsim 1$ are much more complicated because geometric resonance and surface scattering effects had to be considered (see Fig. 1). With the additional assumption $|kl| \ll 1$ $(l = v_F\tau$, the electron mean free path, τ the lifetime) $\underline{\Lambda}(\underline{B})$ can be computed. For a discussion of nonreciprocal effects only its antisymmetric part $\underline{A}(\vec{B}) = \underline{\Lambda}(\vec{B}) - \underline{\Lambda}(-\vec{B})$ has to be considered. The real (′) and imaginary (″) parts of the relevant components read

$$\begin{aligned} A'_{xzx} &= \tfrac{2}{5}\eta\,\Omega_c f(\Omega_c)\,kR \ , \\ A''_{xzz} &= \tfrac{2}{5}\eta\,\Omega_c f(\Omega_c)\,kR \ , \\ A'_{zzz} &= \tfrac{2}{5}\eta\,\Omega_c[2 + f(\Omega_c)]\,kR \ , \\ A''_{zzx} &= \tfrac{2}{5}\eta\,\Omega_c[2 - f(\Omega_c)]\,kR \ , \end{aligned} \tag{2}$$

with $\eta = zs(mv_F/Mv_t)|k|$, $f(\Omega_c) = (1 + 4\Omega_c^2)^{-1}$, $\Omega_c = \omega_c\tau$, $\kappa_{t,l} = (k^2 - \omega^2/v_{t,l})$. Here M is the ionic mass, z, the number of conduction electrons/atom, and $s = v_s/v_t$; v_t, v_l are the transverse and longitudinal bulk velocities, v_s the SAW velocity for $B = 0$. The SAW penetration depth is given by $\kappa_{t,l}$. The boundary condition (1) leads to the SAW dispersion relation

$$(k^2 + \kappa_t^2)^2 - 4k^2\kappa_t\kappa_l - \Delta(k,B) = 0 \ , \tag{3}$$

where $\Delta(k,B) = F(\underline{\Lambda}(k,B))$ is a linear function and describes the field-dependent coupling of SAW and conduction electrons. Defining

$$\Delta v_s = v_s(k,B) - v_s(k,-B) = v_s(k,B) - v_s(-k,B)$$

the nonreciprocal part of v_s is finally given by

$$\frac{\Delta v}{v_s} = \frac{4}{5}\frac{zmv_F}{Mv_t}\Omega_c(1 + 4\Omega_c^2)^{-1} kRD(s,r) \ , \tag{4}$$

with

$$D = (\nu_l + \nu_t)(1 - \nu_l\nu_t)[4s(\nu_l/\nu_t + r^2\nu_t/\nu_l - 2 + s^2)]^{-1}$$

and $r^2 = v_t^2/v_l^2$, $\nu_l = (1 - r^2s^2)^{1/2}$, $\nu_t = (1 - s^2)^{1/2}$. As required $\Delta v_s/v_s$ changes sign under field or propagation reversal for the geometry shown in Fig. 2(a). Equation (4) is only valid for $kR \ll 1$. It therefore does not describe the low-field limit where geometric resonances occur. In the high-field limit $\Omega_c \gg 1$

the nonreciprocal effect goes to zero with $\Delta v_s/v_s \sim B^{-2}$ in qualitative agreement with experiment [Figs. 2(a) and 3].

For a numerical analysis we took literature values for Al.[7] The numerical prefactor in (4) is $\frac{4}{5}(zmv_F/Mv_t)\ D(s,r)=0.005$ and $R=1.1 \times 10^{-3}|B|^{-1}$ with B in teslas, giving $|kR|=0.27|B|^{-1}$ for SAW (12.4 MHz) and $0.251|B|^{-1}$ for bulk waves (10.5 MHz). For the latter we get $kR=2$, 2.5, 3, 3.5 for $B=0.04$, 0.032, 0.027, 0.022, respectively. These values agree rather well with the peaks in the bulk velocity [Fig. 2(c)] identifying them indeed as geometric resonances.[5] The only remaining fit parameter in (4) is the lifetime τ. In Fig. 3 we plot $\Delta v_s/v_s$ normalized to its value at $B=1$ T. The best fit for $B>0.4$ T is obtained for $1=0.53\times10^{-3}$ cm or $\tau=0.26\times10^{-11}$ s. This value leads to an absolute SAW nonreciprocity of $\Delta v_s(1\ \mathrm{T})/v_s=3.36\times10^{-4}$ which is in excellent agreement with the experimental value $\Delta v_s(1\ \mathrm{T})/v_s=3.32\times10^{-4}$. Another independent estimate of τ is given by the onset of de Haas–Shubnikov oscillations in the SAW or bulk velocities[5] for $B\simeq1$ T. Taking $\omega_c(1\ \mathrm{T})\tau\simeq1$ leads to $\tau\simeq0.6\times10^{-11}$ s, a value close to the one used above. The theoretical fit shown in Fig. 3 gives the salient features of the experiment. The fit is not perfect because in the interesting field region $0.25 \leqslant B \leqslant 1$ T we have $0.25 \leqslant |kR| \leqslant 1$ and $0.1 \leqslant \omega_c\tau \leqslant 0.4$ rather than $|kR| \ll 1$. Finally, for the electromagnetic skin depth $\delta=(c^2/2\pi\omega\sigma_0)^{1/2}$ we have $|k\delta|=3.4\times10^{-5}$, $|kl|=0.11$ justifying our initial assumptions $|k\delta| \ll 1$, $|kl| \ll 1$.

In conclusion we have found for the first time nonreciprocal SAW propagation in an external magnetic field. The effect demonstrated here is due to a coupling of conduction electrons moving on cyclotron orbits to the SAW displacement field, mediated by impurity scattering. Outside the geometrical resonance regime the theory agrees well with experimental results in Al. According to Eq. (4) the nonreciprocal SAW effect should be even bigger for metals with heavy electrons such as Bi. Experiments on this metal are in progress.

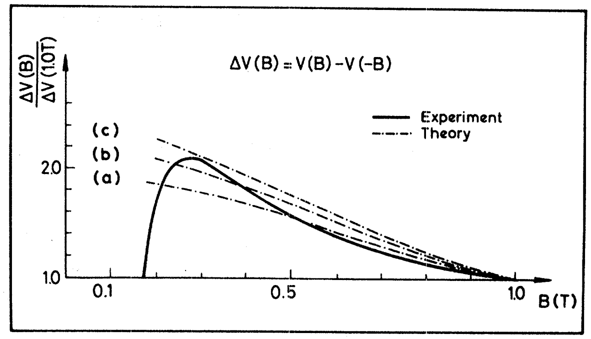

FIG. 3. Comparison of theory and experiment for nonreciprocal effects for the geometry given in Fig. 2(a). Full line is experimentally determined $\Delta v(B)/\Delta v(1\ \mathrm{T})$, where $\Delta v(B)=v(B)-v(-B)$. Dotted lines are theoretical fits for different mean free paths [curve (a): $l=0.53\times10^{-3}$ cm, (b): 0.6×10^{-3} cm, (c): 0.65×10^{-3} cm].

We would like to thank Professor J. F. Koch, Technical University München for providing us with high-quality Al single crystals. Discussions with Dr. R. E. Camley are gratefully acknowledged. This research was supported in part by Sonderforschungsbereich 65 Frankfurt-Darmstadt.

[1]For a review see A. A. Maradudin, in *Festkörperprobleme (Advances in Solid State Physics)*, edited by J. Treusch (Vieweg, Braunschweig, 1981), Vol. XXI, Chap. 25.
[2]P. Grünberg and F. Metawe, Phys. Rev. Lett. 39, 1561 (1977); J. R. Sandercock and W. Wettling, J. Appl. Phys. 50, 7784 (1979).
[3]B. W. Roberts, in *Physical Acoustics*, edited by W. P. Mason (Academic, New York, 1968), Vol. 4B.
[4]C. Lingner and B. Lüthi, Phys. Rev. B 23, 256 (1981).
[5]J. Heil, B. Lüthi, and P. Thalmeier (unpublished).
[6]M. S. Steinberg, Phys. Rev. 111, 425 (1958).
[7]$z=3$, $m=1.06m_0$, $v_F=2.02\times10^8$ cm/s, $M=0.448\times10^{-22}$ g, $v_t=0.34\times10^6$ cm/s, $v_l=0.65\times10^6$ cm / s.