Brief Reports

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Experimental evidence for mutual friction between ^{3}He and superfluid ^{4}He

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In the theoretical description of ^{3}He flowing through superfluid ^{4}He it is generally assumed that below 500 mK the ^{4}He can be treated as a "mechanical" vacuum. With the use of dilution refrigerators it is shown in various ways that this assumption is an oversimplification. In one experiment the effects are demonstrated with a double mixing chamber. In another experiment a flow impedance is installed in the dilute outlet tube of a single mixing chamber. Furthermore, the invalidity of the mechanical-vacuum model is demonstrated by experiments in which the properties of these systems are substantially modified by shunting a flow impedance by a superleak.

In the theoretical description of ^{3}He flowing through superfluid ^{4}He it is generally assumed that below 500 mK the ^{4}He component can be treated as a "mechanical" vacuum,[1] i.e., the force F_{34} due to mutual friction between the two components of the mixture is zero:

$$F_{34}=0 \; . \tag{1}$$

This means in particular that the viscosity η_m of the ^{3}He quasiparticle gas, measured in an experiment where the mixture was flowing as a whole,[2] also determines the flow properties when only the ^{3}He component is moving (as in dilution refrigerators).

A further consequence is that the hydrodynamic equation for superfluid ^{4}He in the stationary state obtains the familiar form

$$\vec{\nabla}\mu_4=0 \; . \tag{2}$$

Equations (1) and (2) constitute the starting point of the hydrodynamics of ^{3}He- ^{4}He mixtures as in the dilute side of dilution refrigerators. In this paper we will show that in many experimental situations the ^{4}He *cannot* be considered as a mechanical vacuum. This would imply that a mutual friction has to be taken into account. The evidence is supplied by two experiments. In the first experiment the effects are demonstrated using a double mixing chamber[3] (DMC); in the second experiment a small tube is installed in the dilute outlet tube of a (single) mixing chamber. In both experiments the violation of Eqs. (1) and (2) will also be demonstrated by shunting a tube by a superleak, using the general property of superleaks that $\Delta\mu_4$ is zero. If $\Delta\mu_4$ was originally equal to zero, the properties of the system will not be affected by the superleak. On the other hand, if installing a superleak does change the characteristics of a system, it may be concluded that $\Delta\mu_4$ was originally unequal to zero.

The theoretical prediction of the behavior of a double mixing chamber, based on Eqs. (1) and (2), is described extensively by Coops *et al.*[3] It is derived that

$$g\Delta\rho\Delta h=\Pi_0(T_1^2-T_2^2) \tag{3a}$$

and

$$g\Delta\rho\Delta h=Z_1\dot{n}_1V_d\eta_m \; . \tag{3b}$$

In these equations g is the gravitational acceleration, $\Delta\rho$ is the difference between the densities of the dilute and concentrated phases, Δh is the difference in level of the phase boundaries in the two mixing chambers, Π_0 is a coefficient determined in such a way that Π_0T^2 is the temperature-dependent term in the osmotic pressure along

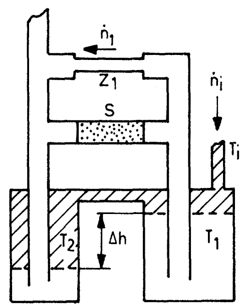

FIG. 1. Schematic diagram of a double mixing chamber with a superleak. It shows the impedance Z_1, the superleak S, and the level difference of the phase boundaries Δh.

the phase-separation curve, T_1 and T_2 are the temperatures of the first and second mixing chamber, respectively, Z_1 is the flow impedance of the dilute outlet tube of the first mixing chamber, $\dot{n}_1$ is the number of ^{3}He moles diluted per second in the first mixing chamber, and V_d is the volume of 1 mole of ^{3}He in the dilute phase (see also Fig. 1).

The value of Δh is measured by two capacitive level detectors, placed vertically in the mixing chambers. Each detector consists of two thin-walled concentric tubes with an annular space of 0.5 mm. A vertical, 0.5-mm-wide slit is machined in the tubes to ensure free accesss of the liquids to the annular space. The capacitive coupling between the two tubes is a linear function of the level of the phase boundary.

From Eq. (3a), values of Δh in the order of 10—20 cm were expected. However, the *measured* values of Δh turned out to be very small. They were in order of 1 cm and practically flow and temperature independent. These observations cannot be satisfactorily explained on the basis of Eqs. (3a) and (3b). Changing the dimensions of the various tubes in the systems (also Z_1) did not change the observations essentially. However, shunting Z_1 by a superleak, as depicted in Fig. 1, resulted in a larger Δh, which was flow and temperature dependent. Furthermore, the flow and temperature distribution in the DMC was changed significantly (Fig. 2), clearly demonstrating that Eq. (2) was not satisfied in Z_1.

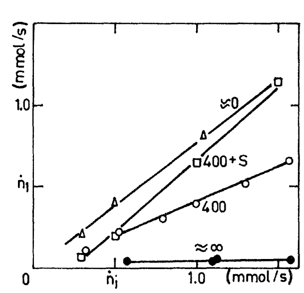

FIG. 2. Typical $\dot{n}_1$-$\dot{n}_i$ dependences calculated from the measured $\dot{n}_i$, T_i, and T_2 for several different impedances Z_1: Δ, $Z_1 << 400$ mm^{-3}; ●, $Z_1 >> 400$ mm^{-3}; ○, $Z_1 = 400$ mm^{-3}; □, $Z_1 = 400$ mm^{-3}, shunted by a superleak.

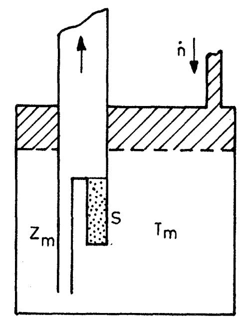

FIG. 3. Schematic diagram of a single mixing chamber showing the locations of Z_m and S.

In order to investigate the eventual invalidity of Eqs. (1) and (2) in a different way, we performed a series of experiments modifying the dilute exit tube of a single mixing chamber. First we measured the mixing chamber temperature T_m as a function of the flow rate $\dot{n}$ without flow impedance in the exit tube (Fig. 4, curve 0). In the next step we measured the T_m-$\dot{n}$ dependence with a tube Z_m (length, 23 mm, i.d., 0.8 mm, $Z_m = 2300$ mm^{-3}) installed. If Eq. (1) would be valid there would be a pressure drop in Z_m in the order of 200 Pa. Since this is much smaller than the osmotic pressure in the mixing chamber (2000 Pa), it is expected from Eq. (2) that Z_m will have no harmful effect on the operation of the machine; only a minor decrease of the ^{3}He concentration in the still liquid is expected. However, the installation of Z_m completely disor-

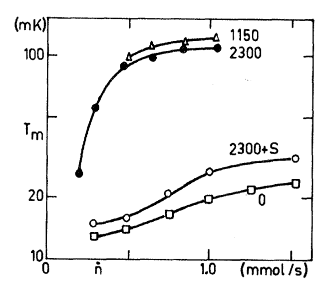

FIG. 4. Measured T_m-$\dot{n}$ dependences of a single mixing chamber with several different impedances Z_m: □, open exit tube; △, $Z_m = 1150$ mm^{-3}; ●, $Z_m = 2300$ mm^{-3}; ○, $Z_m = 2300$ mm^{-3}, shunted by a superleak.

dered the operation of the machine. The temperature T_m was as high as 100 mK when $\dot{n}=1$ mmol/s. The still temperature was high and about 50% ^{4}He was circulated with the ^{3}He ($\dot{n}$ in Fig. 4 represents the total flow rate), indicating a very low ^{3}He concentration in the liquid of the still. Similar to the case of a double mixing chamber we shunted Z_m by a superleak. This situation is depicted in Fig. 3. As can be seen in Fig. 4, the refrigerator operated in practically the same way as without flow impedance in the exit tube.

From the experiments described above we conclude that Eqs. (1) and (2) have a limited applicability to the hydrodynamics of ^{3}He-^{4}He mixtures. A mutual friction term has to be included. The magnitude of the effect is large. The invalidity of Eqs. (1) and (2) has important implications for the calculations concerning the dilute side of a ^{3}He-circulating dilution refrigerator such as tube dimensions and the influence of these dimensions on the low-temperature limit of dilution refrigerators. The interpretation of ^{4}He-circulating dilution refrigerators (in which ^{4}He is used to drive ^{3}He through the mixing chamber) will probably also have to be revised. The mutual friction between ^{3}He and ^{4}He is presently the subject of more systematic investigations in our laboratory.

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[1] I. M. Khalatnikov, *An Introduction to the Theory of Superfluidity* (Benjamin, New York, 1965), Chap. 24; J. C. Wheatley, Am. J. Phys. 36, 181 (1968).

[2] K. A. Kuenhold, D. B. Crum, and R. E. Sarwinski, *Low Temperature Physics* LT-13, edited by K. D. Timmerhaus, W. J. O'Sullivan, and E. F. Hammel (Plenum, New York, 1972), Vol. 1, p. 563.

[3] G. M. Coops, A. T. A. M. de Waele, and H. M. Gijsman, Cryogenics 19, 659 (1979); G. M. Coops, thesis, Eindhoven, 1981 (unpublished).