Exact solution for space-charge broadened packets in semiconductors

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(Received 28 December 1981)

The transport equation for charge packets in time-of-flight measurements of drift velocity in bulk semiconductors including full account of space-charge effects is derived. It is found to be a nonlinear, partial, integro-differential equation. Two key transformations permit finding an *exact, analytical solution* by decomposing the problem into separate equations for the shape function, the broadening function, and the velocity function. The solution consists of a rectangular-shaped charge packet which broadens linearly in time and propagates with an exponentially increasing velocity until it begins entering the electrode at which time the velocity can be expressed in terms of modified Bessel functions. The observed current pulse in the external circuit is also calculated and it is shown that experimenters have made a mistake in interpreting the pulse and so have deduced too high a drift velocity. The error, however, has typically been small ($\sim 1\%$).

I. INTRODUCTION

Drift-velocity measurements of carriers subjected to an electric field in bulk semiconductors have been made by a time-of-flight technique for 30 years.[1] The study of space-charge effects on the current in such media is equally old,[2] though the analytical study of transient effects dates from the work of Many and Rakavy[3] and of Helfrich and Mark[4] in 1962. These papers and most of the later work[5–11] have been concerned with the transient approach to a steady state arising from an abrupt turn-on. Recent work by Henson[12] and by Johnson and Lonngren[13] has addressed the more general problem involving an arbitrary initial condition. They have been able to obtain explicit expressions for the charge density and the electric field in terms of a single function. It, however, is the solution of a differential equation that is exceedingly complicated in the general case. None of the papers has dealt with the propagation of packets of charge used in time-of-flight experiments in semiconductors. It is for the purpose of accounting for space-charge effects on packet propagation that we undertake this analysis of the transport process.

The time-of-flight technique involves forming a thin sheet of excess charge just inside the surface of the semiconductor and measuring the time it takes to reach the opposite electroded face of the sample under a given applied electric field. The sheet of charge is formed either by injection from a junction or by some ionizing radiation which creates both holes and electrons. In the latter case one sign of charge is collected by the adjacent electrode, while the other drifts through the semiconductor to the far electrode. The current pulse observed at that electrode is a rather flat-topped pulse lasting for the entire time of flight because it consists of both displacement current and particle current. Alternately stated, the current pulse consists of the flow of image charge, which is initially on the input face electrode, around the external circuit to the output face electrode throughout the transit time. The falltime of the current pulse is noticeably longer than its risetime because of charge-packet broadening during transit.

Our analysis proceeds by first deriving the equation of transport for a propagating, broadening, conserved charge packet. This turns out to be a nonlinear, partial, integro-differential equation. In spite of its forbidding appearance we find that two key transformations reduce it to a problem admitting an *exact, analytical solution*. The effect of the transformations is to break the equation into three equations, one each for the shape function, the broadening function, and the velocity function. The first two become trivial first-order linear differential equations. The third must be solved in two intervals, one corresponding to the packet being entirely between the two electrodes, and the other to the packet entering the far electrode. In the first interval, this third equation is also a first-order linear differential equation, while in the second interval it is a generalized Riccati equation. An additional series of four transformations then allows the latter equation to be solved exactly in terms of modified Bessel functions of index zero.

The exact solution found takes the form of a rectangular density packet that broadens linearly in time and drifts with a velocity that increases exponentially in time while the packet is entirely between the electrodes and with a more complicated dependence involving modified Bessel functions while the packet enters the electrode.

The solution is then used to obtain a simple expression for the detected current pulse. The analysis shows that the points on the current pulse chosen by experimenters to measure the drift interval have been incorrect, though the resulting percentage error on the velocity is small. Our solution concerns only the transport process within the semiconductor, and the current pulse derived should be used as a current source in whatever external circuit is used for its detection. The risetime and falltime of that circuit must be considered in experimental evaluation of the current pulse.

II. ELECTRIC FIELD

A planar geometry, shown in Fig. 1, is used in bulk time-of-flight experiments. A thin sheet or packet of charge having a density $n(x,t)$ (measured in electrons per unit distance in the direction of travel) is initialized by bombardment of the left surface with an ionizing radiation such as high-energy electrons. This creates a thin sheet ($\sim 1\ \mu$m) of charge which then drifts in a uniform applied field to a collecting electrode whose current is observed. What complicates the problem, of course, is the field of the charge packet itself, that is, the space-charge field.

An explicit expression for the space-charge field can be obtained for an arbitrary density function from Poisson's equation,

$$\frac{\partial E}{\partial x}=-\frac{en(x,t)}{\epsilon_0 \kappa A}\ , \tag{1}$$

where E is the electric field, e the magnitude of the electron charge, ϵ_0 the permittivity of free space, κ the dielectric constant of the semiconductor medium, and A the area of the electrodes. Integrating (1) twice and applying the voltage conditions $V(0)=0$, $V(L)=V_0$ (see Fig. 1) yields the voltage

$$V(x,t)=\frac{xV_0}{L}+\frac{e}{\epsilon_0 \kappa A}\left[\int_0^x \int_0^{x'} n(x'',t)dx''\,dx'-\frac{x}{L}\int_0^L \int_0^{x'} n(x'',t)dx''\,dx'\right], \tag{2}$$

from which the electric field is found to be

$$E(x,t)=-\frac{V_0}{L}-\frac{e}{\epsilon_0 \kappa A}\left[\int_0^x n(x',t)dx'-\frac{1}{L}\int_0^L \int_0^{x'} n(x'',t)dx''\,dx'\right]. \tag{3}$$

III. DERIVATION OF TRANSPORT EQUATION

Under many experimental conditions used in bulk time-of-flight experiments the transit time is too short for significant trapping or recombination to occur. Thus the charge density satisfies the charge-conservation equation,

$$\frac{\partial n}{\partial t}+\frac{\partial j}{\partial x}=0\ , \tag{4}$$

where j is the flux (electrons/s). For typical applied electric fields drift is more important than diffusion and so we drop the latter for simplicity and write the flux as

$$j=nv\ , \tag{5}$$

where v is the drift velocity.

In a semiconductor such as silicon the drift velocity is known to be a saturating function of the electric field, that is, at low fields the velocity is proportional to the electric field but at high fields it becomes a constant independent of the field. Measurement of this function is, of course, the object of the time-of-flight experiments. Our previous study[14] of time-of-flight experiments on surface transport in MOS (metal-oxide—semiconductor) structures showed that the transport can be quite accurately characterized by using a first-order Taylor-series expansion of the velocity as a function of field. Since the velocity of an electron and the electric field are opposite in direction, we write

$$\begin{aligned} v(-E) &= v(V_0/L) + \left[-E - \frac{V_0}{L} \right] \frac{\partial v}{\partial E} \\ &= v_0 + a \left[\int_0^x n(x',t)dx' - \frac{1}{L} \int_0^L \int_0^{x'} n(x'',t)dx''dx' \right] , \end{aligned} \tag{6}$$

where

$$v_0 \equiv v(V_0/L) , \tag{7}$$

$$a \equiv \frac{e}{\epsilon_0 \kappa A} \frac{\partial v}{\partial E} \bigg|_{E=+V_0/L} , \tag{8}$$

and we assume $a > 0$. If we consider a constant mobility μ region where $v = \mu E$, then (6) is exact with the alternate definitions $v_0 \equiv \mu V_0/L$ and $a \equiv e\mu/\epsilon_0 \kappa A$.

Combining (4)—(6) leads to the transport equation

$$\frac{\partial n}{\partial t} + v_0 \frac{\partial n}{\partial x} + a \frac{\partial}{\partial x} \left[n \left[\int_0^x n\, dx' - \frac{1}{L} \int_0^L \int_0^{x'} n\, dx''\, dx' \right] \right] = 0 , \tag{9}$$

which is a nonlinear, partial, integro-differential equation.

IV. SOLUTION OF TRANSPORT EQUATION

A. Shape and broadening functions

The first step in solving the transport equation (9) is the realization that the double integral in it is a function of time only, can be brought outside the space derivative, and so can be seen to be a time-dependent velocity change. We are thus led to introduce a traveling coordinate

$$\xi = x - \int_0^t v_T(t')dt' , \tag{10}$$

where

$$v_T(t) = v_0 - \frac{a}{L} \int_0^L \int_0^{x'} n\, dx''\, dx' . \tag{11}$$

Since the trailing edge of the packet begins at $t = 0$, $x = 0$, which gives $\xi = 0$, we see that $\xi = 0$ represents the trailing edge of the packet and $v_T(t)$ is the velocity of that point in the packet. The trans-

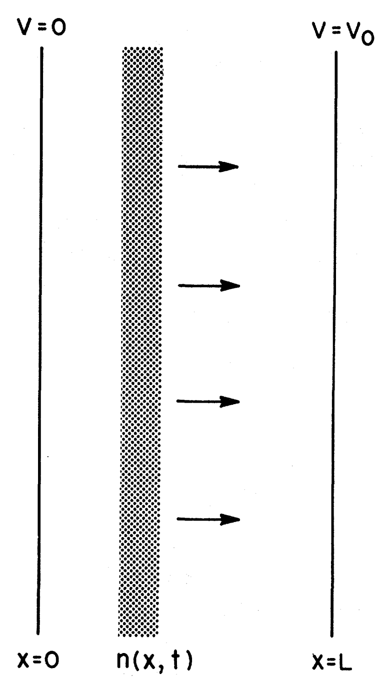

FIG. 1. Planar geometry of bulk time-of-flight experiments.

port equation then becomes

$$\frac{\partial n}{\partial t}+a\frac{\partial}{\partial \xi}\left[n\int_0^{\xi} n\,d\xi'\right]=0\ . \tag{12}$$

We now look for a solution of the form

$$n=\frac{F(z)}{w(t)}\ , \tag{13}$$

$$z=\xi/w(t)\ , \tag{14}$$

which represents a propagating, broadening, conserved packet which retains its same functional form during broadening. We call F the shape function and w the broadening function. Substitution of this into (12) leads to separation of variables,

$$\frac{dw}{dt}=\frac{a\dfrac{d}{dz}\left[F\int_0^z F\,dz'\right]}{\dfrac{d}{dz}(zF)}=c_1\ , \tag{15}$$

thus allowing each side to be set equal to a constant c_1.

Integration with respect to time then yields the broadening function

$$w=w_0+c_1t \tag{16}$$

and integration with respect to z yields

$$c_1zF=aF\int_0^z F\,dz'+c_2\ . \tag{17}$$

The integration constant c_2 must vanish as can be seen by setting $z=0$. It is then apparent that

$$F=c_1/a,\ \ 0\le z\le 1\ . \tag{18}$$

Outside this interval $F=0$ is a solution. If we denote the total number of electrons in the packet as N, we see that $N=c_1/a$. Thus we have found that the packet is rectangular in shape,

$$n(\xi,t)=\frac{N}{w(t)}[S(\xi)-S(\xi-w(t))]\ , \tag{19}$$

where $S(\xi)$ is the step function,

$$S(\xi)=\begin{cases}0, & \xi\le 0\\ 1, & \xi>0\end{cases} \tag{20}$$

and the packet broadens linearly in time[15]

$$w(t)=w_0+aNt\ , \tag{21}$$

both remarkably simple results. This solution applies if the packet is initialized with a width w_0 at $t=0$ with its trailing edge at $x=0$. This would appear to be an adequate characterization of the ionizing radiation bombardment usually used.

B. Velocity function

We now must substitute the density function into (11) and solve for the velocity function,

$$v_T\equiv\frac{dx_T}{dt}=v_0-\frac{aN}{2Lw}[(L-x_T)^2S(L-x_T)-(L-x_T-w)^2S(L-x_T-w)]\ , \tag{22}$$

where x_T denotes the trailing-edge position. Consideration of this differential equation is naturally divided between two different intervals of time: (a) the packet being entirely between the electrodes, $0\le x_T<x_T+w\le L$, and (b) the packet entering the collecting electrode, $0\le x_T\le L<x_T+w$.

In interval (a) we have

$$S(L-x_T)=S(L-x_T-w)=1$$

and (22) becomes

$$\frac{dx_T}{dt}-\frac{aN}{L}x_T=v_0-aN\left[1-\frac{w_0}{2L}\right]+\frac{a^2N^2}{2L}t\ , \tag{23}$$

which is a first-order linear differential equation that is easily solved for

$$x_T=\left[\frac{Lv_0}{aN}-\frac{L}{2}+\frac{w_0}{2}\right](e^{aNt/L}-1)-\frac{aNt}{2}\ , \tag{24}$$

where we have required $x_T=0$ at $t=0$. The velocity in interval (a) is then

$$v_T=\left[v_0-\frac{aN}{2}+\frac{aNw_0}{2L}\right]e^{aNt/L}-\frac{aN}{2}\ . \tag{25}$$

Thus we see that the packet trailing-edge velocity depends exponentially on time because of its own space-charge effect. Of course, as $N\to 0$, $v_T\to v_0$, which is the drift velocity at the particular applied field. The velocity of the leading edge is just (25) plus aN as seen from (21).

In interval (b) we have $S(L-x_T)=1$ and $S(L-x_T-w)=0$ and (22) becomes

$$\frac{dx_T}{dt}=v_0-\frac{aN(L^2-2Lx_T+x_T^2)}{2L(w_0+aNt)}\ . \tag{26}$$

This can be recognized as a generalized Riccati equation,[16] that is, a first-order differential equation containing a quadratic nonlinearity. The generalized Riccati equation can be transformed to a second-order linear differential equation. In this case the required dependent variable transformation is

$$x_T = \frac{2L(w_0+aNt)}{aNy}\frac{dy}{dt}\ , \tag{27}$$

with the result

$$\frac{d^2y}{dt^2} - \left[\frac{v_0 aN}{2L(w_0+aNt)} - \frac{a^2N^2}{4(w_0+aNt)^2}\right]y = 0\ . \tag{28}$$

We are led next to transform the independent variable to τ by

$$\tau = \frac{v_0(w_0+aNt)}{2aNL}\ , \tag{29}$$

which leads to

$$\frac{d^2y}{d\tau^2} + \frac{1-4\tau}{4\tau^2}y = 0\ . \tag{30}$$

We recognize this as the normal form[17] of the modified Bessel-Clifford equation[18] of index zero. Transformation of the dependent variable,

$$y = \tau^{1/2} f\ , \tag{31}$$

converts it to the conventional form of the modified Bessel-Clifford equation of index zero,

$$\tau\frac{d^2f}{d\tau^2} + \frac{df}{d\tau} - f = 0\ . \tag{32}$$

Transformation of the independent variable,

$$z = 2\tau^{1/2}\ , \tag{33}$$

now converts it to the modified Bessel equation[19] of index zero,

$$z\frac{d^2f}{dz^2} + \frac{df}{dz} - zf = 0\ . \tag{34}$$

The general solution of (34) is a linear combination of modified Bessel functions of index zero,

$$f = a_1 I_0(z) + a_2 K_0(z)\ . \tag{35}$$

This can now be substituted into (31) and thence into (27). Since (27) is a ratio of dy/dt and y, only the ratio of the constants, $R = a_1/a_2$, will enter (27). Thus there is only one arbitrary constant in x_T as expected since it originated from a first-order differential equation. Thus we obtain

$$x_T = L\frac{R[I_0(z)+zI_1(z)] + K_0(z) - zK_1(z)}{RI_0(z)+K_0(z)} \tag{36}$$

as the general solution for the trailing edge position in case (b). In (36) z should be regarded as a function of time through (33) and (29). The velocity is then

$$v_T = \frac{v_0}{z[RI_0(z)+K_0(z)]^2}\{Rz^2[I_0^2(z)-I_1^2(z)] + z[K_0^2(z)-K_1^2(z)] + 2RzI_0(z)K_0(z) + 2RzI_1(z)K_1(z)\}\ . \tag{37}$$

The positions (24) and (36) for intervals (a) and (b) must be joined at the time t_1 when the leading edge of the packet reaches the electrode, that is, when

$$\left[\frac{Lv_0}{aN} - \frac{L}{2} + \frac{w_0}{2}\right](e^{aNt_1/L} - 1) + \frac{aNt_1}{2} + w_0 = L\ . \tag{38}$$

Equating (24) and (36) at time t_1 leads to a value for the arbitrary constant R,

$$R = \frac{Lz_1K_1(z_1) - w(t_1)K_0(z_1)}{w(t_1)I_0(z_1) + Lz_1I_1(z_1)}\ , \tag{39}$$

where $z_1 \equiv z(t_1)$.

The solution is now complete. The velocity function v_T given by (25) and (37) for time intervals (a) and (b), respectively, can now be substituted into the definition (10) of the traveling coordinate ξ in the terms in which the density solution (19) is expressed. Note that the solution to the original very complicated equation (9) is reduced to three separate and, for the most part, simple solutions for the shape, broadening, and velocity functions through the use of two key substitutions (10), (11) and (13), (14).

V. CURRENT PULSE

The observed quantity in the bulk time-of-flight experiment is the current in the external circuit joining the electroded faces of the semiconductor. It is equal to the current in the semiconductor, which consists of the displacement current, which flows throughout the time of flight and to the particle current which flows only during the time that the packet is entering the electrode. The total current is

$$I = A\left[\epsilon_0 \kappa \frac{\partial E}{\partial t} + i\right], \tag{40}$$

where i is the particle current density. The latter is given in terms of the particle flux j of (4) by

$$\begin{aligned} i &= -\frac{e}{A} j \\ &= +\frac{e}{A}\frac{\partial}{\partial t}\int_0^x n(\xi,t)dx , \end{aligned} \tag{41}$$

where the last expression is obtained by integrating (4). Substitution of the functional dependence (13) into (41) gives

$$i = -\frac{e}{A} n(x - x_T, t)\left[v_T + \frac{x - x_T}{w(t)}\frac{dw}{dt}\right]. \tag{42}$$

Combining (40) and (42) and inserting the expressions (3) and (19) for E and n, respectively, into the combination yields the total current

$$\begin{aligned} I = \frac{eaN^2}{w^2} &\left[(x - x_T)S(x - x_T) - (x - x_T - w)S(x - x_T - w) \right. \\ &\left. - \frac{1}{2L}[(L - x_T)^2 S(L - x_T) - (L - x_T - w)^2 S(L - x_T - w)]\right] \\ - \frac{eN}{w} &\left[(v_T + aN)S(x - x_T - w) - v_T S(x - x_T) \right. \\ &\left. - \frac{1}{L}[(L - x_T - w)(v_T + aN)S(L - x_T - w) - (L - x_T)v_T S(L - x_T)]\right] \\ - \frac{eN}{w} &[S(x - x_T) - S(x - x_T - w)]\left[v_T + \frac{(x - x_T)aN}{w}\right]. \end{aligned} \tag{43}$$

In time interval (a) (24) and (25) are used here for x_T and v_T and in time interval (b) (36) and (37) are used.

This general form for I is less illuminating than its specializations to intervals (a) and (b). They are easily obtained to be

$$I = \begin{cases} -\dfrac{eN}{L}\left[v_T + \dfrac{aN}{2}\right], & x_T < x_T + w \le L \quad (44a) \\[2ex] -\dfrac{eN(L - x_T)}{Lw}\left[v_T + \dfrac{aN(L - x_T)}{2w}\right], & x_T \le L < x_T + w . \quad (44b) \end{cases}$$

In this form it is apparent that I is independent of x, as expected, because from Maxwell's equations the divergence of I vanishes.

VI. DISCUSSION

Since time-of-flight experiments are aimed at measuring v_0, we now must determine how v_0 is to be extracted from a current pulse measurement.

First, it is apparent that small packets should be used to reduce space-charge effects. More precisely, packet sizes obeying $aN << v_0$ should be used. However, the limit $aNL << w_0 v_0$, which corresponds to minimal packet broadening, need not be met. In the $aN << v_0$ limit (44) becomes

$$I = \begin{cases} -\dfrac{eNv_0}{L}, & x_T < x_T + w \le L \quad (45a) \\[2ex] -\dfrac{eN(L - v_0 t)v_0}{L(w_0 + aNt)}, & x_T \le L < x_T + w . \quad (45b) \end{cases}$$

Thus a simple test of the observance of the $aN << v_0$ limit is the constancy of the current in time interval (a). This has been understood by time-of-flight experimenters.[1,20]

Second, it is apparent from this analysis and from simple physical reasoning that the only part of the packet that travels the whole sample thickness L under drift conditions is the trailing edge. It begins at $t=0$, the beginning of the observed current pulse at $x_T=0$, and reaches the electrode, $x_T=L$, when the current pulse returns to zero as seen from either (44b) or (45b). This fact does not seem to have been realized by time-of-flight experimenters since they have measured the arrival time as being that instant at which the current pulse has dropped to one-half of maximum. This makes their deduced velocity high, but because they have used small packets and long transit times the error is small ($\sim 1\%$). Under less favorable conditions, however, the error could be substantial. We should, however, remark that the solution found here has not considered the external circuit risetime and falltime which must be considered in an experiment. Equation (44) should be regarded as the current source for that circuit.

Figure 2 presents a plot of the calculated electric field and density at four times during transit for conditions applying to a particular measurement made by Norris and Gibbons.[20] Figure 3 plots the

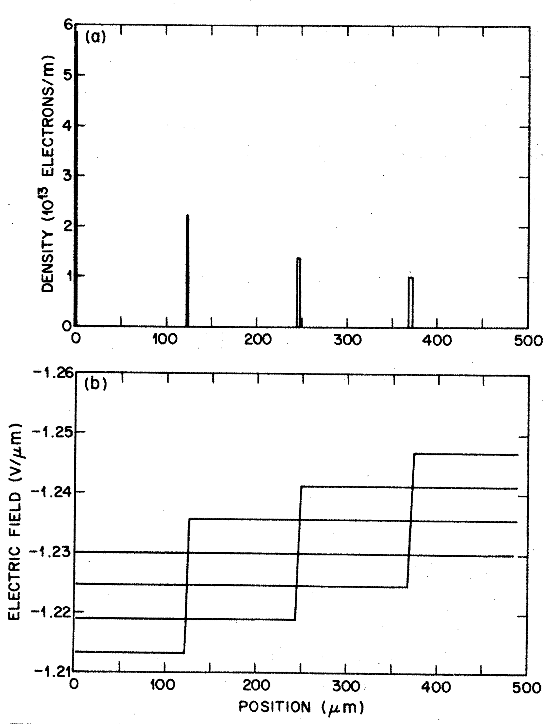

FIG. 2. (a) Density vs position at four different times. The conditions correspond to a particle measurement (Ref. 20), where $a=8.15\times10^{-6}$ m/s, $L=488$ μm, $N=5.85\times10^{7}$, $v_0=3.65\times10^{4}$ m/s, and $w_0=1.0$ μm. (b) Electric field vs position for the same four times as in part (a).

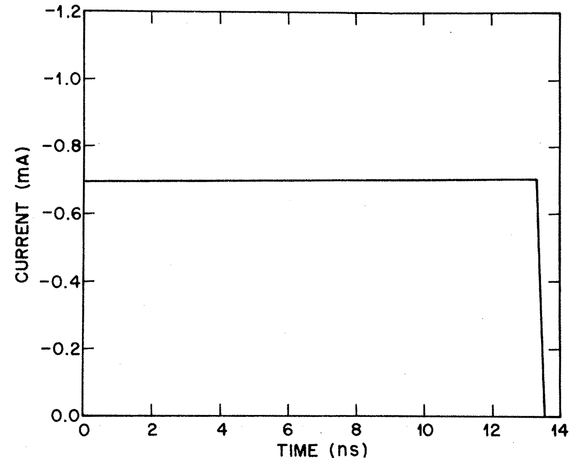

FIG. 3. Current vs time for conditions of Fig. 2.

resulting current pulse. It is apparent that they have worked in the small-packet limit where the space-charge electric field causes only a minor perturbation to the applied field and where the current pulse is essentially flat-topped indicating a nearly constant velocity during transit.

Figures 4–6 show a similar case except that the total number of carriers in the packet is 20 times larger. Figure 4 shows the velocity versus time. The exponential rise in interval (a) is apparent here because of the larger value of N, while the leveling off of the velocity in interval (b), caused by the entrance of the charge packet into the electrode, can also be seen. In Figs. 5 and 6 note the greater packet broadening, the substantial space-charge field, and the noticeable lack of a flat-topped current which indicates an accelerating charge packet.

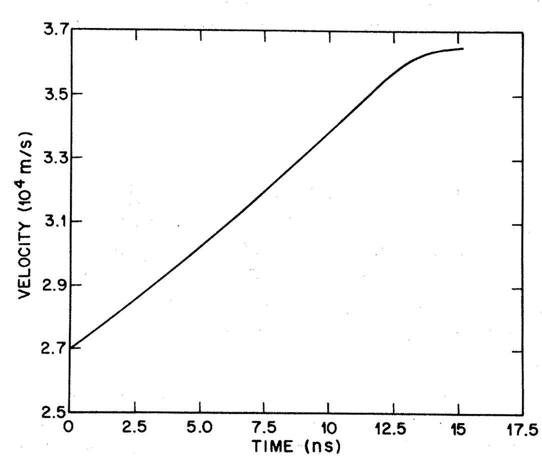

FIG. 4. Velocity vs time for $N=11.7\times10^{8}$ and other parameters the same as in Fig. 2.

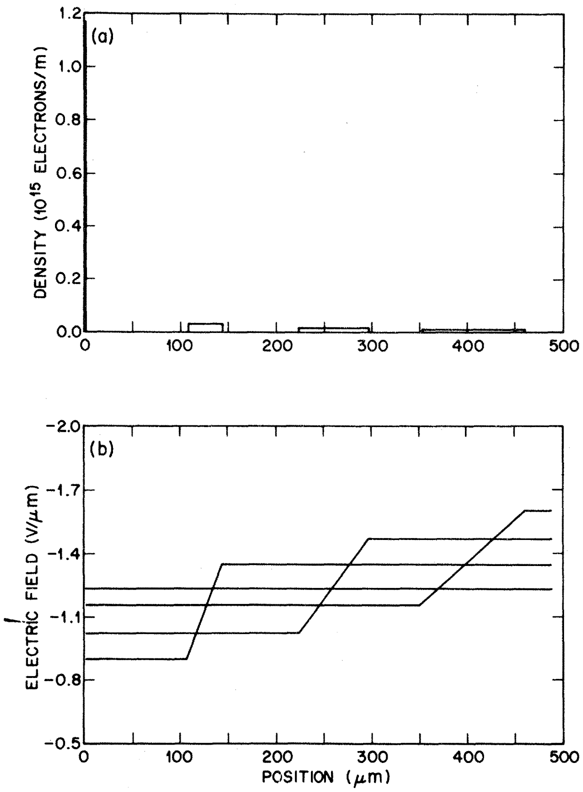

FIG. 5. (a) Density vs position at four different times. Conditions same as Fig. 4. (b) Electric field vs position for the same four times as in part (a).

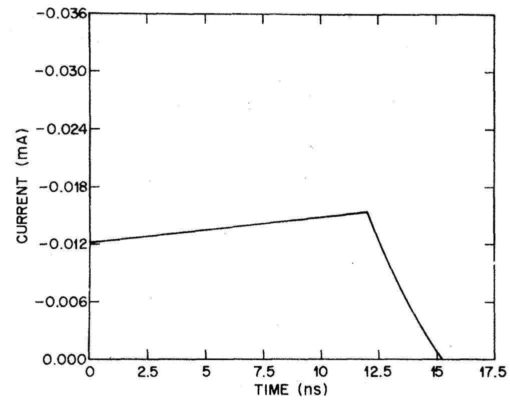

FIG. 6. Current vs time for conditions of Fig. 4.

We conclude with a few remarks on the linear broadening of the charge packet in this geometry of bulk time-of-flight experiments. Such a simple dependence would raise few questions were it not for the comparison to recent derivations[14,21] of the broadening of charge packets in MOS structures which is proportional to the cube root of time. The essential difference in the geometries that causes this difference in time dependences is the location of the image charges of the packet and the consequent difference in the space-charge fields. In the MOS structure the image charge resides on the gate with the electric field lines between it and the packet forming a capacitor field which is perpendicular to the direction of travel. On the other hand, in the parallel electrode geometry of Fig. 1 the image charge of the packet resides on the two electrodes, changing its proportion on each one during the transit, and the electric field joining the image charge and the packet is parallel to the direction of travel. Thus the altered space-charge field causes the altered broadening dependence on time.

ACKNOWLEDGMENTS

The author wishes to thank D.A. Kleinman and K. K. Thornber for a number of helpful conversations.

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[1]For a general discussion of the time-of-flight technique and an extensive list of references, see, L. Reggiani, in *Physics of Nonlinear Transport in Semiconductors,* edited by D. K. Ferry, J. R. Barker, and C. Jacobini (Plenum, New York, 1980), p. 243.

[2]W. Shockley and R. C. Prim, Phys. Rev. 90, 753 (1953).

[3]A. Many and G. Rakavy, Phys. Rev. 126, 1980 (1962).

[4]W. Helfrich and P. Mark, Z. Phys. 166, 370 (1962).

[5]I. P. Batra, B. H. Schechtman, and H. Seki, Phys. Rev. B 2, 1592 (1970).

[6]M. Zahn, C. F. Tsang, and S.-C. Pao, J. Appl. Phys. 45, 2432 (1974).

[7]M. Zahn, IEEE Trans. Electr. Insul. EI-12, 176 (1977).

[8]L. DeOliveira and G. Ferreira, J. Electrostat. 1, 371 (1975); 2, 187 (1976); 2, 249 (1976).

[9]L. DeAlmeida and G. Ferreira, Rev. Brasil. Fis. 5, 349 (1975).

[10]P. Camargo and G. Ferreira, Rev. Brasil. Fis. 6, 231 (1976).

[11]K. E. Lonngren, J. Appl. Phys. 48, 2630 (1977).

[12]B. L. Henson, J. Appl. Phys. 51, 4078 (1980).

[13]S. F. Johnson and K. E. Lonngren, J. Appl. Phys. 52, 5763 (1981).

[14]D. F. Nelson (unpublished).

[15]K. K. Thornber has informed the author of a simple argument by which he concluded earlier that linear broadening occurred in this geometry (private communication).

[16]E. D. Rainville, *Intermediate Differential Equations,* 2nd ed. (McMillan, New York, 1964), p. 31.

[17]*Intermediate Differential Equations,* Ref. 16, p. 5.

[18]*Intermediate Differential Equations,* Ref. 16, p. 183.

[19]*Intermediate Differential Equations,* Ref. 16, p. 182.

[20]C. B. Norris, Jr. and J. F. Gibbons, IEEE Trans. Electron Dev. ED-14, 38 (1967).

[21]K. K. Thornber, D. F. Nelson, and J. A. Cooper, Appl. Phys. Lett. 39, 843 (1981).