Effective stopping-power charges of swift ions in condensed matter

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The effective charge of energetic ions as it pertains to the stopping power of solids is calculated in a dielectric-response approximation. The density distribution of N electrons bound in an ion of atomic number Z_1 is given by a variational statistical approximation. The effective charge $Z_1^* e$ is always larger than the ionic charge $Q_1 = (Z_1 - N)e$, because of close collisions. A comprehensive low-velocity formula predicts $Z_1^* e$ for given Q as a function of the ratio between the ion size and the mean electron spacing in the medium. At high velocities one obtains a partition rule of stopping powers for the effective charge of ionic projectiles. The results are compared with new precision stopping-power measurements on C, Al, and Au with ${}_7$N ions.

I. INTRODUCTION

Consider an atomic projectile moving with velocity v_1 in a dense medium. For the moment, let the medium be the valence electrons of a solid of density $3/4\pi r_s^3 a_0^3$, where r_s is the radius of the average volume occupied by each electron in units of $a_0 = \hbar^2/me^2 = 0.529$ Å. Values are typically $r_s \simeq 2$, but they range from $r_s = 1.5$(Au, W) to $r_s = 5.88$(Cs). Two things happen. First, the onrushing electrons eject bound electrons from the projectile. This leaves the projectile with an ionic charge $Q_1 = (Z_1 - N)e$, where Z_1 is the atomic number of the projectile and N the number of electrons still bound to the projectile nucleus. The degree of ionization or, for short, the ionization $q = (Z_1 - N)/Z_1$ is v_1 dependent, of course, for the higher the relative velocity v_r between the ion and the electrons in the medium the more electrons are stripped from the ion. The ionization cross section for bound electrons drops to zero when the ionization energy exceeds the kinetic energy of the electrons hurled at the ion. It follows that $N(v_r)$ comprises electrons which in steady state are bound so tightly to the ion that—in the language of the correspondence principle—their orbital velocities are larger than v_r. Second, the projectile through Coulomb interaction transfers momentum p to electrons of the medium. Leaving screening in the medium aside, this momentum transfer, $\sim 2Z_1^*(b)e^2/bv_1$, is proportional to an "effective charge" $Z_1^*(b)e$ of the ion as seen by an electron of charge $-e$ at impact parameter b. The energy loss of the projectile in this encounter $\sim 2Z_1^{*2}(b)e^4/mv_1^2 b^2$, when integrated over electrons with all possible impact parameters b, yields the rate of energy loss of the ion in the medium. That is, the stopping power, $S = -dE/dx$, of the medium for an ion is, in this first approximation, proportional to a mean-square effective-ion charge.

The difficulty arises in defining the effective charge. Obviously, electrons that approach the ion with impact parameters b that are larger than the ion radius Λ see the ion as a moving point charge Q_1 irrespective of the internal ion structure. But if $b < \Lambda$, medium electrons penetrate the screening cloud of N bound electrons. The energy transfer is then governed by an effective charge $Z_1^*(b)e$ that is larger than Q_1. Thus on general grounds, close collisions should make the mean-square effective stopping-power charge $(Z_1^* e)^2$ larger than Q_1^2.

The description of $Z_1^* e$ has eluded satisfactory treatments and the entire concept of an effective charge has been in question. It is the purpose of this paper to derive the effective stopping-power charge of a projectile of given ionization $q(v_r)$. The approach is to remain so transparent throughout the development that the salient parameters that characterize the ion are always kept in sight. This conveys an intimate physical understanding and still yields reliable formulas for the interaction of heavy ions with condensed matter, if judged by new precision measurements in this laboratory designed to test some of the details of the work presented here.

There has been a vigorous resurgence of interest in the problem of effective stopping-power ion charges recently, spurred by the needs of fusion research, space exploration, and materials development. This has stimulated large-scale compilations of literature data. They reveal striking disagreements in the experimental evidence of what "best" stopping power data might be. The theoretical tools available for clarification were restricted and blunt. All this has given focus to new measurements. An understanding of the source of earlier discrepancies is emerging. The data begin to converge toward a consistent base by which to gauge new theoretical developments.

The early history of the effective-charge concept was reviewed by Betz,[1] and complications of early data were published by Ziegler *et al.*[2] and by Yarlagadda *et al.*[3] The stripping of ions to the charge $Q_1(v_r)$ has been formulated in the statistical approximation[4] and applied to new data.[5,6] A systematic attack on the effective-charge problem was launched recently in this laboratory.[7] The present report delineates the theoretical results of this effort.

Section II states the technical steps encountered in the calculation of the stopping power of an ion in an electron gas, and prepares the ground by formulating, through variational methods, a statistical model of the ion that is analytically suitable for our (BK) needs. It resembles closely the Lenz-Jensen (LJ) model[8] in important respects. The LJ model, also based on the variational principle, was conceived as an improvement over the Thomas-Fermi (TF) model. Section III uses the BK model to calculate in a dielectric-response approximation the stopping power of an electron gas for an ion of ionization q, with new results. The discussion, in Sec. IV, makes contact with experiment and summarizes new trends. Atomic units $e=\hbar=m=1$ are used hereinafter, except when stated otherwise.

II. STATEMENT OF PROBLEM

The charge density of a projectile of nuclear charge Z_1 moving with N bound electrons at velocity $\vec{v}_1$ in a medium is given by

$$\rho_{ne}(\vec{r},t)=Z_1\delta_n(\vec{r}-\vec{v}_1t)-\rho_e(\vec{r}-\vec{v}_1t) \quad , \tag{1}$$

where $Z_1\delta_n$ and ρ_e denote the nuclear and electronic charge densities of the ion, subject to the condition $\int\rho_e\,d\vec{r}=N$. The stopping power $S=-dE/dx$ of the medium for this ion

$$S=\frac{\vec{v}_1}{v_1}\int\rho_{ne}(\vec{r},t)\vec{E}(\vec{r},t)d\vec{r} \tag{2}$$

is determined by the electric field $\vec{E}$ which acts on the ion in the medium. In terms of the wave vector $\vec{k}$ and frequency ω in Fourier space, the field

$$\vec{E}(k,\omega)=-i\vec{k}\phi(k,\omega) \quad , \tag{3}$$

set up by the potential $\phi(k,\omega)$ according to

$$k^2\epsilon(k,\omega)\phi(k,\omega)=4\pi\rho_{ne}(k)\delta(\omega-\vec{k}\vec{v}_1) \tag{4}$$

is the response of the medium with dielectric response function $\epsilon(k,\omega)$ to the moving ion. The right-hand side of Eq. (4) is the Fourier transform of $\rho_{ne}(\vec{r},t)$, since

$$\rho_{ne}(k,\omega)=2\pi\rho_{ne}(k)\delta(\omega-\vec{k}\cdot\vec{v}) \quad , \tag{5}$$

where $\rho_{ne}(k)$ is the Fourier transform of $\rho_{ne}(\vec{r})$ in the rest frame of the ion. Inserting Eqs. (3) to (5) into Eq. (2) yields

$$S=\frac{2}{\pi v_1^2}\int\frac{dk}{k}|\rho_{ne}(k)|^2 \times\int_0^{kv_1}d\omega\,\omega\,\mathrm{Im}\left[\frac{-1}{\epsilon(k,\omega)}\right] \quad . \tag{6}$$

For spherically symmetric charge distributions $\rho_{ne}(k)$ is real, i.e., $|\rho_{ne}(k)|^2=\rho_{ne}^2(k)$, and Eq. (6) reduces to a formula given by Ferrell and Ritchie.[9]

We wish to use Eq. (6) for the derivation of the effective stopping-power charge of the ion. To maintain contact with parameters that describe the state of the atoms, we seek forms of ρ_e and ϵ that permit analysis without exclusive recourse to numerical methods. To this end, we chose for $\rho_e(R)$ the simple form

$$\rho_e(R)=\frac{N}{4\pi\Lambda^3}\frac{\Lambda}{R}e^{-R/\Lambda} \quad . \tag{7}$$

We treat the screening length Λ as a variational parameter. The internal energy of the ion

$$E=E_{ne}+\lambda E_{ee}+E_{\mathrm{kin}} \quad , \tag{8}$$

with potential energy from electron-nucleus interactions

$$E_{ne}=-Z_1\int d\vec{R}\,\rho_e(R)R^{-1}=-Z_1N/\Lambda \quad , \tag{9}$$

electron-electron interactions

$$E_{ee}=\frac{1}{2}\int d\vec{R}\int d\vec{R}'\frac{\rho_e(R)\rho_e(R')}{|\vec{R}-\vec{R}'|}=N^2/4\Lambda \quad , \tag{10}$$

weighed for correlation in an average manner by the variational parameter λ, and with the electron kinetic energy

$$E_{\mathrm{kin}}=\frac{3}{10}\int d\vec{R}\,\rho_e(R)[3\pi^2\rho_e(R)]^{2/3}=aN^{5/3}/\Lambda^2 \quad , \tag{11}$$

where $a=\frac{1}{2}(\frac{3}{4}\pi)^{2/3}(\frac{3}{5})^{5/3}\Gamma(\frac{4}{3})=0.240$. The energy fulfills the conditions

$$\frac{\partial E}{\partial\Lambda}=0,\quad \left.\frac{\partial E}{\partial N}\right|_{N=Z_1}=0 \quad . \tag{12}$$

One obtains

$$E=-\frac{Z_1^{7/3}}{4a}\left[\frac{N}{Z_1}\right]^{1/3}\left[1-\frac{\lambda}{4}\frac{N}{Z_1}\right]^2 \quad . \tag{13}$$

The screening radius Λ calculated according to

$$\frac{1}{\Lambda}=\frac{1}{N}\int d\vec{R}\,R^{-1}\rho_e(R) \tag{14}$$

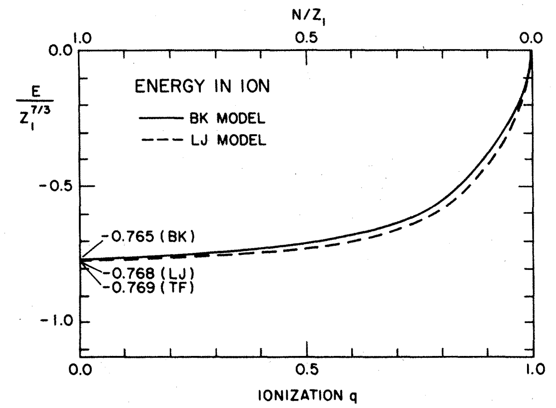

FIG. 1. Comparison of energies in the BK statistical ion model, Eq. (13), with those in the LJ model at all degrees of ionization q. In neutral atoms, $q=0$, the energies in both models are very close to those in the TF model.

becomes

$$\Lambda=\frac{2a(N/Z_1)^{2/3}}{Z_1^{1/3}[1-\frac{1}{4}\lambda(N/Z_1)]}a_0 \tag{15}$$

with $\lambda=4/7$ and $a_0=1$ a.u. $=0.529\times10^{-8}$ cm.

Under these constraints, the BK model performs remarkably well when compared with results based on the LJ model. For example, as shown in Fig. 1, the energies for different ionizations $q=1-N/Z_1$ resemble each other closely. In neutral atoms, $q=0$, the energies of the BK, LJ, and TF models virtually coincide. Similarly the BK and LJ screening radii displayed in Fig. 2 are numerically close for all q.

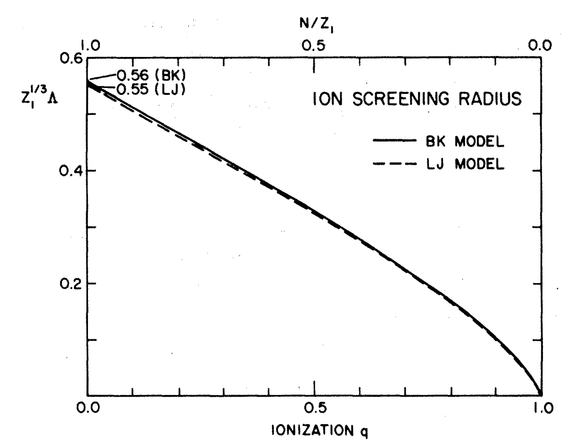

FIG. 2. Comparison of screening lengths, Eq. (14), in BK statistical model and those in the LJ model at all degrees of ionization q.

Equation (7) gives immediately

$$\rho_{ne}(k)=Z_1\frac{q+(k\Lambda)^2}{1+(k\Lambda)^2} \tag{16}$$

to be inserted in Eq. (6). With suitable approximations for $\mathrm{Im}(-1/\epsilon)$ we can then calculate the fractional effective charge of an ion of given ionization q as

$$\zeta\equiv\frac{Z_1^*}{Z_1}=\left[\frac{S_q}{S_{q=1}}\right]^{1/2}, \tag{17}$$

where $S_{q=1}$ is the stopping power for the bare nucleus.

III. EFFECTIVE STOPPING-POWER CHARGE

At high velocities where the ion can excite plasmons in the medium, we employ the plasmon-pole approximation of the dielectric function[10]

$$\epsilon(k,\omega)=1+\frac{\omega_p^2}{\omega_g^2+\beta^2k^2+k^4/4-\omega(\omega+i\gamma)} . \tag{18}$$

The plasmon energy $\omega_p=3^{1/2}r_s^{-3/2}$ and the effective band-gap energy ω_g in semiconductors and insulators give a collective resonance frequency $\Omega_0=(\omega_p^2+\omega_g^2)^{1/2}$.[11,12] Dispersion is included through the term containing $\beta^2=\frac{3}{5}k_F^2$ where $k_F=1.919r_s^{-1}$ is the Fermi momentum. Contributions from single-particle excitations are accounted for through the square of the kinetic energy $k^2/2$ of a free electron of momentum k. The small constant γ represents damping processes. It follows that in the limit $\gamma\to0$

$$\mathrm{Im}\left[-\frac{1}{\epsilon(k,\omega)}\right]=\frac{\pi\omega_p^2}{2A}\delta(\omega-A) , \tag{19}$$

where $A^2=\Omega_0^2+\beta^2k^2+k^4/4$.

The upper and lower integration limits in k are the maximum and minimum momentum transfers k_+ and k_- to target electrons

$$k_\pm=\{2(v_1^2-\beta^2)\pm2[(v_1^2-\beta^2)^2-\Omega_0^2]^{1/2}\}^{1/2} , \tag{20}$$

which gives a threshold for v_1, viz.,

$$v_{\text{thr}}=(\beta^2+\Omega_0^2)^{1/2} , \tag{21}$$

below which plasmon contributions subside. Then,

$$S=\frac{\omega_p^2}{2v_1^2}\int_{k_-^2}^{k_+^2}\frac{dy}{y}\rho_{ne}^2(y) . \tag{22}$$

Equations (17) and (22) yield

$$\zeta^2=q^2+(1-q)\left[(1-q)\left[\frac{1}{(k_+\Lambda)^2+1}-\frac{1}{(k_-\Lambda)^2+1}\right]+(1+q)\ln\frac{(k_+\Lambda)^2+1}{(k_-\Lambda)^2+1}\right](2L)^{-1}, \tag{23}$$

where

$$L = \ln \frac{k_+}{k_-} \qquad (24)$$

At high velocities $v_1 >> v_{\text{thr}}$, Eq. (20) gives $k_+ \simeq 2v_1$ and $k_- \simeq \Omega_p / v_1$ so that one retrieves the Bethe formula for the stopping number L,

$$L \simeq \ln 2v_1^2/\Omega_p \ . \qquad (25)$$

In this limit, Eq. (23) becomes

$$\zeta^2 \simeq \tfrac{1}{2} + \tfrac{1}{2} q^2, \quad v_1 >> v_{\text{thr}} \ . \qquad (26)$$

Equation (26) accounts for the well-known fact that at high velocities approximately one-half of all energy losses are suffered in distant collisions and the rest in close collisions. Neutral projectiles $q = 0$ have an effective stopping-power charge $Z_1^* \simeq 0.7 Z_1$, i.e., the stopping power of the medium for neutral atoms is approximately one-half as large as the stopping power for the bare nuclei.[13] Contributions from target core electrons must be included for the assessment of the total random stopping power of the medium.

At low velocities $v_1 < v_F$, we describe $\text{Im}(-1/\epsilon)$ through a screening constant $k_D = (4k_F/\pi)^{1/2}$ as[14]

$$\text{Im}\left(\frac{1}{\epsilon(k,\omega)}\right) \simeq \begin{cases} \dfrac{2k\omega}{(k^2+k_D^2)^2} & \text{for } k \leq 2k_F \\ 0 \ \text{otherwise} \ . \end{cases} \qquad (27)$$

Integration of Eq. (5) yields

$$S = \frac{4}{3\pi} v_1 \int_0^{2k_F} dk \, \frac{k^3}{(k^2+k_D^2)^2} \rho_{ne}^2(k)$$
$$= \frac{2Z_1^2}{3\pi} \zeta^2 v_1 I(\pi k_F) \ , \qquad (28)$$

where in terms of the functions

$$I(z) = \ln(1+z) - \frac{z}{1+z} \ , \qquad (29)$$

$$K(k_F,\Lambda) = \frac{1}{1-(k_D\Lambda)^2} \{\ln[1+(2k_F\Lambda)^2] - k_D^2\Lambda^2 \ln(1+\pi k_F)\} \ , \qquad (30)$$

the effective charge fraction, Eq. (17), is given by

$$\zeta^2 = \left(1 - \frac{N}{Z_1}\frac{1}{1-(k_D\Lambda)^2}\right)^2 + \left(\frac{N}{Z_1}\right)^2 \frac{1}{[1-(k_D\Lambda)^2]^2}\frac{I(4k_F^2\Lambda^2)}{I(\pi k_F)} + 2\frac{N}{Z_1}\frac{1}{1-(k_D\Lambda)^2}\left(1-\frac{N}{Z_1}\frac{1}{1-(k_D\Lambda)^2}\right)\frac{K(k_F,\Lambda)}{I(\pi k_F)} \qquad (31)$$

The expression $I(\pi k_F)$, Eq. (29), accounts for screening; it is already contained in the paper by Ferrell and Ritchie.[9] Since $2k_F$ is the largest momentum transfer inside the Fermi sea, the parameter $2k_f\Lambda \sim 4\Lambda r_s^{-1}$ is a measure of the ion radius Λ relative to the spacing, expressed through r_s, between the electrons in the medium. Treating $2k_F\Lambda$ as small, we expand ζ and obtain (see Appendix)

$$\zeta \simeq q + C(k_F)(1-q)\ln[1+(2k_F\Lambda)^2] \ , \qquad (32)$$

where

$$C(k_F) = \frac{\pi k_F}{(1+\pi k_F) I(\pi k_F)} - \frac{2}{\pi k_F} \qquad (33)$$

depends only weakly on k_F (or r_s) as demonstrated in Table I. Figure 3 exhibits Eq. (32) in the form

$$\frac{\zeta - q}{1-q} = C(k_F)\ln[1+(2k_F\Lambda)^2] \ , \qquad (34)$$

drawn as curves for different r_s values. The figure compares the curves with numerical values of Eq. (31), shown as points, calculated for a large variety of projectiles ($_6$C, $_7$N, $_{18}$Ar, $_{53}$I, $_{92}$U), with set ionization values $q = 0.197$, 0.447, and 0.852 for each, in targets of different r_s values as indicated in the legend.

Figure 3 illustrates that Eq. (32) is a nearly universal function for all ion-target combinations. We have verified the utility of Eq. (18) in our context: Equation (32) agrees closely with Eq. (6) after double numerical integration when based on the complete Lindhard dielectric function.

TABLE I. Values of $C(k_F)$, Eq. (32), for r_s values and k_F values resembling those in the valence electron gas of various target materials. The Fermi momentum k_F and the one-electron radius r_s, in atomic units, are related as $k_F = (9\pi/4)^{1/3} r_s^{-1} = 1.919 r_s^{-1}$.

~ material	r_s	k_F	$C(k_F)$
Au	1.49	1.29	0.49
C	1.66	1.16	0.50
Al	2.12	0.91	0.52
Cs	5.88	0.33	0.59

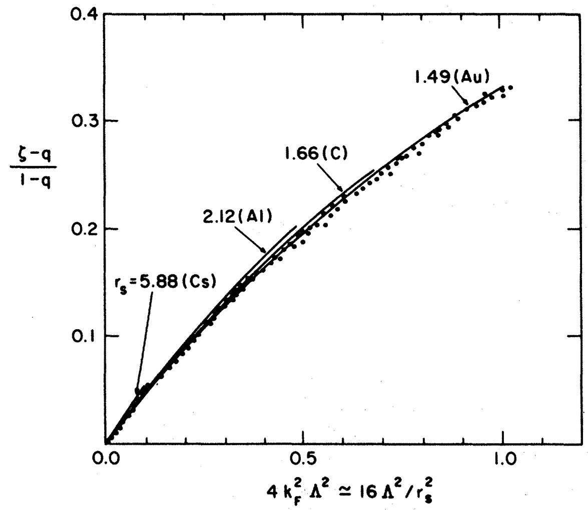

FIG. 3. Demonstration of the equivalence of the approximate formula, Eq. (32), for ζ with the full expression Eq. (31) over the relevant validity range of $2k_F\Lambda \sim 4\Lambda r_s^{-1}$ in the form of Eq. (34). The solid curves cover the validity range of Eq. (32) for a statistical description of the ions ($Z_1 > 4$). The points are numerical values of Eq. (31) for projectiles $Z_1 = 6, 7, 18, 53, 92$ each at ionizations $q = 0.197$, 0.447, and 0.852 in targets with $r_s = 1.49$ (Au), 1.66(C), 2.12(Al), 5.88(Cs).

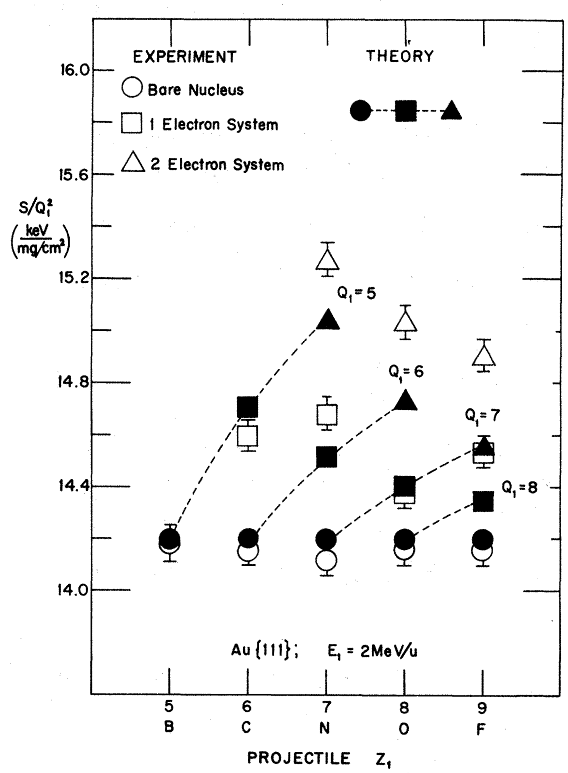

FIG. 4. Comparisons of the predictions of Eq. (35) with stopping-power data as deduced by a "$\frac{1}{10}$ height" criterion (Ref. 15) for fixed ionic charges Q_1 in Au{111} crystal channels.

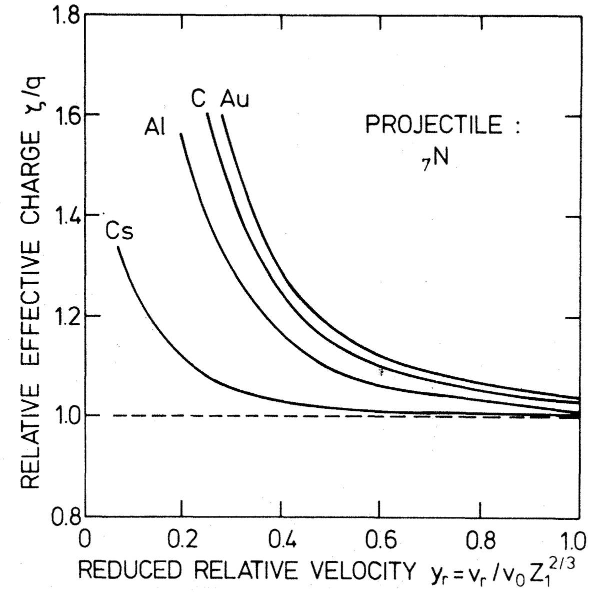

FIG. 5. The ratio ζ/q for $_7$N projectiles in various targets, as a function of the relative projectile velocity variable y_r; according to Eq. (32) with $q(y_r)$ as given in Table II.

IV. DISCUSSION

An important consequence of effective-charge theory as developed here is contained in data on the stopping power of {111} channels in single Au crystals for projectiles of fixed ion charges $Q_1 = (Z_1 - N)$ but different Z_1 for $N = 0, 1, 2$.[15,16] The result is that the stopping power S for given ion charge Q_1 and ion velocity v_1 increases with Z_1 (Fig. 4). The experi-

TABLE II. The mean ionization $q(v_r)$ of ions, atomic number Z_1, moving with velocity v_r, or reduced velocity $y_r = v_r/Z_1^{2/3} v_0$, relative to the electrons in an electron gas, as calculated by a velocity-stripping criterion for the Thomas-Fermi model of the ion (Ref. 4). The table is calculated for the stripping parameter $b = 1.33$. For other b values at given q, replace y_r by $y_r' = (b/1.33)y_r$. Expressions for $v_r(v_1, r_s)$ are given in Refs. 5 and 6.

y_r	q	y_r	q
0.024	0.012	0.771	0.515
0.053	0.034	0.862	0.554
0.098	0.070	0.975	0.598
0.136	0.101	1.116	0.645
0.200	0.155	1.302	0.698
0.251	0.197	1.556	0.756
0.330	0.256	1.928	0.819
0.396	0.305	2.208	0.852
0.456	0.344	2.567	0.886
0.532	0.391	3.098	0.919
0.633	0.447	3.977	0.952
0.697	0.480	5.932	0.981

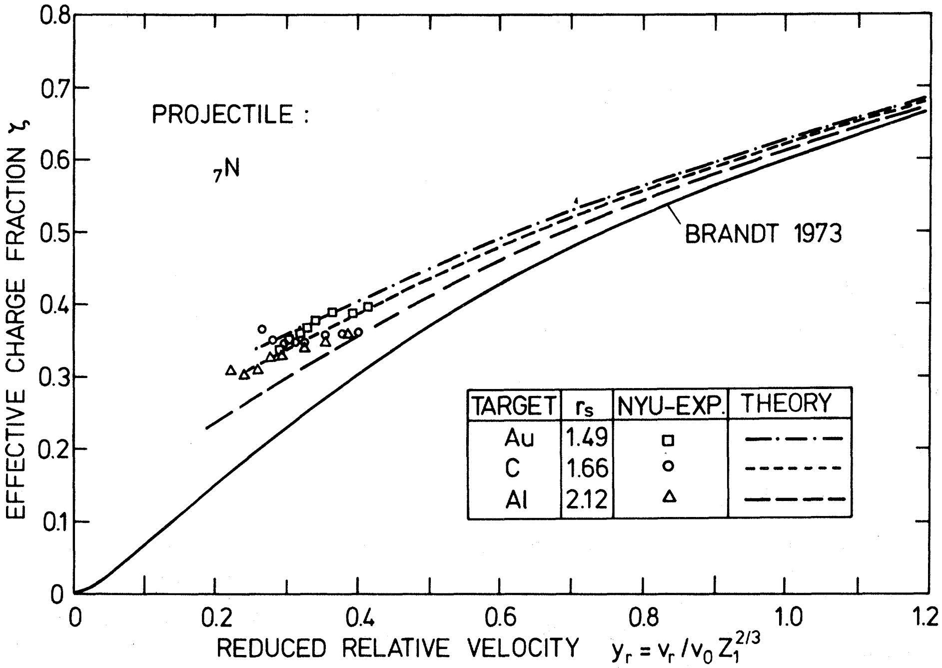

FIG. 6. Comparison of theoretical, Eq. (32) with Table II, and experimental (Ref. 17) stopping powers in terms of the effective-charge fraction ζ of $_7$N ions in Au, C, and Al.

ments were performed so that $2k_F\Lambda << 1$ and reported as $S/Q_1^2 = (\zeta/q)^2 S_0$, where S_0 is a constant. Already the low-velocity approximation Eq. (32), when written in the form

$$\left(\frac{\zeta}{q}\right)^2 = \left[1 + C(k_F)\left(\frac{Z_1}{Q_1} - 1\right)\ln[1-(2k_F\Lambda)^2]\right]^2 , \quad (35)$$

predicts the trends of these experiments for $r_s = 1.49$ (Au) and $C(1.919/r_s) = 0.49$; one sets for the K-shell electrons $\Lambda_{1K} = a_0/Z_{1k}$ with $Z_{1k} = Z_1$ when $N=1$ and $Z_{1K} = Z_1 - 0.3$ when $N=2$.

New precision stopping-power experiments were performed with a variety of projectiles and target materials to test Eq. (32).[17] The magnitude of the predicted effect $\zeta/q > 1$ is demonstrated in Fig. 5 as a function of the relative ion-velocity variable $y_r = v_r/Z_1^{2/3} v_0$, where $v_0 = e^2/\hbar = 2.18 \times 10^8$ cm/sec. The curves in Fig. 5, when multiplied with $q(y_r)$ as tabulated in Table II, appear as curves of dashes in Fig. 6.[18] They converge toward the solid curve for large Z_1, r_s, or v_r as $2k_F\Lambda \to 0$. The solid curve represents $q(y_r)$. It is well known to give an excellent description of $Z_1^{-1}(S/S_p)^{1/2}$ for heavy ions.[3,6] The new experimental data for the light ion $_7$N in various metals indicated by the open symbols support the effective-charge theory as presented here.

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APPENDIX: LOW-VELOCITY EFFECTIVE-CHARGE APPROXIMATION

We derive Eq. (32) from Eq. (31) by introducing the variable $\Gamma \equiv \ln[1+(2k_F\Lambda)^2] = \ln[1+(\pi k_D \Lambda/2)^2]$. In the limit of small $2k_F\Lambda$ or $\Gamma \to 0$, we expand Eq. (31) as

$$\zeta = \zeta_{\Gamma=0} + \Gamma \left.\frac{d\zeta}{d\Gamma}\right|_{\Gamma=0} . \quad (A1)$$

From Eq. (31) follows $\zeta_{\Gamma=0} = q$ in the limit $\Gamma = 0$ where $I = K = 0$. Differentiation of Eq. (31) at $\Gamma = 0$ leads to

$$\left.\frac{d\zeta}{d\Gamma}\right|_{\Gamma=0} = (1-q)\left[\frac{1}{I(\pi k_F)}\left(1 - \frac{1}{\pi k_F}\ln(1+\pi k_F) - \frac{1}{\pi k_F}\right)\right] \quad (A2)$$

which, in terms of I as given in Eq. (29), can be expressed as

$$\left.\frac{d\zeta}{d\Gamma}\right|_{\Gamma=0} = (1-q)C(k_F) , \quad (A3)$$

where $C(k_F)$ is given by Eq. (33). Collecting Eqs. (A1), (A2), and (A3) yields Eq. (32).

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[1]H. D. Betz, Rev. Mod. Phys. 44, 74 (1972).

[2]J. F. Ziegler, in *Helium: Stopping Powers and Ranges in All Elemental Matter,* The Stopping and Ranges of Ions in Matter, Vol. 4, edited by J. F. Ziegler (Pergamon, New York, 1977), and other volumes in this series.

[3]B. S. Yarlagadda, J. E. Robinson, and W. Brandt, Phys. Rev. B 17, 3473 (1978).

[4]W. Brandt, in *Atomic Collisions in Solids,* edited by S. Daty, B. R. Appleton, and C. D. Moak (Plenum, New York, 1975), Vol. 1, p. 261.

[5]S. Kreussler, C. Varelas, and W. Brandt, Phys. Rev. B 23, 82 (1981).

[6]A. Mann and W. Brandt, Phys. Rev. B 24, 4999 (1981).

[7]For preliminary reports see M. Kitagawa and W. Brandt, Bull. Am. Phys. Soc. 26, 602 (1981); F. Schulz and W. Brandt, *ibid.* 26, 602 (1981); J. Shchuchinsky, F. Schulz, and W. Brandt, *ibid.* 26, 1308 (1981).

[8]P. Gombás, *Die Statistische Theorie des Atoms und ihre Anwendungen,* (Springer, Vienna, 1949); W. Lenz, Z. Phys. 77, 713 (1932); H. Jensen, *ibid.* 77, 722 (1932).

[9]T. L. Ferrell and R. H. Ritchie, Phys. Rev. B 16, 115 (1977).

[10]P. M. Echenique, R. H. Ritchie, and W. Brandt, Phys. Rev. B 20, 2567 (1979).

[11]W. Brandt and J. Reinheimer, Phys. Rev. B 2, 3104 (1970).

[12]W. Brandt and R. H. Ritchie, in *Physical Mechanisms in Radiation Biology,* proceedings of a conference held at Airlie, Virginia, 1972, edited by R. D. Cooper and R. W. Wood (U.S. AEC, Technical Information Center, Oak Ridge, Tennessee, 1974), p. 20.

[13]In the limit of small Z_1, Allison and collaborators [Phys. Rev. 127, 792 (1962); 135, A335 (1964)] measured in elegant experiments the stopping powers of H_2 gas for neutral and charged ${}_1H$ and ${}_2He$ projectiles from which ζ/q values of light projectiles in gases can be deduced.

[14]M. G. Calkin and P. J. Nicholson, Rev. Mod. Phys. 39, 361 (1967); M. Kitagawa and Y. H. Ohtsuki, Phys. Rev. B 9, 4719 (1974).

[15]S. Datz, J. Gomez dei Campo, P. F. Dittner, P. D. Miller, and J. A. Biggerstaff, Phys. Rev. Lett. 38, 1145 (1977).

[16]J. A. Golovchenko, A. N. Goland, J. S. Rosner, C. E. Thorn, H. E. Wegner, H. Kreudsen, and C. D. Moak, Phys. Rev. B 23, 957 (1981).

[17]F. Schulz and W. Brandt (unpublished).

[18]Such trends were first uncovered in the course of a literature compilation (Ref. 3) with great uncertainties. The theoretical developments presented here and recent analysis of low-velocity proton data (Ref. 6) now leave no room for an effective proton charge in solids less than one, as speculated in Ref. 3 on heuristic grounds. This conclusion is consistent with the contention (Ref. 4) that protons in metals have no stable neutral-bound state at any velocity.