Quantized Hall effect at low temperatures

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We have studied the quantized Hall effect of the two-dimensional electron gas in GaAs-$Al_xGa_{1-x}As$ heterojunctions at low temperatures to 50 mK. We observe in the small-current and low-temperature limit sharp steps connecting the quantized Hall resistance plateaus. The diagonal resistivity ρ_{xx} decreases with decreasing T at the Shubnikov–de Haas peaks, as well as at the dips, and is vanishingly small at magnetic fields above 40 kG.

The quantized Hall effect is unique to degenerate two-dimensional (2D) electron systems in the presence of a strong perpendicular magnetic field B. It has been observed in the transport coefficients of electron inversion layers in Si-MOSFET's (metal-oxide-semiconductor field-effect transistors[1]) and in GaAs-$Al_xGa_{1-x}As$ heterojunctions[2] at temperatures $T=4.2$ to 1.2 K. In the latter, the effect is observed as a series of flat plateaus in the Hall resistance (ρ_{xy}) plotted as a function of B. Concomitantly, the diagonal resistance ρ_{xx} along the current path becomes vanishingly small in these Hall plateau regions. These regions correspond to the situation that all the extended states in a finite number of Landau levels are completely filled and the Fermi energy E_F is pinned in the gap between the extended states of two Landau levels. The quantized Hall resistance is given by

$$\rho_{xy} = h/ie^2 \ , \qquad (1)$$

where h and e are, respectively, the Planck constant and electronic charge, and the quantum number i is the number of completely filled Landau levels. Despite electron-electron interactions and random impurity potentials, Eqs. (1) is accurate for samples with different electron and impurity concentrations.[1,2]

In this Communication we report several novel phenomena which have appeared below 1 K in the quantized Hall effect in GaAs-$Al_xGa_{1-x}As$ heterojunctions and which will clarify our understanding of Eq. (1) and quantum states of the 2D electron gas. For lower Landau levels above the 40-kG field ρ_{xx} is thermally activated at the Shubnikov–de Haas (SdH) peaks and extrapolates to 0 at $T=0$. Simultaneously, at 50 mK the transitions between quantized values of Hall resistance are sharp as a function of B. Our interpretation is that over 95% of Landau level states are localized. Therefore, one-electron theories treating the number of localized states as a small perturbation parameter cannot explain Eq. (1) convincingly. The narrow band of extended states is explained qualitatively by the sharp percolation level of classical 2D percolation theory. For higher Landau levels, $n \geq 4$, ρ_{xx} displays $\ln T$ dependence at the SdH peaks, which can be explained by Coulomb interactions in the high-B limit as treated recently by Girvin *et al.*[3]

The GaAs-$Al_xGa_{1-x}As$ heterostructures were described in Ref. 2. The specimens were made into standard "Hall bridges," each having six side-arms symmetrically placed to facilitate measurements of ρ_{xx} and ρ_{xy}. The two samples studied had mobilities $\mu=8.6$ and 4.7 m^2/Vs and constant densities $N=4.0$ and $3.2\times10^{15}/m^2$, respectively. The samples were attached by Apiezon N grease on a silver plate, which was in direct metallic contact with the mixing chamber of a dilution refrigerator capable of reaching a 10-mK temperature at 100 kG. Small measuring currents down to 10^{-10} A were used with lock-in detection techniques at 11 Hz. We were able to take data at equilibrium temperatures only to 50 mK.

Figure 1 shows ρ_{xx} and ρ_{xy} plotted against B at 50 mK for the higher mobility sample. The numbers and the arrows above the peaks in ρ_{xx} refer to the quantum number and spin polarization of the Landau levels involved. The spin-splitting seen in the $n=1$ and 2 (the small structure at $B\approx32$ kG) levels results from exchange enhancement of the effective electron g factor.[4] We note that ρ_{xy} is in the Ohmic regime at the low-current densities indicated. However, non-Ohmic behavior was observed in ρ_{xx} for the $n=1$ and 2 Landau levels at 2.6×10^{-6} A/m which was used to obtain sufficiently large signals in our measurements.

The most striking features of our data are the exceedingly wide Hall plateaus in ρ_{xy} and the vanishing of ρ_{xx} at the SdH peaks, as well as at the dips, for $B \gtrsim 40$ kG. The transition from one Hall plateau to the next takes place in an extremely narrow range of B corresponding approximately to a half-filled Landau level at E_F. However, the values of B at the midpoint of the resulting steps are somewhat lower than those at the ρ_{xx} peaks.[5] We studied in detail the T dependence of the width of the $\sim6.5\,k\Omega$ ($\rho_{xy}=h/4e^2$) plateau and also the step at $n=1\uparrow$. To an accuracy better than 1% of the quantized value given by Eq. (1), the plateau width reaches 93% of the

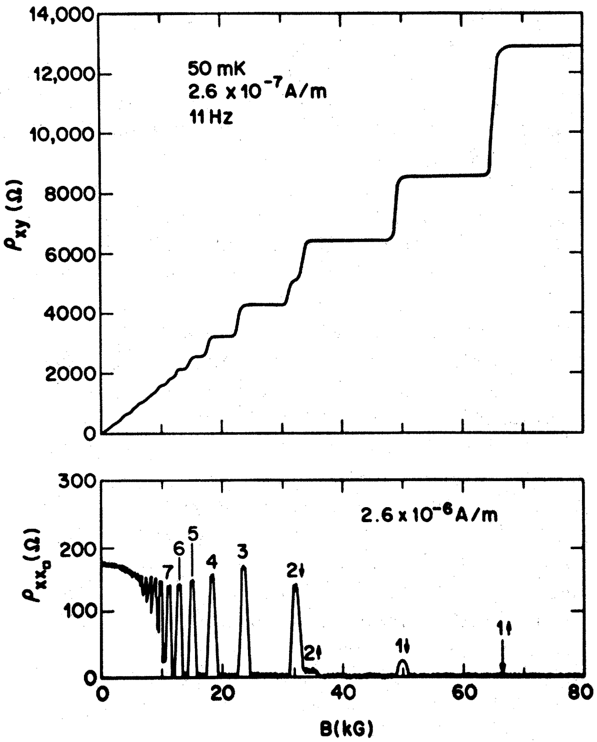

FIG. 1. ρ_{xx} and ρ_{xy} as a function of B. The numbers and the arrows above the ρ_{xx} maxima refer to the Landau quantum number and the spin polarization of the levels.

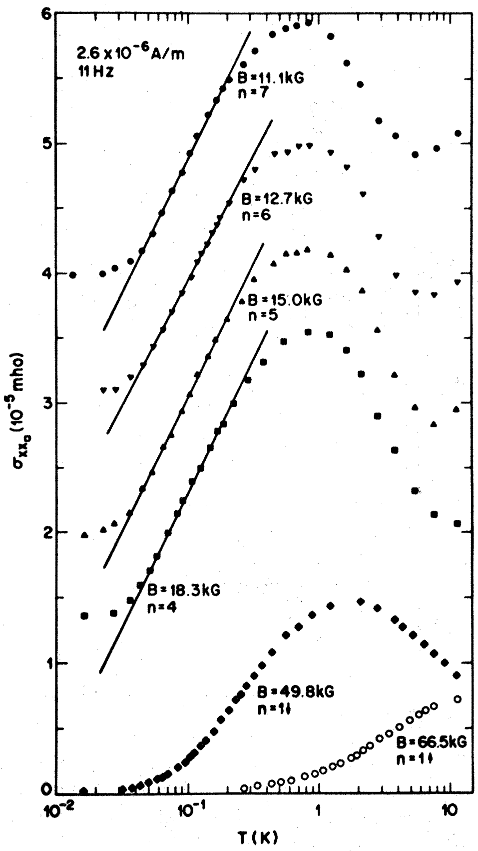

FIG. 2. ρ_{xx} at half-filled Landau levels as a function of T.

maximum possible width as determined from the midpoints of the two neighboring steps at $T=50$ mK and, by extrapolation, approaches at least ~97% as $T\rightarrow 0$. The transition at $n=1\uparrow$ occurs in ~3% of the plateau width at $T=50$ mK and possibly extrapolates to infinitely sharp at $T=0$.

At $T\gtrsim 1.2$ K, ρ_{xx} is known to vanish in the Hall plateau regions,[6] while at the SdH peaks ρ_{xx} increases with decreasing T. We find at lower T that ρ_{xx} decreases with decreasing T even at the SdH peaks. This is particularly clear for the $n=1\uparrow$ peak, which decreases below our resolution at 50 mK. Previously, Kawaji and Wakabayashi[7] reported the vanishing of the lowest-spin- and valley-split peak in Si-MOSFET's. However, the quantized plateau was not observed in their experiment.

Figure 2 shows the T dependence of the diagonal conductivity σ_{xx} at fixed values of B at several ρ_{xx} peaks. It is obtained from ρ_{xx} and ρ_{xy} through $\sigma_{xx}=\rho_{xx}/(\rho_{xx}^2+\rho_{xy}^2)$.[5] In general, σ_{xx} for all Landau levels is smaller than that predicted by Ando and Uemura[8] for short-range scattering. Except for the $n=1\uparrow$ level, the peak value of σ_{xx} increases with decreasing T in the interval from ~10 to 1 K as expected for T higher than the Dingle scattering temperature, while the small increase for $T\gtrsim 10$ K is not understood at present. Below 50 mK, σ_{xx} shows saturation probably caused by electron heating. In the range of T from ~50 to ~300 mK, the data for the quantum levels $n\geqslant 4$ from both samples show a logarithmic dependence of T given by $\Delta\sigma_{xx}=(0.9\pm 0.05)\times 10^{-5}\ln T$, with units in siemens and degrees Kelvin. This result is similar to that observed in Si-MOSFET's at $B=0$ in the weak localization limit.[9] It can be explained by the effect of Coulomb interaction in 2D systems in the high-B limit, as recently treated by Girvin *et al.*[3]

The stronger-T dependence for σ_{xx} peaks of the lower Landau levels is more clearly demonstrated in Fig. 3 where the data of Fig. 2 is replotted on logarithmic σ_{xx} and inverse T scales. Both the $n=1\downarrow$ and $1\uparrow$ levels show thermally activated σ_{xx}. For our lower mobility sample this activated behavior was even more evident at the $n=0\downarrow$ peak, observed at $B=87.5$ kG with an activation energy of ~2.2 K.

Several recent papers discussed theoretical models for the quantized Hall effect. The model by Baraff and Tsui[10] explains the effect in GaAs-$Al_xGa_{1-x}As$ heterojunctions observed at 4.2 K. In this model the donor impurities in $Al_xGa_{1-x}As$ act as an electron reservoir to keep the relative motion of E_F continuous through the energy gaps between Landau levels.

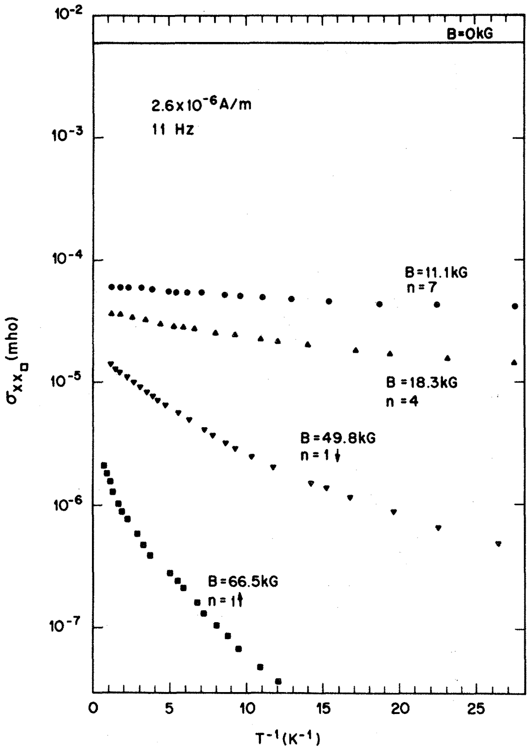

FIG. 3. Data from Fig. 2 plotted on logarithmic σ_{xx} and inverse temperature scales. The activation energy for σ_{xx} is 0.3 K for the $n=1\uparrow$ level below ~0.3 K.

This model, however, does not have strong-T dependent σ_{xx} and can explain only about 30% of our plateau widths. Other one-electron theories[11–15] invoke localized states in the tails of the Landau levels. Prange's model calculations[12] demonstrated explicitly that the Hall current lost to the localized states is compensated by an extra current carried by the rest of the extended states left in the Landau level. Consequently, Eq. (1) holds whenever all the extended states in an integral number of Landau levels are completely filled and when E_F is moved through the localized states. In our experiments, the observed plateau width indicates a localization of more than 95% of the states in the $n=1\uparrow$ and $1\downarrow$ Landau levels. To preserve Eq. (1) with this large fraction of states localized becomes conceptually difficult with Prange's model. However, Laughlin's gauge invariance arguments,[13] being independent of the number of localized states in the Landau level, should hold even if only an infinitesimally small fraction of states are extended.

Arguments based on classical percolation in 2D have also been used, by Tsui and Allen,[15] to show that quantized Hall plateaus result from the presence of localized states at E_F. Very recently, Iordanski[16] and also Kazarinov and Luryi[17] pointed out that in the high-B limit, when the magnetic length $[l\equiv(h/eB)^{1/2}]$ is much smaller than the range (d) of the potential fluctuations, the percolation level in a 2D system is infinitely narrow at the half-filled Landau level.[18] The wide Hall plateaus can be explained with Laughlin's theory by the existence of such a narrow band of extended states at the center of the broadened Landau levels. The strong-T dependence of σ_{xx}, seen in the lower levels in Fig. 3, results from thermal activation into the percolation level and hopping across potential barriers. For the higher Landau levels, observed at lower B, the condition $l \ll d$ may not be met and the percolation energy level may be considerably wider and thermal activation becomes unimportant when E_F is at the center of the percolation level. As a result, it becomes possible to observe the effect of Coulomb interaction in the Landau levels of a 2D system, which gives rise to the logarithmic correction to σ_{xx} seen in Fig. 2.

It appears that this classical model explains qualitatively the striking features of our experiments. However, several problems must be mentioned. First, l is ~100 Å in this experiment and the average donor impurity separation is ~200 Å if we assume uniform distribution) in our samples. It is not clear what causes potential fluctuations with $d \gg 100$ Å in these samples. In this regard, it is desirable to perform the experiment on Si-MOSFET's in the density range where the dominance of short-range scattering is well known. Second, when E_F is at the percolation level, σ_{xx} must remain finite or approaches zero logarithmically as $T\rightarrow 0$. Such characteristics are not yet apparent in σ_{xx} for the $n=1$ levels to 35 mK. Finally, the possibility of a charge-density wave (CDW) ground state of 2D electrons in high B was suggested by Fukuyama, Platzman, and Anderson[19] several years ago. A qualitative explanation of our data based on the pinning and melting of CDW has now been given by Fukuyama and Platzman.[20] Whether such a CDW state indeed exists under our experimental conditions clearly deserves more quantitative work in theory as well as in experiment.

ACKNOWLEDGMENTS

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[5]ρ_{xx} and ρ_{xy} are measured at slightly different places on the sample and, for example, a constant concentration gradient could explain why ρ_{xy} step and ρ_{xx} maximum do not occur at the same value of B. If the maxima in ρ_{xx} had appeared at "half quantized values" of σ_{xy}, then σ_{xx} in Figs. 2 and 3 should be multiplied at 50 mK by 1.44, 1.30, 1.23, 1.19, and 1.14 for $n = 1\uparrow$, $1\downarrow$, 4, 5, 6, and 7, respectively. This small multiplicative factor should decrease smoothly to unity at $T = 10$ K and does not affect the conclusions based on Figs. 2 and 3.

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