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Crossover to mean-field behavior at marginal dimensionality

R. Frowein and J. Kötzler
Institut für Festkörperphysik, Technische Hochschule Darmstadt, D-6100 Darmstadt, Federal Republic of Germany

B. Schaub and H. G. Schuster
Institut für Theoretische Physik der Universität Frankfurt, D-6000 Frankfurt/Main, Federal Republic of Germany
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The effective critical exponent $\gamma_{\text{eff}} = d\ln\chi^{-1}/d\ln t$, where χ is the magnetic susceptibility and $t = (T-T_c)/T_c$ is the reduced temperature, has been measured for the first time in the broad region $0.001 \leq t \leq 25$ in the uniaxial dipolar Ising ferromagnets $LiTbF_4$ and $Dy(C_2H_5SO_4)_3 \cdot 9H_2O$. For $LiTbF_4$ a distinct maximum is found in the t dependence of γ_{eff} which remains to be explained theoretically in a satisfactory way.

Near the Curie temperature, uniaxial ferromagnets like $GdCl_3$,[1] $LiTbF_4$,[2,3] $Dy(C_2H_5SO_4)_3 \cdot 9H_2O$,[4] and TbF_3 (Ref. 5) are observed to obey mean-field laws enhanced by logarithmic corrections which are characteristic for a system near marginal[6–8] dimensionality. We measured for the first time the effective critical exponent of the (internal) magnetic zero-field susceptibility:

$$\gamma_{\text{eff}} = \frac{d\ln\chi^{-1}(t)}{d(\ln t)} \quad , \tag{1}$$

for $LiTbF_4$ and $Dy(C_2H_5SO_4)_3$ (DyES) in the large region of reduced temperature, $0.001 \lesssim t \equiv (T-T_c)/T_c \lesssim 40$. Moreover, γ_{eff} turns out to pass through a maximum which remains to be explained in a satisfactory way by theory.

We employed precise low-field magnetization measurements described elsewhere in some detail.[3,4] High-resolution SQUID detection of magnetic flux changes caused by very small temperature increments (60–100 μK near T_c) allow for the small error bars in the differential quantity γ_{eff}, shown in Figs. 1 and 2.[9] Each plotted value of γ_{eff} represents the mean of ten measuring points for $\chi(T)$ inbetween two adjacent values of t. The Curie temperatures were determined previously by adjusting the field and temperature dependence of the magnetization on both sides of T_c for $LiTbF_4$ (Ref. 4) and DyES (Ref. 3) to the Landau expansion modified by the logarithmic corrections. For $LiTbF_4$, the high-temperature data include small correction, for the occupation of the first excited crystalline field level at $\Delta/k_B = 176$ K (Ref. 3) by the replacement

$$T \rightarrow T\,[1 + \tfrac{29}{18}\exp(-\Delta/k_B T)] \quad .$$

The most accurate result for the susceptibiltiy of an uniaxial dipolar ferromagnet has been given by Brézin and Zinn-Justin[7,8]:

$$\chi^{-1}(t) \propto T\,t\,|\ln t|^{-1/3}(1 + c\ln|\ln t|/|\ln t|) \quad , \tag{2}$$

where $c = (41 + 108\ln\frac{4}{3})/243$ and we have explicitly noted the factor T^{-1} which comes from the fluctuation dissipation theorem. However, formula (2) cannot describe our experiments because it does not lead to the correct mean-field limit $\chi^{-1}(t >> 1) = (t)(\text{const})$. It seems also very hard to improve Eq. (2) by a straightforward computation of higher-order terms because one obtains $|\ln t|^{-1}$ corrections with nonuniversal coefficients.[8]

We have found a way to extrapolate (2) to higher temperatures by observing that in the Ginzburg-Landau Hamiltonian of Brézin and Zinn-Justin,[8] according to Wilson and Kogut,[6] the actual temperature variable is $(T-T_c)/T$ and not $(T-T_c)/T_c$. There-

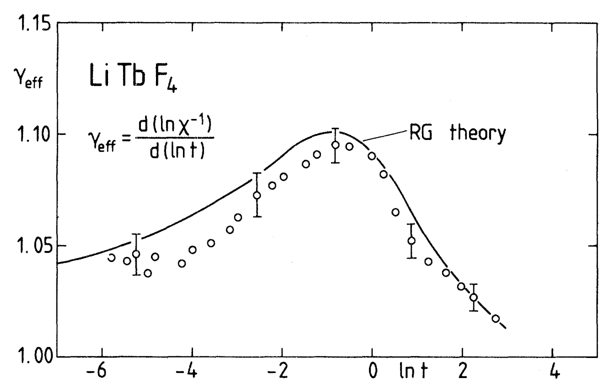

FIG. 1. Effective critical exponent of the zero-field susceptibility of $LiTbF_4$ for $0.001 \leq t \equiv (T-T_c)/T_c \leq 20$ [$T_c = 2.87075(3)$ K (Ref. 3)] compared with a calculation from Eq. (5) using $t_0 = 0.3$ (full line).

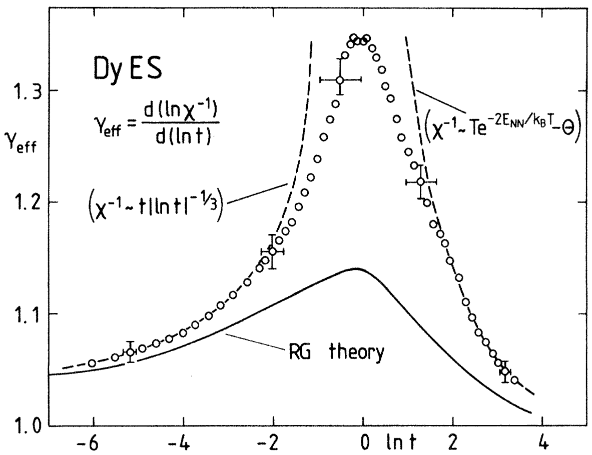

FIG. 2. Effective exponent of the magnetic susceptibility of DyES for $0.004 \leq t \leq 30$ [$T_c = 0.11817(3)$ K (Ref. 4)] compared to extrapolations from the dipolar critical region and from the mean-field region, where one-dimensional short-range order is expected to be important. Full line represents calculation using Eq. (5).

fore we have at least to replace $t \to t/(1+t)$ in Eq. (2). With this replacement $\chi^{-1}(t)$ shows the correct mean-field limit and we obtain

$$\gamma_{eff} = 1 + \tfrac{1}{3}(1+t)^{-1} \times \left[1 + c\left(1 + \left| \ln \left| \frac{t}{t_0(1+t)} \right| \right| \right) \right] \times \left[1 + \left| \ln \left| \frac{t}{t_0(1+t)} \right| \right| + c \ln \left(1 + \left| \ln \left| \frac{t}{t_0(1+t)} \right| \right| \right) \right]^{-1} , \tag{3}$$

where we have introduced as in Ref. 10 the temperature scale t_0.

Figure 1 shows that (5) describes the experiment on $LiTbF_4$ quite well if one uses $t_0 \approx 0.3$ as the only fitting parameter. This t_0 value agrees with Ahlers's *et al.*[2] result from specific-heat measurements in $LiTbF_4$.

Despite this success of our method we still feel that the mean-field limit of Eq. (2) should also be obtainable without our replacement $t \to t/(1+t)$. It remains a challenge for theorists to include the relevant perturbations in the starting Hamiltonian and to perform a proper summation of the perturbation series for $\chi(t)$.

In contrast to $LiTbF_4$ for DyES the maximum of γ_{eff} around $t = 1$ is much more enhanced as illustrated by Fig. 2. Our theoretical result, Eq. (2), cannot account for the observed value of $\gamma_{eff} = 1.35$ since the upper limit attainable by Eq. (2) is given by $\gamma_{eff}(t=1) = 1.25$. Therefore, an additional mechanism must be operative in DyES. it is suggestive to relate this effect to some one-dimensional short-range order originating from nearest-neighbor (NN) interactions between Dy^{3+} along the hexagonal c axis, $E_{NN}/k_B = 0.104$ K, which ecceeds the next-nearest-neighbor coupling by almost one order of magnitude. While in a pure one-dimensional system γ_{eff} tends to infinity as $T \to T_c = 0$, the interchain couplings cause a crossover to a finite γ_{eff} and a finite transition temperature, which at the present status of the theory cannot be calculated. Figure 2 shows that to some extent the high-temperature tail of γ_{eff} can be described by $\chi^{-1} \sim Te^{-2E_{NN}/k_BT} - \theta$, where the NN couplings are considered exactly and all other interactions in the mean-field approximation.[11]

In conclusion we have measured the crossover from critical to mean-field behavior for $LiTbF_4$ and DyES in the rather broad temperature regime $10^{-3} \leq t \leq 25$. It seems worth noting that the ultimate mean-field regime is only obtained for $t \gtrsim 40$. Observing that the relevant parameter entering the renormalization-group equations is $(T - T_c)/T$ and not the approximation $(T - T_c)/T_c$, the crossover of $\gamma_{eff}(t)$ in $LiTbF_4$ has been described by an extrapolation of known renormalization-group results. A more satisfactory theoretical explanation is still lacking.

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