# Temperature-dependent effective mass of a self-trapped electron on the surface of a liquid-helium film

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We have used the Feynman path-integral formulation of the polaron problem to compute the temperature-dependent effective mass of a two-dimensional electron on a film of liquid helium for values of the coupling constant from weak to strong. Because of the nature of the ripplon spectrum (very low velocities) we find that the self-trapped structure (found at $T=0$) "melts" for rather small $kT/E_B$. Here $E_B$ is the zero-temperature binding of the localized state.

## I. INTRODUCTION

In a previous paper[1] we formulated the problem of a two-dimensional (2D) electron on the surface of a liquid-helium film as a polaron problem. We used the Feynman[2] path-integral formulation of the polaron to compute the ground-state ($T=0$) energy and effective mass of the system for all values of the coupling constant (from strong to weak). The definition of the coupling constant arises from the coupling of the electron to the surface ripplons of the helium in the presence of a perpendicular pressing field. We found that the effective mass undergoes an extremely rapid transition from an electronic value to a value on the order of several helium masses for easily attainable (experimentally) coupling constants. A "quasilocalization" transition occurs. The extremely strongly coupled localized state is a self-trapped dimplelike state as described by Sander.[3] We also found that in the strong-coupling limit, the mass is inversely proportional to the square of the ripplon frequency at the capillary wave number, $k_c$.[4] As this ripplon frequency vanishes, the strong-coupling mass goes to infinity, as does the jump in mass, and the transition looks more and more like a real localization transition.

It is of obvious interest to investigate what happens at finite temperatures. Does the dimplelike state "melt" at some temperature? If so, how rapidly does this occur? In this paper we answer these questions. Because of the nature of the ripplon spectrum (very low velocities), we find that the self-trapped structure melts for rather small values of $kT/E_B$. $E_B$[5] is the zero-temperature binding of the localized state.

## II. FORMULATION

As in the $T=0$ problem the Hamiltonian is

$$H=\frac{P^2}{2m}+\sum_{\vec{k}} a^{\dagger}_{\vec{k}} a_{\vec{k}} \hbar\omega_{\vec{k}}+U \quad , \tag{1}$$

where

$$U=\frac{1}{A^{1/2}}\sum_{\vec{k}}(a_{\vec{k}}+a^{\dagger}_{\vec{k}})e^{i\vec{k}\cdot\vec{r}}Q(k) \tag{2}$$

with

$$Q(k)=\left(\frac{\hbar k \tanh kd}{2\rho\omega_{\vec{k}}}\right)^{1/2} e\mathcal{E} \quad . \tag{3}$$

Here $\mathcal{E}$ is a perpendicular pressing field, either externally applied or arising from the image potential from the substrate. The ripplon frequency is given by

$$\omega_k=\left[\left(g'k+\frac{\sigma}{\rho}k^3\right)\tanh kd\right]^{1/2} \quad , \tag{4}$$

where as before $\rho$, $\sigma$, and $g'$ are the density, surface tension, and acceleration of the liquid due to its van der Waals coupling to the substrate. As in the $T=0$ problem,[1] we linearize the ripplon dispersion relation

$$\omega_k=\sqrt{g'd}\,k=sk \quad . \tag{5}$$

In path integral form, the free energy is defined by

$$e^{-\beta F}=Z_r\int D\,\vec{r}(\tau)\,e^{S} \quad , \tag{6}$$

where $Z_r$ is the free ripplon partition function, and $S$ is the action arising from the Hamiltonian in Eq. (1). If $S_0$ is some approximate quadratic action, then

$$F=F_r-kT\ln\left(\int D\,\vec{r}(\tau)e^{S_0}e^{S-S_0}\right)$$

$$\leq F_r+F_0-kT\langle S-S_0\rangle \quad , \tag{7}$$

where

$$\langle S\rangle=\int e^{S_0}S\,D\,\vec{r}(\tau)\Big/\int e^{S_0}D\,\vec{r}(\tau) \quad . \tag{8}$$

Equation (7) is a variational principle for the free energy. We again pick $S_0$ as arising from the Lagrangian

$$L=\tfrac{1}{2}m\left(\frac{d\vec{x}}{dt}\right)^2+\tfrac{1}{2}M\left(\frac{d\vec{y}}{dt}\right)^2-\tfrac{1}{2}\kappa(\vec{x}-\vec{y})^2 \quad , \tag{9}$$

 

which describes the interaction of the electron with a particle of mass $M$ via a spring with spring constant $\kappa$. Again the variational parameters are defined by $\kappa/m=(v^2-w^2)$ and $M/m=(v^2-w^2)/w^2$. The trial free energy is then

$$F=F_0-B-A \ , \tag{10}$$

where

$$F_0=\frac{2}{\beta}\ln\left[\frac{\sinh(\beta v/2)}{\sinh(\beta w/2)}\right]-\frac{2}{\beta}\ln\left[\frac{v}{w}\right] \ , \tag{10a}$$

$$B=\frac{(v^2-w^2)}{2v}\left[\coth\left[\frac{\beta v}{2}\right]-\frac{2}{\beta v}\right] \ , \tag{10b}$$

$$A=\int_0^\beta d\tau\int\frac{d^2\vec{K}}{(2\pi)^2}|Q(K)|^2\exp\left[-\frac{k^2F(\tau)}{2}\right]\times[(N+1)e^{-\omega_k\tau}+Ne^{\omega_k\tau}] \ , \tag{10c}$$

and where

$$N=1/(e^{\beta\omega_k}-1) \ .$$

Here

$$F(\tau)=\frac{w^2}{v^2}\tau(1-\tau/\beta)+\frac{v^2-w^2}{v^3}\left[1-e^{-v\tau}+\frac{2(1-\cosh v\tau)}{(e^{\beta v}-1)}\right] \tag{11}$$

is the temperature- and time-dependent response function of the system. It should be noted that we do not include in Eq. (10) the free particle free energy, and free ripplon free energy ($F_r$) because they do not depend on the variational parameters ($v,w$) which determine the effective mass (see below). However, in order to make any detailed comparison of the total free energy with the ground-state energy, they would have to be accounted for.

It is possible to define an effective electron mass for this system in the following way. It is well known[6,7] that if a free particle is put into a weak constant external field, $f$, then the free energy changes (to order $f^2$) by an amount

$$F^{(m)}=-\frac{\hbar^2\beta^2f^2}{24m^*} \ . \tag{12}$$

If we then apply a constant external field to our system, we can define an "effective" electron mass as

$$\frac{1}{m^*}=\frac{-24}{\beta^2}\frac{1}{\hbar^2}\lim_{f^2\to0}\frac{\partial F}{\partial f^2} \ , \tag{13}$$

which becomes, when the path integrals are performed,

$$\frac{m^*}{m}=\left\{\frac{w^2}{v^2}+\left[1-\frac{w^2}{v^2}\right]\frac{6}{\beta v}\left[\coth\left[\frac{\beta v}{2}\right]-\frac{2}{\beta v}\right]\right\}^{-1} \ . \tag{14}$$

Equation (14) reduces to the model mass $v^2/w^2$ at $T=0$.

As in the ground-state problem, the capillary constant is $k_c=(\rho g'/\sigma)^{1/2}$,[4] and we define a coupling constant, $\alpha=(e\mathcal{E})^2/(8\pi\sigma k_c^2/2)$ ($\hbar=m=1$), with energy measured in units of $k_c^2/2=v_c$. We also define a reduced inverse temperature $x_0=\beta v_c$. We minimize the free energy in Eq. (10) and use the values of $v$ and $w$ so obtained to calculate the effective mass from Eq. (14) for various values of $x_0$ and $\alpha$.

## III. RESULTS AND DISCUSSION

The results of our minimization are given in Figs. 1 and 2, where we have plotted the effective mass [from Eq. (14)] versus $\alpha$ for various temperatures, $x_0$; and versus $x_0$ for various values of $\alpha$. The value $x_0=100$ corresponds to $T=17$ mK since $v_c=1.7$ K for a 100-Å-thick helium film. We also show, for comparison, our earlier results for the zero-temperature mass in Fig. 3. The figures show that the mass decreases toward the free electron mass for all values of the coupling constant as the temperature increases (as expected). For high temperature ($x_0=1$) the mass remains free electronlike even as the coupling constant increases. Figure 1 indicates that the strongly coupled dimplelike state has already "melted" for $kT/E_B<1$ [$E_B$ (Ref. 5) is the zero-temperature binding energy of the localized state and at $x_0=100$, $kT/E_B\sim0.75$]; and the sharp transition at $\alpha\approx\frac{1}{2}$ has been considerably broadened and reduced in magnitude (compare Figs. 1 and 3 ).

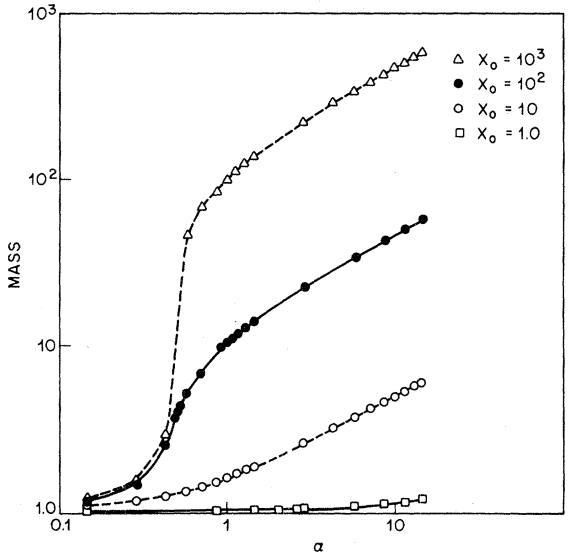


FIG. 1. Temperature-dependent effective mass vs coupling constant $\alpha$ for various temperatures. Here $x_0=k_c^2/(2/kT)$. Points are numerical results, lines are guides to the eye.

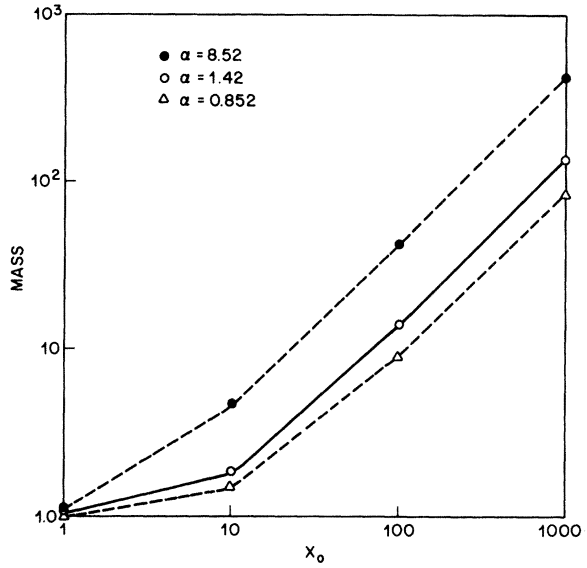


FIG. 2. Effective mass vs temperature for various coupling constants. Points are numerical results, lines are guides to the eye. In both figures, the effective mass is given by

$$\left\{\frac{w^2}{v^2}+\left[1-\frac{w^2}{v^2}\right]\frac{6}{\beta v}\left[\coth\left(\frac{\beta v}{2}\right)-\frac{2}{\beta v}\right]\right\}^{-1} .$$

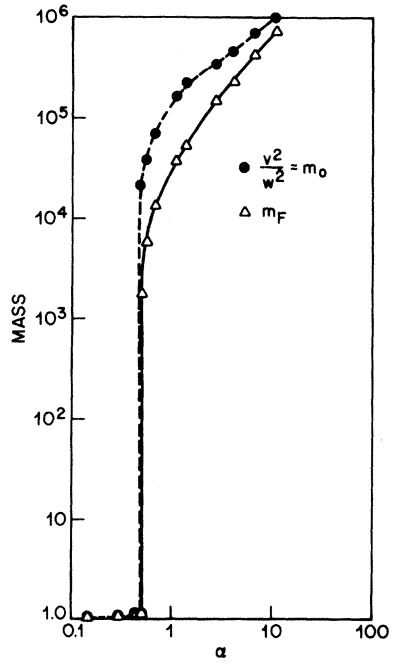


FIG. 3. Model mass ($m_0$) vs coupling constant $\alpha=(T=0)$. Feynman mass ($m_F$) vs coupling constant $\alpha(T=0)$. Mass in units of free-electron mass. Points are numerical results, lines are guides to the eye.

We can obtain an approximate expression for the effective mass in the strongly coupled state, at very low temperature, by using in Eq. (14) the values of the variational parameters obtained from the $T=0$ results[1] with corrections from recoil and internal excited states included. We obtain

$$\frac{v}{v_c}=\sqrt{\alpha} \tag{15}$$

or

$$\beta v = x_0\sqrt{\alpha} \tag{16}$$

and

$$\lambda^2=\frac{w^2}{v^2}=\frac{9}{4}\frac{\eta^2}{\alpha}\left[1-3\frac{\eta}{\sqrt{\alpha}}\right] , \tag{17}$$

where $\eta=\omega_c/(k_c^2/2)$ ($\omega_c=sk_c$ for the linearized ripplon dispersion relation). Then

$$\frac{m^*}{m}\cong\frac{1}{(\lambda^2+6/x_0\sqrt{\alpha})} . \tag{18}$$

Even though the range of validity of this expression is for a somewhat lower temperature than we have investigated numerically, it does show the following. The zero-temperature model mass $\lambda^{-2}$ (being proportional to $\alpha/\eta^2$) $\rightarrow\infty$ as $\eta\rightarrow 0$. However, for finite temperature, the effective mass as we have defined it here remains finite because of the temperature-dependent correction term indicated in Eq. (18). Therefore, the jump in mass, at $\alpha\sim\frac{1}{2}$, remains finite as $\eta\rightarrow 0$ (i.e., as the ripplon velocity vanishes) and the "localization" features of the transition are smeared.

The fact that the dimplelike structure has begun to melt at $T=17$ mK (for a 100-Å film), diminishing the jump in mass, indicates that the temperature has a significant effect and that experiments must be done at quite low temperatures. However, this temperature is attainable, suggesting that experiments to look for the "localization" transition described here and in our earlier papers[1] are difficult, but possible.

## ACKNOWLEDGMENTS

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[4]$k_c = (\rho g'/\sigma)^{1/2}$, where $\rho$, $g'$, $\sigma$, are, respectively, the density, the acceleration of the liquid due to the van der Waals attraction by the substrate and the surface tension.
[5]$E_B$ is defined as the difference in the ground-state energy just above and just below the sharp transition in mass at $T=0$. Although this definition appears arbitrary, $E_B$ defined in this way is always $\sim 22.4$ mK.
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