X-ray scattering from liquid ^{3}He-^{4}He mixtures

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Structure factor of liquid He mixtures of ^{3}He concentration $x=0.11$, 0.20, 0.24, 0.26, 0.31, 0.35, and 0.40 at $T=1.7$ K have been obtained experimentally by x-ray scattering. The structure factor shows a lowering of the peak height and a shift to smaller momenta in its position for the increasing ^{3}He concentration. Furthermore, it presents some fixed points of scattering momentum at which the values of the structure factors are not influenced by the variation of ^{3}He concentration. In order to aid the understanding of the origin of these fixed points, a simple model which is an extension of the Bogoliubov theory to the mixture has been constructed. The structure factor calculated on this model shows a good agreement with the experiment. The fixed points in the model turn out to be at interaction-free momenta.

I. INTRODUCTION

A liquid He mixture provides us with an interesting boson-fermion system both as a Bose system which exhibits superfluidity in the presence of fermions and as a Fermi system with variable Fermi temperature. It forms an actual stage on which the validity of the many-body theories of strongly interacting bosons and fermions are tested. Various thermodynamical and hydrodynamical experiments have been made on this system, but we are still missing some basic information such as the dispersion relation or the structure factor of the system. As for the dispersion, there has been a discrepancy in the experimental evidences between that derived from indirect measurements, such as the velocity of fourth sound,[1] the normal fluid density,[2] and the ion mobility,[3] and that observed by a more direct one, such as Raman scattering[4,5] and neutron scattering.[6,7] The indirect measurements suggested a shift[8] of the ^{4}He roton energy by mixing ^{3}He atoms while the latter denied such a shift. Theoretical work has attributed the erroneous roton energy shift to the unsuitable assumed shape of the ^{3}He excitation spectrum[9–12] used in the analysis and/or to the hybridization effect of ^{3}He and ^{4}He excitations.[13] Recently Lücke and Szprynger[14] have constructed a semiphenomenological theory concerned with the density fluctuation spectrum of a dilute mixture of ^{3}He in He II in which they stressed the significance of the change in the structure caused by mixing. By using our experimental $S(k)$ data[15] as an input, they succeeded in the quantitative explanation of the neutron result on the energy shift of the excitation spectrum from that of pure ^{4}He. Structure factor of the liquid He mixture, therefore, can be partly an important quantity which gives us useful information on the excitations of the system.

In a previous paper[15] we reported on our study of the x-ray scattering from the 6.3 at.% and 13.2 at.% liquid He mixtures. The study has been developed to a wider range of ^{3}He concentrations and we will report on its details in this paper. In Sec. II we describe our experimental setup and measuring procedures. Results obtained are discussed in Sec. III. In order to understand the behavior of the obtained structure factors we carry out a model calculation and it is compared to the experimental result in Sec. IV.

II. APPARATUS

The apparatus is basically the same as that used in our previous study[15] except for several refinements such as a sample cell or a data storage system.

In Fig. 1 we show the cryostat and the gas handling system. The stainless-steel metal Dewar contains about 2 l of liquid helium and the temperature of the sample cell can be lowered to about 1.6 K by pumping the small pot just above the cell. The beryllium x-ray windows of thickness 0.2 mm were attached to the Dewar with epoxy resin. The sample cell (Fig. 2) is constructed with a beryllium cylinder (thickness 0.3 mm, 15 mm o.d.) which was attached to the oxygen-free highly-conductivity copper (OFHC) lids with supertight epoxy resin.[16]

The x-ray beam was derived from the line source of a Rigaku-denki D-9C (copper) x-ray tube. Its automatic-voltage regulator (AVR) has a stability of output voltage and current fluctuation within ±0.1% against the 10% deviation of the input voltage and output fluctuation within ±0.1% during the continuous operation of 24 h. The incident x-ray beam was Cu $K\alpha$ ($\lambda=1.54$ Å) produced by the tube operation at 35 kV, 14 mA and was detected by a scintillation counter after passing a 15-μm nickel filter with which

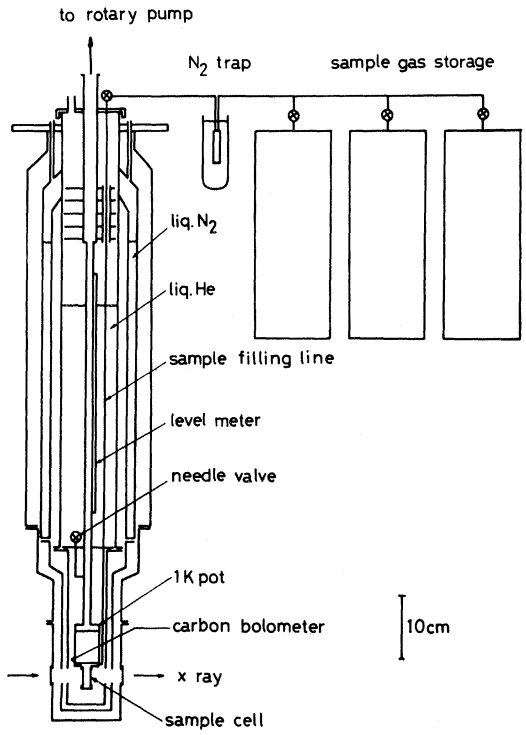

FIG. 1. Cryostat assembly.

the $K\beta$ contribution was reduced to 1% of the $K\alpha$ contribution. Collimation was accomplished by means of a 4.9° Soller slit,[17] a 1° divergence slit, a 1° scatter slit, and a 0.4-mm receiving slit. The angular resolution was measured to be 0.17 Å^{-1} at $q=0.6$ Å^{-1}, 0.10 Å^{-1} at $q=2.0$ Å^{-1}, and 0.08 Å^{-1} at $q=3.7$ Å^{-1}.

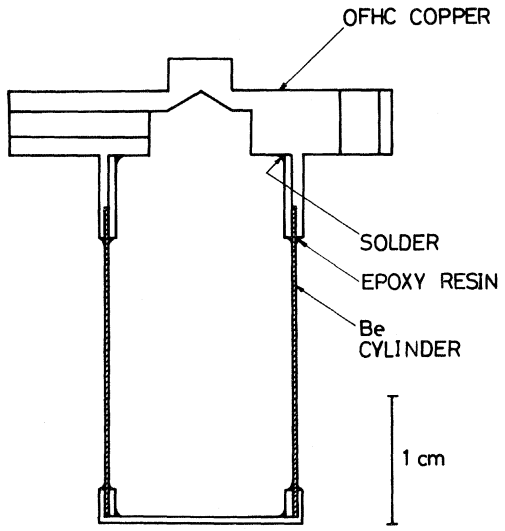

FIG. 2. Sample cell. A beryllium cylinder (thickness 0.3 mm, 15-mm o.d.) is attached to the OFHC lids with super-tight epoxy resin.

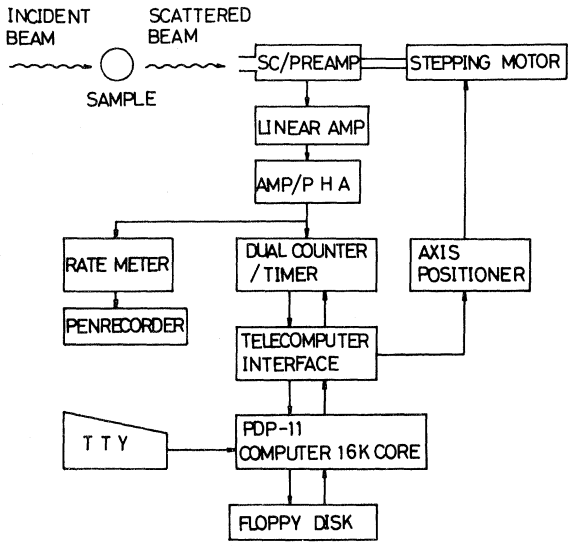

FIG. 3. Data collection system.

The data collection system is shown in Fig. 3. The scattered beam detected by a scintillation counter (SC) was converted to voltage pulses, which were then amplified to enter the pulse-height analyzer (PHA). The following dual counter/timer has two operating modes: fixed time mode (FTM) and fixed counts mode (FCM). We made use of the FCM and the data were collected four times at each angle. Collected data were sent to a Canberra PDP-11 minicomputer and were stored in its core-memory. After this the scattering angle was changed by use of a stepping motor where the scattering angles were programmed previously.

III. RESULTS AND DISCUSSION

The liquid structure factor is related to the measured x-ray scattering intensity I through the expression[18]

$$S=\frac{IT_1\rho_1}{I_1T\rho}\left(S_1+\frac{\sigma_i}{\sigma_e}\right)-\frac{\sigma_i}{\sigma_e}\ , \tag{1}$$

where ρ and T are the number density and the transmission coefficient of the liquid under study, and σ_e and σ_i are the elastic and inelastic scattering cross sections. I_1, ρ_1, and T_1 are the same respective quantities for pure liquid ^{4}He whose structure factor S_1 is known to us. In Eq. (1) the scattering cross sections are assumed to be equal both for ^{3}He and ^{4}He atoms, and they are obtained from the work of Kim and Inokuti.[19] Transmission coefficients T and T_1 are calculated from the x-ray mass absorption coefficient ($\mu/\rho=0.383\,12$ cm^2/g) of He atoms and the number density of the sample liquid.

One experiment consists of three stages: (a) empty-cell scattering, (b) liquid ^{4}He scattering, and (c) sample liquid scattering. These three steps were followed for every sample liquid, and for the first two, not only their relative differences but also each of their absolute values appeared reproducibly within statistical errors. In Fig. 4 we show the obtained net intensity, (c)−(a), of the liquid He mixture (open circles). The height of the peak is lowered and its position is shifted to the smaller momentum transfer for increasing ^{3}He concentrations. The dashed and solid lines represent the smoothed value for each liquid He mixture and liquid ^{4}He, respectively. In order to obtain a good approximation to the functions, the data were classified into three regions regarding momentum, i.e., a low-momentum, a middle-momentum around the peak, and a high-momentum region. In each region, a fifth-order polynomial was computed to fit the data using the least-square method. As we can see in the figure the mismatching of the neighboring functions was within statistical errors, and they are negligible.

The smoothed values are then put into the I and I_1 in Eq. (1) to give us the structure factor in Fig. 5, where the obtained structure factors are shown for three representative ^{3}He concentrations. For the values of S_1 we adopted the neutron data by Cowley and Woods.[20] Error bars on the plot of $x=0.11$ structure factor were estimated from the statistical deviation with each of the (c)−(a) intensities described above and do not contain the systematic ones propagated from the S_1 data. Because the ine-

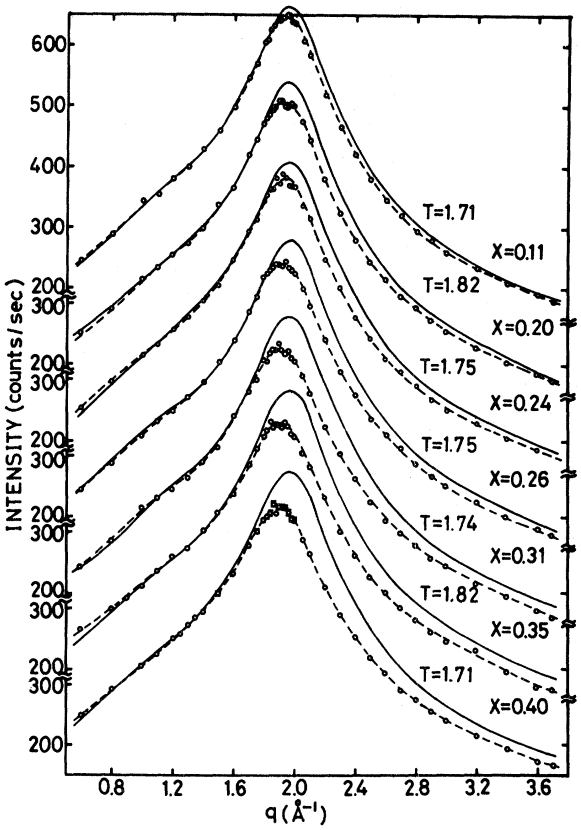

FIG. 4. Net scattered intensity from liquid He mixture. The dashed and solid line represent the smoothed value of the intensity for each liquid He mixture and each liquid ^{4}He, respectively.

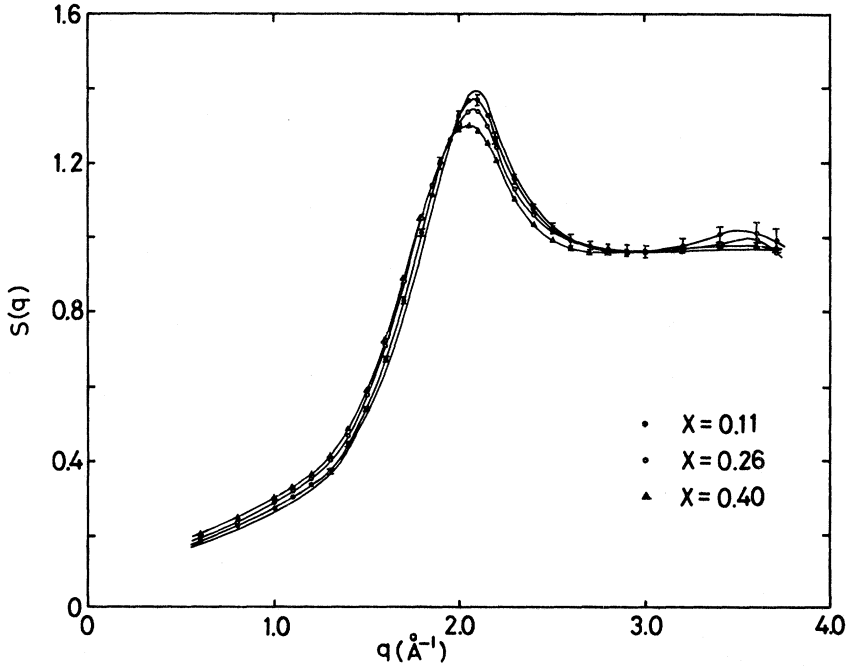

FIG. 5. $S(q)$ for liquid He mixture. $S(q)$ value for $x=0$ (solid line) is normalized to the neutron result for pure liquid ^{4}He by Cowley and Woods (Ref. 20).

lastic scattering becomes dominant in the total intensity for $q \gtrsim 3.0$ Å^{-1}, the structure factor had to be calculated from the small difference between large quantities which resulted in an increase of errors. As pointed out at Fig. 4, effects of the ^{3}He atoms can be characterized as the lowered peak and the shift towards the smaller momentum transfer of the peak position. The former can be understood by the larger zero-point energy of the ^{3}He atoms while the latter by the increase of the interatomic spacing due to the extended volume of ^{3}He atoms. In Fig. 6 we show the x dependence of the peak height $S_{\max}$ and its position $q_{\max}$. The solid line represents the calculated results of the uniform dilation (UD) approximation in which the system is assumed to be enlarged uniformly by a factor of $[\rho(0)/\rho(x)]^{1/3}$ in length. In this approximation a replacement of q with a new scaled momentum transfer $q' = (\rho'/\rho)^{1/3}q$ will give us a new structure factor for the dilated system, $S'(q') = S(q)$. It follows that

$$S'_{\max} = S_{\max} \ , \tag{2a}$$

$$q'_{\max} = (\rho'/\rho)^{1/3} q_{\max} \ . \tag{2b}$$

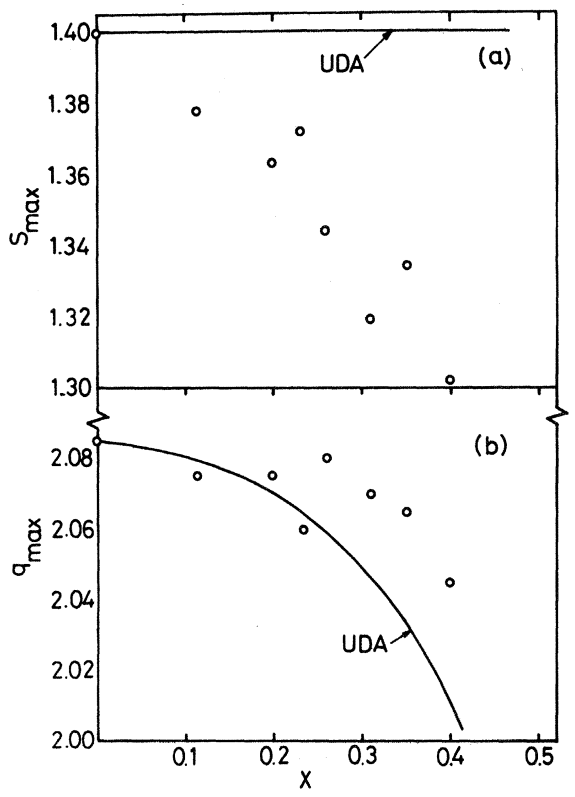

FIG. 6. $S(q)$ peak height (a) and its position (b) as a function of x. Experiment (open circles) showed a notable decrease in peak height and a slight shift to the smaller momenta in its position. The solid lines represent the calculated results of the uniform dilation approximation (UDA) in which the system is assumed to be enlarged uniformly by a factor of $[\rho(0)/\rho(x)]^{1/3}$ in length.

As can be seen from the figure, however, the experimental results of not only $q_{\max}$ but also $S_{\max}$ turned out to be decreasing with respect to x. In addition, the experimental results of $q_{\max}$ seem less dependent on x than that of the UD theory. These tendencies of $S_{\max}$ and $q_{\max}$ are quite analogous to those in pres-

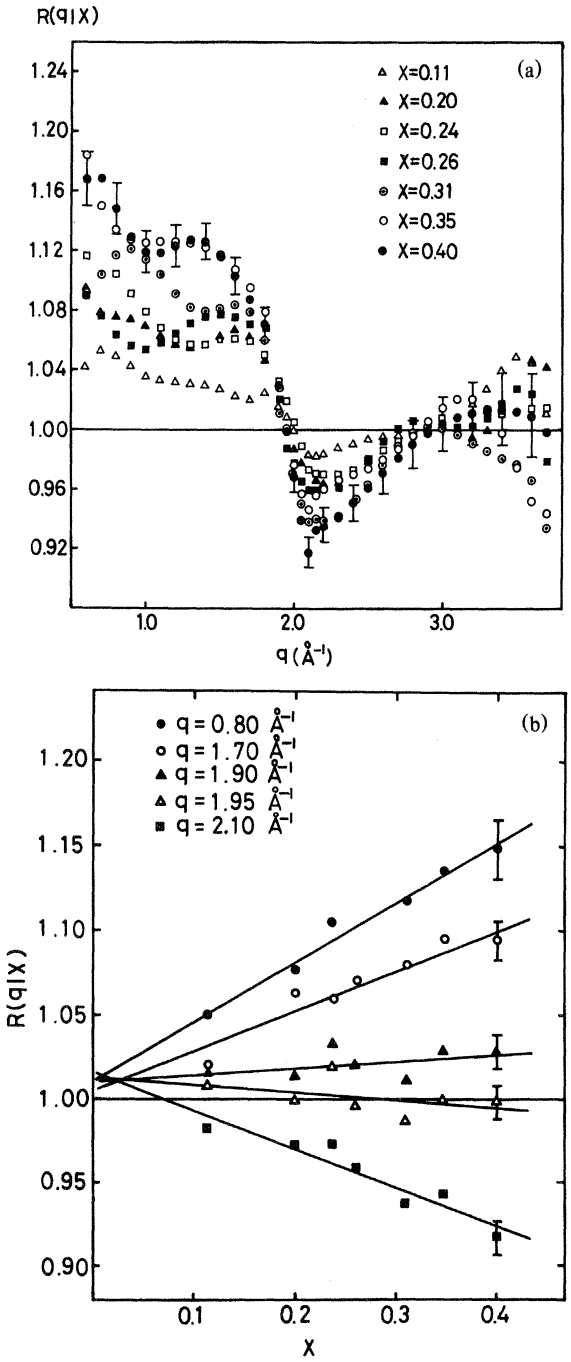

FIG. 7. $R(q|x)$ for liquid He mixture. (a) $R(q|x)$ vs q. One can see an oscillatory behavior about the nodes of fixed points. (b) $R(q|x)$ vs x. Solid lines are drawn by linear approximation of the data.

TABLE I. Table of the observed structure factor $S(q|x)$ and the ratio $R(q|x)=S(q|x)/S(q|x=0)$.

x	0.11		0.20		0.24		0.26		0.31		0.35		0.40	
q	$S(q\|x)$	$R(q\|x)$	$S(q\|x)$	$R(q\|x)$	$S(q\|x)$	$R(q\|x)$	$S(q\|x)$	$R(q\|x)$	$S(q\|x)$	$R(q\|x)$	$S(q\|x)$	$R(q\|x)$	$S(q\|x)$	$R(q\|x)$
0.60	0.183	1.043	0.193	1.096	0.196	1.116	0.192	1.090	0.192	1.092	0.210	1.191	0.205	1.168
0.70	0.199	1.053	0.204	1.079	0.210	1.113	0.203	1.076	0.209	1.104	0.218	1.153	0.221	1.168
0.80	0.227	1.049	0.232	1.076	0.238	1.104	0.230	1.063	0.241	1.117	0.245	1.134	0.248	1.148
0.90	0.255	1.042	0.263	1.074	0.267	1.091	0.258	1.056	0.274	1.121	0.276	1.127	0.276	1.129
1.00	0.280	1.036	0.289	1.070	0.291	1.079	0.285	1.054	0.301	1.115	0.304	1.125	0.302	1.119
1.10	0.304	1.033	0.313	1.063	0.314	1.068	0.311	1.058	0.325	1.104	0.331	1.126	0.329	1.118
1.20	0.334	1.032	0.342	1.057	0.343	1.060	0.344	1.064	0.353	1.091	0.364	1.126	0.363	1.123
1.30	0.377	1.031	0.386	1.055	0.387	1.057	0.392	1.071	0.396	1.082	0.412	1.125	0.412	1.127
1.40	0.443	1.030	0.455	1.057	0.455	1.057	0.463	1.076	0.465	1.079	0.483	1.122	0.485	1.126
1.50	0.539	1.027	0.558	1.063	0.556	1.060	0.565	1.077	0.568	1.081	0.586	1.116	0.587	1.117
1.60	0.667	1.023	0.696	1.067	0.692	1.061	0.701	1.075	0.707	1.084	0.722	1.107	0.719	1.103
1.70	0.828	1.020	0.862	1.063	0.859	1.059	0.868	1.070	0.876	1.079	0.888	1.094	0.882	1.087
1.80	1.018	1.025	1.039	1.046	1.042	1.050	1.062	1.070	1.052	1.060	1.071	1.079	1.063	1.071
1.90	1.196	1.015	1.193	1.013	1.215	1.032	1.201	1.020	1.191	1.011	1.210	1.028	1.211	1.028
1.95	1.272	1.008	1.261	0.999	1.287	1.019	1.257	0.996	1.247	0.987	1.263	1.000	1.262	0.999
2.00	1.330	0.999	1.314	0.987	1.337	1.005	1.300	0.977	1.287	0.967	1.298	0.975	1.289	0.968
2.05	1.373	0.990	1.356	0.978	1.370	0.989	1.337	0.965	1.317	0.950	1.325	0.956	1.302	0.939
2.10	1.372	0.982	1.359	0.972	1.359	0.973	1.341	0.959	1.311	0.938	1.320	0.944	1.281	0.917
2.15	1.336	0.982	1.314	0.966	1.321	0.971	1.305	0.959	1.279	0.940	1.298	0.954	1.269	0.933
2.20	1.270	0.984	1.244	0.964	1.253	0.970	1.237	0.958	1.212	0.939	1.237	0.958	1.207	0.935
2.30	1.157	0.988	1.131	0.965	1.137	0.970	1.126	0.961	1.103	0.941	1.130	0.964	1.104	0.942
2.40	1.079	0.991	1.058	0.972	1.060	0.973	1.056	0.969	1.035	0.950	1.056	0.969	1.035	0.951
2.50	1.028	0.994	1.016	0.982	1.012	0.979	1.014	0.981	0.996	0.963	1.008	0.974	0.994	0.961
2.60	0.995	0.995	0.992	0.992	0.986	0.986	0.992	0.992	0.976	0.976	0.980	0.980	0.971	0.971
2.70	0.976	0.996	0.980	1.001	0.973	0.993	0.981	1.001	0.969	0.989	0.968	0.988	0.962	0.981
2.80	0.967	0.997	0.974	1.005	0.969	0.999	0.975	1.006	0.967	0.998	0.966	0.997	0.960	0.990
2.90	0.964	0.999	0.969	1.004	0.969	1.005	0.971	1.006	0.967	1.002	0.971	1.006	0.963	0.998
3.00	0.966	1.002	0.965	1.001	0.972	1.008	0.968	1.004	0.965	1.001	0.978	1.015	0.968	1.004
3.10	0.972	1.008	0.961	0.997	0.974	1.010	0.966	1.002	0.961	0.997	0.984	1.020	0.972	1.008
3.20	0.981	1.017	0.961	0.995	0.976	1.010	0.967	1.002	0.957	0.991	0.985	1.020	0.976	1.011
3.30	0.994	1.028	0.966	1.000	0.977	1.011	0.974	1.008	0.953	0.986	0.979	1.013	0.979	1.013
3.40	1.008	1.041	0.979	1.012	0.979	1.011	0.985	1.018	0.950	0.981	0.965	0.998	0.981	1.014
3.50	1.018	1.050	0.999	1.030	0.982	1.013	0.997	1.028	0.947	0.977	0.947	0.977	0.982	1.013
3.60	1.016	1.045	1.018	1.047	0.987	1.015	0.996	1.024	0.939	0.966	0.929	0.955	0.982	1.010
3.70	0.987	1.011	1.019	1.043	0.992	1.015	0.956	0.979	0.912	0.934	0.925	0.947	0.975	0.999

surized liquid ^{4}He in an opposite sense. Henshaw[21] has studied the pressure dependence of the structure factor of liquid ^{4}He by means of a neutron scattering. In this case S_{max} and q_{max} increased with the increasing pressure and the latter was less dependent on the pressure (or density) than that of the UD theory. The increase and decrease in S_{max} correspond to the localization (pressurized liquid ^{4}He) and delocalization (liquid He mixture) of the atoms in the liquid, respectively. This effect also explains the rather small deviation of q_{max} in both experiments. Therefore effects of pressurizing and mixing ^{3}He atoms on liquid ^{4}He stand opposite to each other with respect to the localization and delocalization of the atoms in the system.

One of the other features of the structure factor in Fig. 5 is the fact that the curves seem to cross each other at several "fixed points." To see the behavior of the structure factor more explicitly, we have calculated the ratio

$$R(q|x)=S(q|x)/S(q|0) \tag{3}$$

shown in Fig. 7(a). The $R(q|x)$ possesses several features as follows:

(1) There exist two characteristic momenta q_1 and q_2, and $R(q)$ presents an oscillation as

$$R(q)\geq 1 \quad \text{for } q\leq q_1 ,$$
$$R(q)\leq 1 \quad \text{for } q_1\leq q\leq q_2 ,$$
$$R(q)\geq 1 \quad \text{for } q\geq q_2 .$$

(2) The values of q_1 and q_2 are nearly independent of x.

(3) The amplitude of the oscillation is an increasing function of x. The third feature can more clearly be seen in Fig. 7(b) where the x dependence of $R(q|x)$ for various values of q is given. The solid lines in the figure were drawn by linear approximation of the data.

The values of $S(q|x)$ and $R(q|x)$ are listed in Table I.

In Sec. IV we present our model calculation with an aim to explain the above-mentioned features of our experimental result.

IV. ELEMENTARY EXCITATION MODEL CALCULATION FOR A BOSE-FERMI MIXTURE

Although various theroretical work has been focused on the static and dynamic structure of this system, no prediction has ever been made concerning our observed fixed points of the structure factor. In this section we construct a simple model called binary-boson approximation (BBA) which is basically an extension of the Bogoliubov[22] theory to the mixture case. It is often pointed out that the Bogoliubov theory corresponds to the lowest-order calculation of dynamical[23] or variational[24] theory. We show that our model well explains the experimental result in spite of its simplicity.

We adopt the same Hamiltonian that we used in our previous paper [Eqs. (4a)–(4d) in Ref. 15]. Next we consider a partial-density Green's function representing the correlation between the partial-density fluctuation of ^{3}He or ^{4}He atoms and the total-density fluctuation defined by

$$\chi_q^\alpha(t)=-i\Theta(t)\langle[\rho_q^\alpha(t),\rho_{-q}(0)]\rangle \quad (\alpha=3,4) , \tag{4}$$

where $\Theta(t)$ is the unit function [$\Theta(t)=1$ for $t>0$, $\Theta(t)=0$ for $t<0$]. The angle brackets represent a ground-state average for $T=0$ and a statistical ensemble average for $T\neq 0$. This partial Green's function satisfies an equation of motion which is Fourier transformed to be

$$\omega\chi^\alpha(q,\omega)=\chi^{\alpha'}(q,\omega) , \tag{5a}$$

$$\chi^{\alpha'}(q,\omega)=\sum_p[\epsilon_\alpha(p+q)-\epsilon_\alpha(p)]\chi_p^\alpha(q,\omega) , \tag{5b}$$

where $\epsilon_\alpha(p)=\hbar^2p^2/2m_\alpha$. In the derivation of Eq. (5) we used the random-phase approximation (RPA) in which only the terms associated with $k=q$ in the interaction summation are left to survive while the product of two parameters in a commutator is replaced by its expectation value.

In order to close the set of equations, we again consider an equation of motion of the new Green's function

$$\omega\chi^{\alpha'}(q,\omega)=\sum_p[\epsilon_\alpha(p+q)-\epsilon_\alpha(p)]\omega\chi_p^\alpha(q,\omega)$$
$$=2\epsilon_\alpha(q)N_\alpha+\sum_p\left(\epsilon_\alpha(q)^2+\hbar^4\frac{\vec{p}\cdot\vec{q}}{m_\alpha}\frac{(\vec{p}+\vec{q})\cdot\vec{q}}{m_\alpha}\right)\chi_p^\alpha(q,\omega)+2\epsilon_\alpha(q)\frac{N_\alpha}{\Omega}v(q)\chi(q,\omega) , \tag{6}$$

where N_α represents the total number of $^\alpha$He atoms. The term $\hbar^4(\vec{p}\cdot\vec{q}/m_\alpha)\cdot(\vec{p}+\vec{q})\cdot\vec{q}/m_\alpha$ on the right-hand side is considered to have the order of $\hbar^2q^2/m_\alpha$ × (average kinetic energy of He atoms). For bosons ($\alpha=4$) we can neglect this term on account of the Bose-Einstein condensation. In fact, for pure bosons this neglect corresponds to the Bogoliubov approximation and the resulting solution has a pole at the bogolon energy. For fermions ($\alpha=3$) no condensation in momentum space can be expected. But even in this situation a comparison with other terms allows us to neglect this term up to $x\simeq 0.4$ in a relatively

low-temperature region.

This approximation consequently squeezes the fermions' particle-hole continuum into a discrete single mode as if it were a boson branch. In this sense we call this a binary-boson approximation (BBA).

The final equations obtained are

$$\chi(q,\omega)=\chi^4(q,\omega)+\chi^3(q,\omega) \ , \tag{7a}$$

$$\omega\chi^4(q,\omega)=\chi^{4'}(q,\omega) \ , \tag{7b}$$

$$\omega\chi^{4'}(q,\omega)=2\epsilon_4(q)N_4+\epsilon_4(q)^2\chi^4(q,\omega) +2\epsilon_4(q)\frac{N_4}{\Omega}v(q)\chi(q,\omega) \ , \tag{7c}$$

$$\omega\chi^3(q,\omega)=\chi^{3'}(q,\omega) \ , \tag{7d}$$

$$\omega\chi^{3'}(q,\omega)=2\epsilon_3(q)N_3+\epsilon_3(q)^2\chi^3(q,\omega) +2\epsilon_3(q)\frac{N_3}{\Omega}v(q)\chi(q,\omega) \ . \tag{7e}$$

They are easily solved to show

$$\chi(q,\omega)=\chi^4(q,\omega)+\chi^3(q,\omega) \ , \tag{8a}$$

$$\chi^4(q,\omega)=2\epsilon_4(q)N_4\frac{\omega^2-\epsilon_3(q)^2}{(\omega^2-\omega_+^2)(\omega^2-\omega_-^2)} \ , \tag{8b}$$

$$\chi^3(q,\omega)=2\epsilon_3(q)N_3\frac{\omega^2-\epsilon_4(q)^2}{(\omega^2-\omega_+^2)(\omega^2-\omega_-^2)} \ , \tag{8c}$$

where

$$\omega_\pm^2=\tfrac{1}{2}(E_4(q)^2+E_3(q)^2 \pm\{[E_4(q)-E_3(q)]^2+4V_3^2V_4^2\}^{1/2}) \ , \tag{9a}$$

$$E_\alpha(q)=[\epsilon_\alpha(q)^2+V_\alpha^2]^{1/2} \ , \tag{9b}$$

$$V_\alpha^2=2\epsilon_\alpha(q)\frac{N_\alpha}{\Omega}v(q) \ . \tag{9c}$$

If we put $\alpha=4$ in Eq. (9b), the bogolon expression reappears. In Fig. 8 dispersions that are obtained tell us that these two modes are a consequence of mutual interactions between mass 4 and mass 3 bogolons. The excess kinetic energy of mass 3 atoms inhibits the formation of a rotonlike minimum in its dispersion at $x=1$.

The structure factor is connected with the density Green's function through

$$S(q)=\frac{1}{N}\int S(q,\omega)\,d\omega \ , \tag{10}$$

$$S(q,\omega)=-\frac{1}{\pi}\frac{1}{1-e^{-\omega/kT}}\mathrm{Im}\chi(q,\omega) \ . \tag{11}$$

On introducing small imaginaries in the denominators of Eqs. (8b) and (8c), the Green's functions are found to have a δ-function singularity in its imaginary part. Hence, the integration in Eq. (10) for

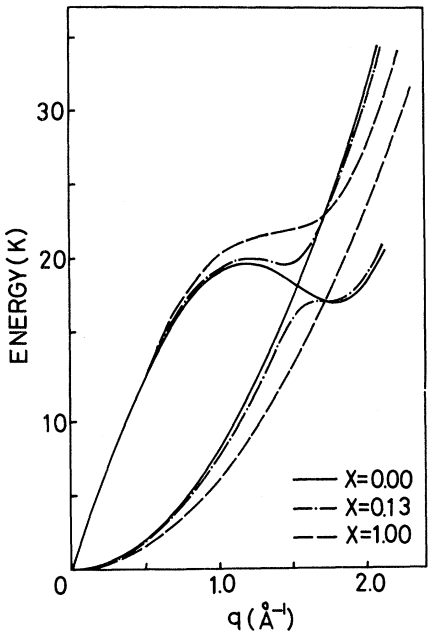

FIG. 8. Dispersions of a Bose-Fermi mixture obtained from BBA calculation.

absolute zero is easily carried out to give the result

$$S(q|x)=\frac{1}{\omega_++\omega_-}\left[[x\epsilon_3(q)+(1-x)\epsilon_4(q)] +\frac{\epsilon_3(q)\epsilon_4(q)}{\omega_+\omega_-}[x\epsilon_4(q)+(1-x)\epsilon_3(q)]\right] . \tag{12}$$

As a check, if we put $x=0$, Eq. (12) is reduced to

$$S(q|0)\equiv S_0(q)=\left[1+4m_4\frac{N_4}{\Omega}v(q)/\hbar^2q^2\right]^{-1/2} \ . \tag{13}$$

This is the simplest theoretical expression for the structure factor of a Bose system which is produced by connecting the Bogoliubov excitation spectrum with the Bijl[25]-Feynman[26] relation. Therefore the expression obtained in Eq. (12) is associated with the extension of both theories to the mixture case. In order to calculate the structure factor we have to settle the interaction potential $v(q)$. Here we adopt the soft-core-square-well potential

$$v(r)=\begin{cases}V_a, & 0<r<a\\ -V_b, & a<r<b\\ 0, & r>b\end{cases} \tag{14}$$

which appeared in the theory of Sunakawa *et al.*[27] Its comparison with the Lennard-Jones 6-12 potential is shown in Fig. 9 for $a=2.5$ Å, $b=4.5$ Å, $V_a=40$ K,

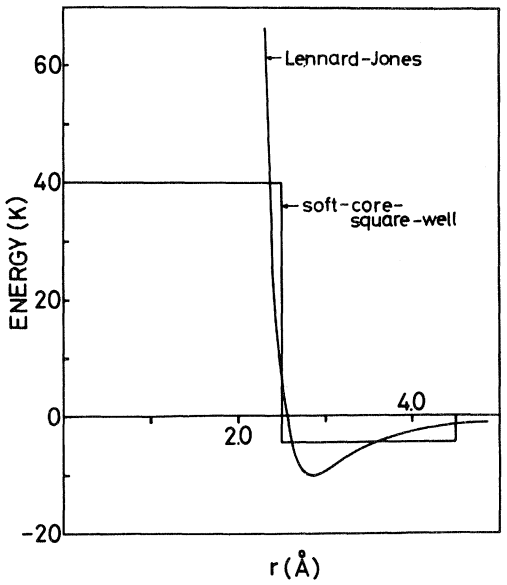

FIG. 9. Interaction potentials of He atoms.

and $V_b = 4.58$ K. This choice of the values of the parameters of the potential gives us the pure boson structure factor of the form shown in Fig. 10 which is in good agreement with the neutron data by Cowley and Woods.[20] Figure 11 shows the calculated structure factors for several values of the fermion concentration. The most characteristic feature of the result is the fixed points below and behind the peak at which the values of the structure factors are all exactly equal to unity.

In Fig. 12 the ratio $R(q|x)$ which is defined by Eq. (3) is shown for this model. Curves are seen to oscillate about the nodes of the fixed points from $R > 1$ to $R < 1$ and again to $R > 1$ with its amplitude being an increasing function of the fermion concentration. When we compare Fig. 7(a) with Fig. 11, we can conclude that this simple BBA model reproduces quite well the qualitative behavior of the structure factor of liquid He mixture for various fermion concentrations.

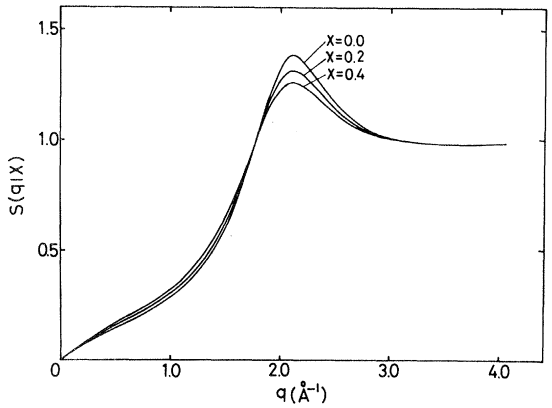

FIG. 11. BBA calculation of $S(q|x)$ for a Bose-Fermi mixture. One can see the fixed points of scattering momentum at which $S(q|x)$ values are independent of x.

Physically, the fixed points which appeared in the BBA model correspond to the zeros of $v(q)$. As we stated at the beginning of this section, our BBA is a kind of lowest-order calculation in which the structure factor is expressed by means of local interactions in the momentum space. Hence, as far as the momenta associated with the zeros of $v(q)$ are concerned, atoms behave like ideal particles to give us an obvious result, that the structure factor is equal to unity which is independent of the fermion concentration.

On the other hand, experimentally obtained values of the structure factor at the fixed points are $S(q_1) = 1.25$ and $S(q_2) = 0.97$, respectively, and they differ from unity. These discrepancies are thought to originate in the absence of higher-order terms in our theory. In fact, the role of the higher-order effects

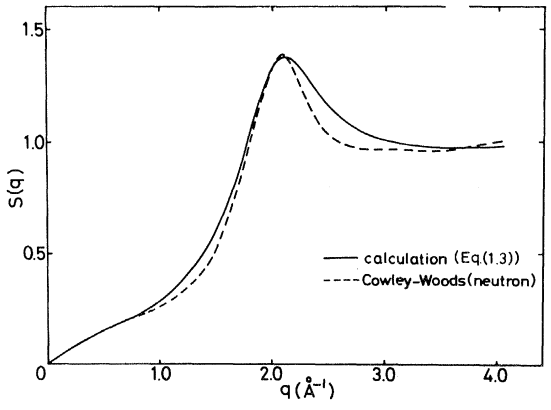

FIG. 10. $S(q)$ for pure liquid ^{4}He.

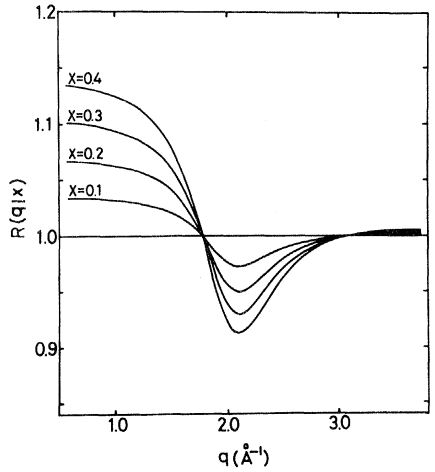

FIG. 12. BBA calculation of the ratio $R(q|x)$.

can clearly be seen in the case of pressurized pure liquid ^{4}He. When we vary the number density $\rho(=N_4/\Omega)$ in the $S_0(q)$ expression [Eq. (13)] as a parameter, we obtain a quite similar behavior of the structure factors as those in the BBA theory; they also exhibit some density-independent fixed points at their interaction-free momenta. Evidence of these fixed points in the pressurized liquid ^{4}He is presented by the inelastic neutron scattering experiment by Dietrich *et al.*[28] although only single-excitation contributions were taken in the integration of $S(q,\omega)$ to $S(q)$. For a more complete $S(q)$ theory, Mihara and Puff[23] have constructed a nonlinear integral equation for the $S(q)$ of ground-state ^{4}He in which nonlocal interactions, namely, the higher-order effects, are included:

$$\left[\frac{\epsilon(k)}{S(k)}\right]^2=\epsilon(k)^2+2\rho\epsilon(k)v(k)$$
$$+\frac{1}{m}\int\frac{d^3q}{(2\pi)^3}\{v(\vec{k}+\vec{q})[\hat{\vec{k}}\cdot(\vec{k}+\vec{q})]^2-v(q)(\hat{\vec{k}}\cdot\vec{q})^2\}[S(q)-1]\ . \tag{15}$$

It is easy to see that the inclusion of the higher-order effects does not influence the position in q of the fixed points but shift the value of $S(q)$, since the ρ appears solely at the local interaction term in the equation. As another nonlocal theory, we consider Massey's[29] variational calculation for the structure of liquid ^{4}He. By using a Jastraw-type wave function and Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) equations, he calculated the $g(r)$ of a Bose liquid by means of a variational method in real space. In Fig. 13 we show our result of the Fourier transformation of the Massey's $g(r)$ for various values of ρ. We can see several fixed points. Moreover, the position of the fixed points at both sides of the maximum shows quantitative agreement with our experimental result as shown in Table II. The interaction potential used in Massey's calculation is the Lennard-Jones 6-12 potential shown in Fig. 9, which is fairly realistic. Then the excellent agreement seems to have some necessity; first, the information extracted from the position of the fixed points for both parameters, x and ρ, are after all related to the two-body interaction potential of helium atoms. And second, this interaction potential is believed to be the same for both helium isotopes.

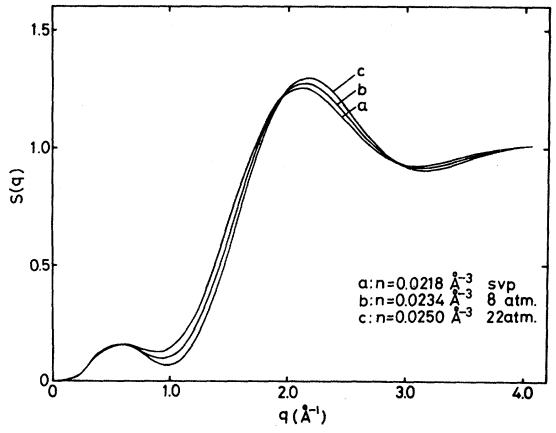

FIG. 13. $S(q)$ for liquid ^{4}He under various values of pressure which has been Fourier transformed from the theoretical result of $g(r)$ by Massey (Ref. 29).

Structure factor theory for a liquid He mixture has been developed by Massey, Woo, and Tan (MWT).[30] They extended the variational method of Massey[29] to a mass 3-mass 4 boson mixture case and obtained partial radial distribution functions $g^{(4,4)}(r)$, $g^{(4,3)}(r)$, and $g^{(3,3)}(r)$ for $x=0.06$. We have calculated the corresponding partial structure factor $S^{(4,4)}(q)$, $S^{(4,3)}(q)$, and $S^{(3,3)}(q)$. The resulting structure factor, however, showed a contrary effect of the ^{3}He atoms to the experimental facts. Figure 14 indicates that the system is more ordered with added mass 3 atoms than in the pure state. This discrepancy may be attributed partly to the MWT's choice of the number density value in the calculation; they used the same ρ value to that of pure mass 4 boson system, while as we mentioned above, the role of the number density in the structure factor expression is very important. As for the presence of the fixed points of the structure factor, we can say nothing from their theory because their calculation is limited to a unique ^{3}He concentration.

TABLE II. Positions and values of the fixed points. ^{3}He-^{4}He: Experimentally observed value in the present work. ^{4}He: Observed in the structure factors which we obtained from Massey's theoretical $g(r)$ values.

	^{3}He-^{4}He (experimental)	^{4}He (theoretical)
q_1	1.95	1.96
$S(q_1)$	1.25	1.23
q_2	2.9	2.92
$S(q_2)$	0.97	0.94

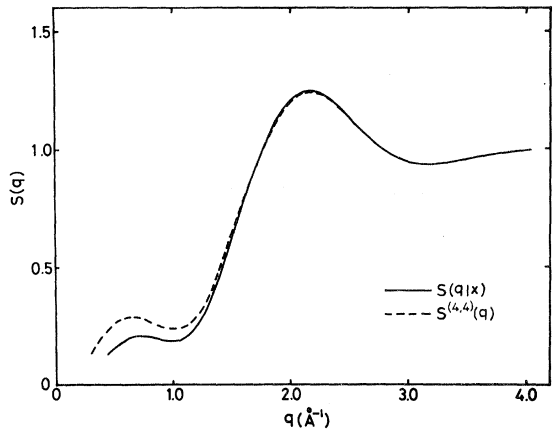

FIG. 14. $S(q|x)$ for mass 3-mass 4 boson mixture for $x=0.06$, which has been calculated from Massey-Woo-Tan's (Ref. 30) theoretical result of the partial radial distribution functions.

Campbell[31] extended the MWT theory to a more complete one including multiphonon interactions. His result, however, does not give us any explicit x dependence of the structure factor. At present, there seems to exist no theory which gives us the x dependence of the structure factor especially concerning the presence of the fixed points. More detailed theories would be necessary for a full understanding on this point.

The evaluation of the condensate fraction n_0 in the mixture is also of great importance. Hyland, Rowlands, and Cummings[32] have argued the role of the condensate and noncondensate atoms in the spatial correlation of the atoms for superfluid ^{4}He and have proposed a simple expression of the temperature variation of the radial distribution function at large r. Their basic idea is that the spatial correlation is formed solely by noncondensate atoms. Recently, Robkoff, Ewen, and Hallock[33] (x ray) and Sears and Svensson[34] (neutron) experimentally obtained the condensate fraction n_0 in superfluid ^{4}He with the aid of this expression. In liquid He mixture, however, ^{3}He atoms which are noncondensate by their nature rather disturb the spatial correlation because of their larger zero-point motion, as we have seen in our experiment. The separation of the two, i.e., condensate-induced and ^{3}He-induced, mechanisms of the delocalization of atoms and the following determination of n_0 is left to the future research.

To conclude, we have first measured the x-ray scattering intensities from liquid He mixture and have obtained its structure factor which showed the following characteristics: with increasing ^{3}He concentration (1) the peak position shifted to lower momenta, (2) the magnitude decreased around the peak and increased below and behind the peak, and (3) there exist at least two fixed points of scattering momentum at which the magnitude is not influenced by the variation of the ^{3}He concentration.

In order to understand this behavior of the structure factors, we have carried out a simple binary-boson-approximation (BBA) model calculation. The model manifested all the three above-mentioned characteristics of the structure factor, and the x-independent fixed points in this model were found to be at interaction-free momenta. Although the lack of information about higher-order effects makes it impossible to compare quantitatively our experimental results with theories, a comparison with liquid ^{4}He theory by Massey for various liquid densities yielded an excellent agreement concerning the position of the fixed points. This fact seems to reflect the fact that the position of the fixed points for both situations are, after all, related to the interaction potential itself and that the interaction potential is identical for both ^{3}He and ^{4}He atoms.

The experimental finding of the fixed points in this experiment suggest, in analogy to the pressure effect on the structure factor of pure liquid ^{4}He, that these fixed points might also correspond to the zero points of the Fourier transformed interaction potential. These observations imply the possibility of directly obtaining important information on the He-He interaction by structure factor experiments.

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