PHYSICAL REVIEW B VOLUME 25, NUMBER 1 1 JANUARY 1982

Kink instability in planar ferromagnets

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(Received 6 October 1981)

The 360° screw kinks in a planar ferromagnet with an in-plane magnetic field are unstable above a critical-field strength. We show that this critical-field strength decreases rapidly as the kink velocity increases toward its maximum value. As a consequence, square-root singularities appear in the dynamic structure factor $S(q,\omega)$ at field-dependent cutoff values of ω/q.

Nonlinear solitonlike modes in one-dimensional (1D) magnetic systems have recently attracted considerable attention.[1–6] Inelastic neutron scattering experiments on the planar ferromagnet $CsNiF_3$ in a in-plane field by Kjems and Steiner[2] were interpreted to confirm Mikeska's prediction[1] of a thermal soliton contribution to the dynamic structure factor $S(q,\omega)$. This interpretation has subsequently been questioned,[3–5] but recent measurements of the out-of-plane structure factor seem to demonstrate[6] that the major part of the quasielastic peak is in fact due to scattering from thermal solitons.

In this Communication we show that these kink-type solitons in 1D planar ferromagents become unstable above a critical strength of the in-plane magnetic field which decreases rapidly with increasing kink velocity. To our knowledge, this is the first study of the instability of moving kinks. The static case has been discussed by Hornreich and Thomas[7] in a slightly different context, and more recently by Kumar.[8] As a result of our anlaysis, we predict square-root singularities in the dynamic structure factor $S(q,\omega)$ at cutoff values of ω/q which are drastically reduced from the sine-Gordon result already for moderate field strengths.

We consider a 1D planar ferromagnet described by the Hamiltonian

$$H=\sum_l[-J\vec{S}_l\cdot\vec{S}_{l+1}+A(S_l^z)^2-g\mu_B BS_l^x] \quad , \qquad (1)$$

where J, $A>0$, and the other notation is standard.[1] Disregarding quantum effects, the spin configurations at low temperatures are given by classical vectors $\vec{S}_l=S\vec{s}_l$ of constant amplitude S ($\vec{s}_l^{\,2}=1$). In the continuum approximation $\vec{s}_l(t)\to\vec{s}(z,t)$ one obtains the energy functional

$$E[\vec{s}]=\int_{-\infty}^{+\infty}[\tfrac{1}{2}\vec{s}\,'^2+\tfrac{1}{2}s_3^2+b(1-s_1)]dz \qquad (2)$$

and the equation of motion

$$\frac{\partial\vec{s}}{\partial t}=\vec{s}\times(\vec{s}\,''-s_3\vec{e}_3+b\vec{e}_1) \quad , \qquad (3)$$

where the prime indicates $\partial/\partial z$. Here, the units of z, t, and E are $\delta_0=a(J/2A)^{1/2}$ (a is the lattice constant), $t_0=\hbar/2AS$ and $E_0=S^2(2AJ)^{1/2}$, respectively, and $b=B/B_a$, where $B_a=2AS/g\mu_B$ is the anisotropy field (54 kG for $CsNiF_3$).

We are interested in solitary waves $\vec{s}(Z)$, $Z=z-ut$ such that $\partial/\partial z=d/dZ$, $\partial/\partial t=-ud/dZ$, with the velocity u in units of $u_0=\delta_0/t_0=aE_0/\hbar S$. It is convenient to represent the spin direction $\vec{s}$ by a "meridional" angle θ ($|\theta|\leq\pi/2$) between $\vec{s}$ and the easy plane, and an azimuthal angle φ ($0\leq\varphi\leq2\pi$) between the planar component of $\vec{s}$ and the field direction, such that $\vec{s}=(\cos\theta\cos\varphi,\cos\theta\sin\varphi,\sin\theta)$. Then, the equations of motion take the form

$$-u\theta'=\varphi''\cos\theta-2\theta'\varphi'\sin\theta-b\sin\varphi \quad , \qquad (4a)$$

$$u\varphi'\cos\theta=\theta''+(\varphi'^2-1)\sin\theta\cos\theta-b\sin\theta\cos\varphi \quad , \qquad (4b)$$

which are the Euler-Lagrange equations of the variational functional

$$K[\theta,\varphi]=E[\vec{s}]+u\int_{-\infty}^{+\infty}\varphi'\sin\theta dZ \quad . \qquad (5)$$

In order to test the stability of a given solitary solution (θ_s,φ_s) of Eqs. (4a) and (4b), we expand the functional K to second order in the deviations (α,β) from (θ_s,φ_s),

$$K[\theta,\varphi]=K[\theta_s,\varphi_s]+\tfrac{1}{2}K_2[\alpha,\beta] \quad ,$$

and determine the eigenvalues λ_n of K_2 as functions of b. The solution is linearly stable if all $\lambda_n>0$; the limits of stability occur at critical fields b_c found as the roots of $\lambda_n(b_c)=0$. The corresponding eigenmodes (α_n,β_n) are for $\lambda_n=0$ solutions of the linearized equations of motion for (α,β), i.e., a critical field b_c is a bifurcation point where the solution (θ_s,φ_s) is coexistent with a solution $(\theta_s+\alpha_n,\varphi_s+\beta_n)$ with infinitesimal (α_n,β_n). The dependence of the amplitude of this bifurcating solution on $b-b_c$ may be studied by a perturbation expansion to higher orders in (α,β).

We are interested in solitary waves starting from and returning to the state of uniform equilibrium magnetization, whose φ variation is a 2π kink, i.e., we have the boundary conditions

$$\theta_K(\pm\infty)=0,\quad \varphi_K(\mp\infty)=0,2\pi \quad . \tag{6}$$

A static solution ($u=0$) is the "azimuthal 2π kink"

$$\theta_K=0,\quad \sin(\varphi_K/2)=\operatorname{sech}\zeta,\quad \zeta=b^{1/2}z \quad , \tag{7}$$

with energy $E_K(0)=8b^{1/2}$ and width $\delta\propto b^{-1/2}$. The stability analysis described above leads to the following eigenvalue problem

$$\frac{d^2\alpha}{d\zeta^2}+(\lambda-1-b^{-1}+6\operatorname{sech}^2\zeta)\alpha=0 \quad , \tag{8a}$$

$$\frac{d^2\beta}{d\zeta^2}+(\lambda-1+2\operatorname{sech}^2\zeta)\beta=0 \quad , \tag{8b}$$

which decouples into two Schrödinger-type equations.

The second equation admits (independent of b) one bound state $\lambda=0$, $\beta=2\operatorname{sech}\zeta\equiv d\varphi_K(z)/d\zeta$, which is the Goldstone mode reflecting the marginal stability of the kink (7) against translations.

Equation (8a) allows two bound-state solutions $\lambda_1=b^{-1}-3$, $\alpha_1=\operatorname{sech}^2\zeta$, and $\lambda_2=b^{-1}$, $\alpha_2=\sinh\zeta\operatorname{sech}^2\zeta$. The first yields the stability limit $b_c=\frac{1}{3}$, i.e., $B_c=B_a/3$ (18 kG for $CsNiF_3$), of the static kink against meridional perturbations α. This instability corresponds closely to the instability of a 360° domain wall described in Ref. 7. It was also noticed recently by Kumar.[8]

In order to study the solution bifurcating at $b=b_c$, we expand the equations of motion to higher order in α and β. To lowest order, the meridional deviation from the kink (7) remains of the form $\alpha=\alpha_0\operatorname{sech}^2\zeta$, but there occurs an azimuthal deviation proportional to α_0^2, determined by

$$\beta''-(b_c\cos\varphi_K)\beta=(2\alpha'\varphi_K'+\tfrac{1}{2}\alpha\varphi_K'')\alpha \quad . \tag{9}$$

By using the fundamental solutions $\beta_1=\operatorname{sech}\zeta$, $\beta_2=\sinh\zeta+\zeta\operatorname{sech}\zeta$ of the corresponding homogeneous equation, we find the bounded solution of Eq. (9):

$$\beta=\tfrac{1}{2}\alpha_0^2(1+2\cosh^2\zeta)\sinh\zeta\operatorname{sech}^4\zeta \quad . \tag{10}$$

In order to determine the dependence of the amplitude α_0 on $b-b_c$, we calculate the energy to order α_0^4. After some tedious algebra in the evaluation of the various integrals, we find

$$E=8b^{1/2}[1-\tfrac{1}{10}\alpha_0^4-\tfrac{3}{4}\alpha_0^2(b-b_c)] \quad .$$

This shows that the instability of the static kink occurs by "inverted" bifurcation of an unstable solitary wave existing for $b<b_c$, whose azimuthal variation is a 2π kink and whose meridional variation is a pulse varying as $\alpha_0\sim\frac{1}{2}[15(b_c-b)]^{1/2}$ for $b\to b_c$. We have shown that this type of unstable solution exists down to $b\to 0$ where its amplitude varies as $\alpha_0\sim\pi/2-(2b)^{1/2}$ and its width diverges as $b^{-1/2}$.

For the moving kink ($u\neq 0$), no general analytic solution is available. General considerations show that the azimuthal variation remains a 2π kink, but there now occurs a meridional component $s_3\neq 0$. Thus the kink has the same symmetry as the expected instability mode, and the stability limit will coincide with its existence limit: at $b_c(u)$, a stable and an unstable solution of the same symmetry merge and annihilate each other.

For weak out-of-plane dynamics, $|s_3|\ll 1$, the equations of motion (3) may by adiabatic elimination of the meridional component, $\theta=\dot\varphi$, be approximately mapped into the sine-Gordon (SG) equation[1] $\varphi''-\ddot\varphi=\sin\varphi$ with the traveling solitary-wave solution ($|u|<1$, $1-u^2=\gamma^{-2}$)

$$\begin{aligned}&\sin(\varphi_K/2)=\operatorname{sech}(\gamma b^{1/2}Z) \quad ,\\ &\theta_K=-2u\gamma b^{1/2}\operatorname{sech}(\gamma b^{1/2}Z) \quad ,\end{aligned} \tag{11}$$

of energy $E_K(u)=8\gamma b^{1/2}$ and width $\delta\propto\gamma^{-1}b^{-1/2}$. The existence limit of this solution is obviously $u=\pm 1$ for all b, but its limit of validity decreases from $b=\infty$ at $u=0$ (where it is exact) to $b=0$ at $u=\pm 1$. Therefore, one may only conclude that $b_c(1)=0$.

The SG solution in the limit $b\to 0$, $u\to 1$ is connected to the unstable kink at $u=0$, $b\to 0$ mentioned above by a branch of unstable π-kink solutions of the form

$$\begin{aligned}&\tan\varphi=u^{-1}\sinh(\gamma^{-1}Z) \quad ,\\ &\sin\theta=\gamma^{-1}\operatorname{sech}(\gamma^{-1}Z) \quad ,\end{aligned} \tag{12}$$

of energy $E(u)=2\gamma^{-1}$ and width $\delta\propto\gamma$.

For intermediate values $0<u<1$ we have determined $b_c(u)$ approximately by extremizing the functional $K[\theta,\varphi]$ given in Eq. (5) for trial functions[9]

$$\begin{aligned}&\sin(\varphi/2)=\operatorname{sech}(Z/\delta)\\ &\tan(\theta/2)=D\operatorname{sech}(Z/\delta)\end{aligned} \tag{13}$$

with δ and D ($|D|\leq 1$) as variational parameters. This ansatz has been chosen such that it allows the reproduction of the kink exactly in both limits $u\to 0$ and $b\to 0$, and K can be calculated explicitly as function of δ and D. The extremum conditions show that for given $u>0$ there exists for small b three solutions: a maximum at $D_1>0$ and a minimum and a maximum at $D_{2,3}<0$.[10] The latter two merge at a critical value $b_c(u)$. The critical curve obtained in this way is plotted in Fig. 1, where it is compared with the result of a numerical integration of Eqs. (4a) and (4b).

The instability field $b_c(u)$ decreases rapidly with increasing kink velocity u. The physical origin of this result is the fact that because of the meridional component $s_3\neq 0$ the field exerts a torque $\vec{s}\times\vec{b}$ which can "throw the spin configuration over" more easily.

The behavior for $|u|\ll 1$ may be found by a Landau-type argument based on the functional K considered as a function of the meridional amplitude

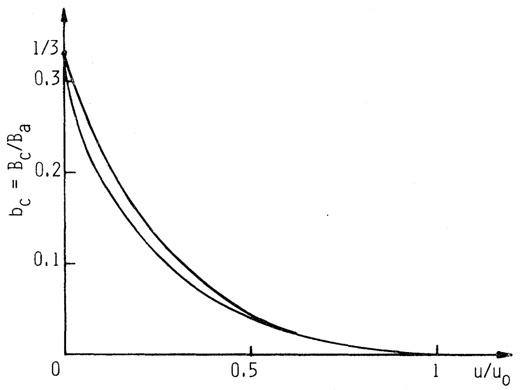

FIG. 1. Dependence of the instability field B_c on the kink velocity u, obtained by variational calculation (upper curve) and numerical integration (lower curve), respectively.

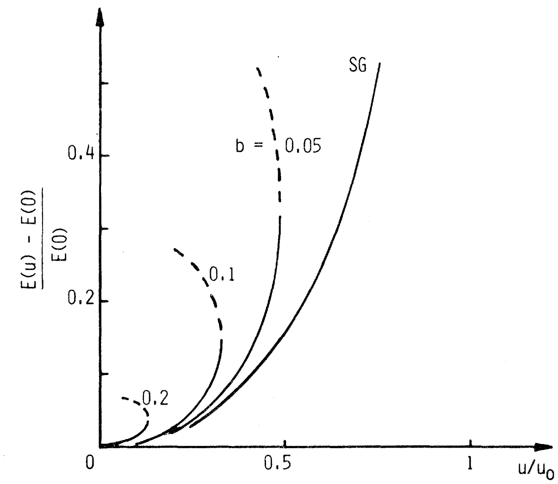

FIG. 2. Dependence of kink energy on kink velocity for various values of field strength $b = B/B_a$, compared with the sine-Gordon result.

θ_0. Because of the symmetry $K(\theta_0,u,b) = K(-\theta_0,-u,b)$, K has near $u=0$, $b=\frac{1}{3}$, the Taylor expansion $(d,e>0)$

$$K(\theta_0,u,b) = K_0 + cu\theta_0 + d(\tfrac{1}{3}-b)\theta_0^2 - e\theta_0^4 \ ,$$

which yields, at negative θ_0 values, a minimum and a maximum merging at

$$b_c(u) - b_c(0) \sim u^{2/3} \ .$$

For $|u| \to 1$, on the other hand, both the variational calculation and the numerical integration yield

$$b_c(u) \sim (1-u^2)^2 \ .$$

Figure 2 shows the kink energy as a function of u for constant values of b, obtained from Eq. (2) with the variational ansatz (13). Each of the curves for stable kinks ends in a square-root cusp at a critical value $u = u_c(b)$, where it merges with the curve for the unstable kink. The value of $u_c(b)$ is considerably reduced already at moderate values of b from the SG value $u_c^{\mathrm{SG}} = 1$.

These results have important consequences for inelastic neutron scattering experiments. In the independent-soliton approximation[1,2] one obtains a dynamic structure factor, which for an arbitrary velocity-dependent kink energy $E_K(u)$ takes the generalized form

$$S(q,\omega) \propto \exp[-\beta E_K(u)][M(q,u)]^2 \frac{1}{u}\frac{\partial E_K}{\partial u}\bigg|_{u=\omega/q} \ ,$$

with $M(q)$ the kink-shape function. For low temperatures, $S(q,\omega)$ still has as a function of ω for constant q a central peak, but the cusps in $E_K(u)$ now give rise to square-root singularities at the cutoff frequencies $\omega = \pm q u_c(b)$. This is to be contrasted with the SG result where the cutoff occurs at $\omega = \pm q u_0$, and the divergence of $\partial E_K/\partial u$ is suppressed by the Boltzmann factor because of the divergence of E_K for $u \to u_0$. Observation of these effects would yield further evidence for the soliton interpretation of the neutron scattering observations.

This work was supported by the Swiss National Science Foundation.

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[10] At small u, there occurs another minimum near $D=-1$ which is an artifact of the variational ansatz (13).