PHYSICAL REVIEW B VOLUME 25, NUMBER 7 1 APRIL 1982

Coherent bremsstrahlung at low energies

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(Received 6 November 1981)

We present a calculation of coherent bremsstrahlung of electrons (or positrons) from crystal targets, which is based on the lowest Born approximation but does not make use of the high-energy or small-angle approximation. Accordingly, the results remain valid down to electron kinetic energies of the order of $\gtrsim 1$ MeV, and are applicable to several experiments that have been carried out in recent years using electrons with kinetic energies of a few MeV or tens of MeV.

I. INTRODUCTION

The coherent emission of bremsstrahlung by electrons (or positrons) passing close to a row of atoms was originally viewed[1] as a high-energy phenomenon which should be significant for electron energies of ~ 200 MeV or more. The effect was actually first verified in the GeV region,[2] but in later years a number of low-energy experiments were carried out[3-8] that have demonstrated the persistance of the coherent bremsstrahlung effect for electron energies as low as[3] 35 keV. The particles and their energies used in these experiments were 15-MeV electrons,[4] 40-MeV electrons and positrons,[5] 7–10-MeV electrons,[7] and 16–28-MeV electrons and positrons.[6,8]

The high-energy calculations of coherent bremsstrahlung[1,9] have been based on the lowest nonvanishing Born approximation, which treats the transition as a two-step process, one step being the first-order interaction of the free electron with the crystal potential, the other the emission of the photon by the electron to first order in powers of e, the electron charge. Second Born-approximation effects in the crystal potential have been considered by Akhiezer *et al.*,[10] as well as quasiclassical corrections to the Born approximation,[11] but the applicability of lowest Born-approximation results has been shown[11] to be more general than what would be indicated by the nominal conditions of applicability of that approximation. In fact, the numerous experimental studies of high-energy coherent bremsstrahlung that have been carried out since 1962 have all given results in essential agreement with the lowest Born-approximation theory.[12,13]

The above, two-step calculation of bremsstrahlung constitutes the lowest-order approximation, in an expansion in powers of Ze (Z being the nuclear charge), to a more exact approach, which includes higher powers of Ze but is still based on the lowest-order treatment of photon emission. For conventional bremsstrahlung, such an approach is based on the use of Coulomb-distorted electron wave functions in the first-order matrix element of photon emission.[14] For the case of coherent bremsstrahlung, one should make use, instead, of Bloch wave functions of the electron in the crystal.[15]

For electrons of kinetic energy $T_0 \gtrsim 1$ MeV propagating close to a major crystal plane or crystal axis, the atoms of that plane or axis may be assumed to form a continuously charged plane, or a continuous line charge, respectively.[16] The required Bloch functions are then periodic in one (two) dimensions only, i.e., in the direction(s) transverse to that plane (line). They describe the quantum mechanical states of electrons, bound in the transverse direction to the plane (line) while propagating along the same; these are known as "channeling" states.[17]

Such Bloch functions may now be employed in a first-order (in e) calculation[14] of photon emission. Here, one must distinguish between the cases where the electron states are "discrete bound" states of the plane (line) potential, i.e., lie in narrow bands, or where they are "free" states, i.e., lie in the continuum. For bound-bound transitions, the corresponding radiation is termed "bremsstrahlung of channeled particles" by Akhiezer *et al.*,[11] but is more commonly known as "channeling radiation."[18-21] If the transverse electron energy lies in the continuum, the emission corresponds to conventional "coherent bremsstrahlung."[20] There is thus no basic difference between these two radiation processes as already implied by the mentioned terminology of Akhiezer, although the literature contains contradictory statements regarding this point.[22,23]

It has been shown that if the electron energy lies high enough in the continuum, the band gaps at the boundaries of the one-dimensional Brillouin zone are very small, and the (transverse) energy levels in an extended-zone scheme essentially become the parabola in the rest frame, $E_{tr} \cong k_{tr}^2/2m + \gamma V_0$, where k_{tr} is the transverse momentum of the electron of total energy γm, and V_0 the zero-order Fourier component of the crystal potential.[20] This fact suggests that the picture of coherent bremsstrahlung as a spontaneous transition between such almost-free states should be very close to one where coherent bremsstrahlung is treated as transitions between plane-wave states induced by the crystal potential, i.e., when use of the Born approximation is made. In fact, Andersen *et al.*[20], after having shown that to lowest order in powers of Ze, the intensity formulas for coherent bremsstrahlung (for emission in the forward direction) become identical whether obtained assuming a spontaneous transition between Bloch states, a two-step transition between plane-wave electrons, or even classical emission of radiation, have demonstrated numerically [for the case of 4-MeV electrons channeled along the (110) plane of Si], that for electrons only a few ($\gtrsim 5$) keV in the continuum, the Born results of the emitted photon intensity agree very closely with the results of an exact calculation using Bloch functions.

Accordingly, we present here a calculation of the coherent bremsstrahlung of electrons (or positrons) from crystal targets based on the lowest nonvanishing Born approximation, which may be considered reliable in view of the arguments presented above. The results of such a calculation include all the "orders" of coherent bremsstrahlung, $\Delta n = 2, 4, 6 \ldots$ (where n is the level number in a reduced Brillouin-zone scheme) as categorized by the Bloch-function picture,[20] but it excludes any (lower-energy) channeling radiation effects.[21] We do not make use of the high-energy or small-angle approximation,[1] so that our results remain valid down to electron energies of the order of 1 MeV, and are applicable to the low-energy electron experiments that have been reported in the literature.[3–8]

It should be noted that Akhiezer *et al.*[11] have advanced criteria for the applicability of Born-approximation analyses to the case of coherent bremsstrahlung which set a lower limit on the angle Θ the electron direction may make with a crystal axis; essentially, $\Theta \geqslant (Ze^2/Ea)^{1/2} \equiv \theta_{ch}$ where E is the incident electron energy, a the spacing of crystal planes, and θ_{ch} the Lindhard critical channeling angle. These arguments, however, only refer to fast particles, where $E >> m$, and the stated criterion is designed to exclude the occurrence of bound states (appropriate only for channeling radiation) in coherent bremsstrahlung. The photon energies alone, however, clearly distinguish the two radiation types [~ 50 keV for channeling radiation, ~ 500 keV for coherent bremsstrahlung[24] of 56-MeV positrons at the (100) plane of Si], and the distinction between bound-bound and free-free transitions within the unified treatment of the two radiation phenomena in the spontaneous-emission picture remains frequently clear[20] on energetic grounds even at $\Theta = 0°$. We may thus accept the applicability of a Born-approximation analysis even for electrons incident along a crystal axis.

II. LOW-ENERGY COHERENT BREMSSTRAHLUNG EXPERIMENTS

Numerous experiments on coherent bremsstrahlung have been done at high electron energies (typically, $\sim 10^2$–10^3 MeV), and their results were satisfactorily described by theory based on the lowest Born approximation and the high-energy and small-angle approximations.[12,13,25,26] A number of Russian experiments have pioneered the study of coherent bremsstrahlung of low-energy electrons.

(1) Korobochko, Kosmach, and Mineev[3] report on experiments in which electrons with kinetic energy T_0 between 35 and 84 keV traversed a thin crystalline LiF target. The observed bremsstrahlung was identified by them as being the coherent type because the peaks in the spectrum could be well understood from the kinematics of coherent bremsstrahlung.

This experiment, unfortunately, would be hard to compare with the theory given in the present paper (as far as intensity is concerned) since, evidently, Coulomb-distortion effects on the electron wave functions are quite severe at these low energies. Certainly one cannot understand these results on the basis of the Born-approximation approach.

(2) Grachyov *et al.*[4] measured coherent bremsstrahlung produced by electrons with $T_0 = 15$ MeV incident on a thin Si crystal, with its (111) plane perpendicular to the electron beam. Peaks were found in the bremsstrahlung spectrum at positions predicted by coherent bremsstrahlung kinematics. When the crystal was misset from the previous direction by 15°, the peaks were washed out, and essentially the spectrum of ordinary bremsstrahlung from a polycrystalline scatterer was obtained.

(3) The work of Komar *et al.*[7] is similar to that of Ref. 4. The authors detected coherent bremsstrahlung from electrons of $T_0 = 7$–10 MeV impinging on thin Si crystals in the [110] direction.

(4) In the United States, the earliest low-energy coherent bremsstrahlung experiments were carried out in 1969 at the Naval Research Laboratory by Godlove and Toms,[5] in which electrons and positrons with $T_0 = 40$ MeV were incident on a thin Si crystal. By rotating the crystal, various peaks were observed in the bremsstrahlung intensity detected within a small solid angle about the forward direction. These peaks could be correlated with the incidence perpendicular to various crystal planes. The peaks were roughly the same for incident positrons and electrons, indicating their coherent bremsstrahlung origin (rather than any connection with channeling).

Further experimental work on low-energy coherent bremsstrahlung was subsequently carried out at the Lawrence Livermore Laboratory, as follows.

(5) Walker *et al.*[6] have observed forward bremsstrahlung intensities from $T_0 = 16-28$-MeV electrons and positrons, axially channeled in the [111] and [110] directions of thin Si crystals, and have noticed a reduction in the bremsstrahlung intensity for positrons but not for electrons, explained by the fact that channeling of positively charged particles tends to keep them away from the crystal axes. In addition, low-energy coherent bremsstrahlung peaks were observed.

(6) Walker, Berman, and Bloom[8] observed a prominent 500-keV peak in the forward bremsstrahlung spectra of $T_0 = 28$ MeV positrons and electrons incident in the (110) plane close to the [111] direction of a thin Si crystal. The peak position was found to be consistent with the kinematics of coherent bremsstrahlung.

Extensive measurements of the forward planar channeling radiation as well as the coherent bremsstrahlung of $T_0 = 4$ MeV electrons in Si crystals, relative to the (100), (110), and (111) planes, have recently been carried out by the Aarhus group.[20] In that study, a unified interpretation of these two phenomena was advanced, in which channeling radiation is described as the bound-bound (and free-bound), and coherent bremsstrahlung as the free-free transitions, between Bloch states of a periodic planar potential. The latter transitions can be distinguished experimentally from channeling radiation by the dependence of the transverse energy peaks on the transverse momentum transfer, which is nearly linear as the difference of two (almost-free) parabolas.

While in the mentioned work,[20] the electron wave functions in the crystal were represented as the states in the electrostatic potential of uniformly charged parallel planes, it was also demonstrated both analytically and numerically that except possibly for the lowest-energy free-free transitions, the lowest Born approximation gave a reliable description of coherent bremsstrahlung intensities, even at the low (4-MeV) primary electron energies involved. In the following sections, we shall therefore adopt the Born approach but retain the exact three-dimensional lattice nature of the crystal potential. In addition, the high-energy and small-angle approximation of high-energy coherent bremsstrahlung studies[1] will not be made here. Our formulas are expected to be accurate only for the lighter elements and for electron kinetic energies of the order of 1 MeV or above. This is a reasonable expectation, since, as already stated, Andersen *et al.*[20] showed that the relevant Born-approximation result was accurate for $T_0 = 4$-MeV electrons, and, on the other hand, the measurements of Rester and Dance[27] at $T_0 = 1$ MeV are in resonably good agreement with the predictions of the Bethe-Heitler formula of ordinary bremsstrahlung for $z = 13$.

III. THEORY

We will deal with a perfect crystal in the harmonic force approximation. By "coherent bremsstrahlung" we will mean free-free one-photon emission by a charged particle in a crystal in which the phonon occupation numbers are unchanged. Bremsstrahlung emission processes in crystals in which these occupation numbers change can be expected to be largely incoherent; such nonzero-phonon processes are background effects which are of no interest in the present study.

Since, by definition, a coherent bremsstrahlung event is a zero-phonon process, it will not give any significant recoil energy to the macroscopic crystal considered, and therefore it will obey the energy conservation equation

$$E_0 = E + k \quad , \tag{1}$$

besides obeying the momentum conservation equation

$$\vec{p}_0 = \vec{p} + \vec{k} + \vec{\tau} \quad , \tag{2}$$

where E_0 and E are the initial and final electron energies, respectively, $\vec{p}_0$ and $\vec{p}$ the respective initial and final electron momenta, and $\vec{k}$ the photon momentum. The recoil momentum $\vec{\tau}$ imparted to the crystal can only take the set of discrete values

$$\vec{\tau} = 2\pi\vec{g} \quad , \tag{3a}$$

where $\vec{g}$ runs over the set of all reciprocal-lattice vectors. For crystals with a cubic space lattice, one has

$$\vec{\tau} = (2\pi/a) \sum_{i=1}^{3} \hat{e}_i n_i \quad , \tag{3b}$$

where $\hat{e}_i$ is a unit vector along the ith cubic axis, a the length of the edge of the primitive unit cell, and the n_i are zero or positive or negative integers.

As in the case of high-energy coherent bremsstrahlung,[1] one also finds for low-energy electrons considered here that for a given longitudinal recoil momentum τ_L, defined by

$$\tau_L = \vec{\tau} \cdot \vec{p}_0 / p_0 \quad , \tag{3c}$$

the photon energy k has to be smaller than a given value $k_\tau(\tau_L)$, being a function of τ_L, above which the coherent bremsstrahlung spectrum cuts off more or less abruptly. This effect gives rise to the peaks in the coherent bremsstrahlung spectrum; it is caused by the fact that energy and momentum conservation lead to a minimum momentum transfer $\tau_{\min} \equiv p_0 - p - k$ to the lattice, which must be equal to or exceeded by the value of τ_L. This condition may be transformed into the inequality

$$k \leq k_\tau(\tau_L), \quad k_\tau \cong \frac{p_0 \tau_L}{E_0 - p_0 + \tau_L} \tag{3d}$$

(assuming $\tau_L << p_0$) as mentioned. Equation (3a) can be satisfied in two ways.

Case A: $\tau_{\min} << 2\pi/a$. In this case, which applies to photon energies $k << E_0$ (typically $x << 0.1$ where $x = k/T_0$), and which is prominent in the high-energy regime,[1] one has significant contributions to coherent bremsstrahlung only from the reciprocal-lattice plane approximately normal to the direction of incidence that contains the origin. Then, since $\tau_L = \tau \sin\Theta \cong \tau\Theta$, where Θ is the angle of the incident electron direction with a lattice axis, one has

$$\Theta \geqslant \tau_{\min}/\tau \quad , \qquad (4)$$

and this condition leads to the well-known minimum of coherent bremsstrahlung for $\Theta = 0$. The reader is referred to Fig. 1 of Ref. 1 for a geometrical overview of this situation, as well as that for Case B.

Case B: $\tau_{\min} \gtrsim 2\pi/a$. In this case, the reciprocal lattice planes approximately normal to the direction of incidence and spaced successively from the plane that contains the origin (but excluding the latter plane) will furnish the prominent coherent bremsstrahlung peaks, and the angle of incidence $\Theta = 0$ is permitted for coherent bremsstrahlung production. This situation will mainly be encountered in the low-energy case considered here, and the peaks will be typically situated at $x \gtrsim 0.1$.

Applying the standard rules of quantum electrodynamics and using Van Hove's time-dependent formalism[28,29] one finds that in the lowest nonvanishing Born approximation the coherent bremsstrahlung cross section summed over photon polarizations is given by

$$\frac{d^2\sigma_{\text{coh}}}{d^3p\,d^3k} = \frac{8\pi\sigma_0 N}{v_0}\delta(E_0 - E - k)\frac{m^2}{p_0 E k}\sum_{\vec{\tau}}\delta(\vec{p}_0 - \vec{p} - \vec{k} - \vec{\tau})\,|\,S(\vec{\tau})|^2\frac{[1 - F(\vec{\tau})]^2}{\tau^4}$$
$$\times\left\{\frac{p^2\sin^2\theta(4E^2 - \tau^2)}{(Ek - \vec{p}\cdot\vec{k})^2} + \frac{p_0^2\sin^2\theta_0(4E_0^2 - \tau^2)}{(E_0 k - \vec{p}_0\cdot\vec{k})^2} - \frac{2}{(Ek - \vec{p}\cdot\vec{k})(E_0 k - \vec{p}_0\cdot\vec{k})}\right.$$
$$\left.\times[\,p p_0 \sin\theta\sin\theta_0\cos\phi(4EE_0 - \tau^2 - 2k^2) + k^2(p^2\sin^2\theta + p_0^2\sin^2\theta_0)\,]\right\} , \qquad (5)$$

for unpolarized incident electrons, where the sum over $\vec{\tau}$ runs over all the points of the reciprocal lattice. In (5), $\sigma_0 = Z^2\alpha(e^2/mc^2)^2$, with $\alpha = \frac{1}{137}$, and Z is the nuclear charge of each atom in the crystal, it having been assumed for simplicity that only one species of atoms is present; N is the number of unit cells in the crystal; v_0 the volume of a unit cell; θ_0 (respectively, θ) the angle between $\vec{p}_0$ and $\vec{k}$ (respectively, $\vec{p}$ and $\vec{k}$); and ϕ the angle between the planes $(\vec{p}_0, \vec{k})$ and $(\vec{p}, \vec{k})$. The quantity $[1 - F(\vec{\tau})]/\tau^2$ is proportional to the Fourier transform of the screened nuclear Coulomb field, with $[1 - F(\vec{\tau})]$ representing the effect of the screening by the atomic electrons. The quantity

$$S(\vec{\tau}) = \sum_{j=1}^{\nu}\exp[\,W_j(\vec{\tau})\,]\exp(-2\pi i\,\vec{\rho}_j\cdot\vec{\tau}) \qquad (6)$$

describes the interference effect of the atoms in a unit cell and also takes account of the thermal motion of the lattice by means of Debye-Waller type factors $\exp[-W_j(\vec{\tau})]$ pertaining to each of the atoms $j = 1, \ldots, \nu$ in a unit cell, $\vec{\rho}_j$ being the position vector of the jth of these atoms.

For crystals of the diamond-type structure ($\nu = 8$) and assuming as usual that all the $W_j(\vec{\tau})$'s are equal, with $W(\vec{\tau})$ their common value, the factor $|\,S(\vec{\tau})|^2$ in (5) is equal to $\exp[-2W(\vec{\tau})]$ times a number equal to 64 if all the $n_i (i = 1, 2, 3)$ are even and their sum is a multiple of 4, to 32 if all the n_i are odd, and to zero otherwise. The calculations reported in Sec. IV, which refer to a special case of the diamond-type structure, namely Si crystals, were carried out under this assumption.

We are interested in integrating $d^2\sigma_{\text{coh}}/d^3p\,d^3k$ over the outgoing electron momentum vector and the photon direction, for a given photon energy. This can be done in a very simple way, but strangely enough no mention of the fact occurs in the published literature.[30] After performing the trivial integration over $\vec{p}$, the integration over $\hat{k}$ can be carried out by a simple trick whose main point is that $\vec{p}_0 + \vec{\tau}$ (*not* $\vec{p}_0$) is taken as polar axis for each $\vec{\tau}$. In more detail, let $\vec{k}$ make an angle θ_1 with $\vec{p}_0 + \vec{\tau}$ and let its azimuthal angle, with respect to an arbitrary direction perpendicular to $\vec{p}_0 + \vec{\tau}$, be ϕ_1. We first integrate over θ_1 using the familiar formula for changes of variables in δ functions. This leaves us with an integral over ϕ_1 which is elementary. The integration of $d^2\sigma_{\text{coh}}/d^3p\,d^3k$ over $\vec{p}$ and $\hat{k}$ yields the following expression for the coherent bremsstrahlung cross-section differential with respect to photon energy:

$$\frac{d\sigma_{\rm coh}}{dk}=\frac{4\pi^2\sigma_0 N}{v_0}\frac{m^2}{p_0k^2}\sum_{\vec\tau}{}'\frac{|S(\vec\tau)|^2[1-F(\vec\tau)]^2}{\tau^4|\vec p_0-\vec\tau|}I(\vec\tau)\ , \tag{7a}$$

where

$$I(\vec\tau)=\sum_{r=1}^{4}I_r(\vec\tau)\ , \tag{7b}$$

with

$$\begin{aligned}
I_1(\vec\tau)&=\frac{[p_\perp(\vec\tau)]^2(4E_0^2-\tau^2)}{\epsilon_\tau^2}\ ,\\
I_2(\vec\tau)&=(4E^2-\tau^2)\left[-1+\frac{2E_0}{\rho_\tau}-\frac{m^2(E_0-\eta_\tau)}{\rho_\tau^3}\right]\ ,\\
I_3(\vec\tau)&=2\frac{4E_0E-\tau^2+2k^2}{\epsilon_\tau}\left[(E_0-\epsilon_\tau)\left(\frac{E}{\rho_\tau}-1\right)-\frac{\sigma_\tau}{\rho_\tau}\right]\ ,\\
I_4(\vec\tau)&=\frac{2k^2}{\epsilon_\tau}\left(\eta_\tau+E+\frac{[p_\perp(\vec\tau)]^2-m^2}{\rho_\tau}\right)\ .
\end{aligned} \tag{7c}$$

Here,

$$\begin{aligned}
p_\perp(\vec\tau)&=[|\vec p_0-\vec\tau|^2-(E-\epsilon_\tau)^2]^{1/2}\ ,\\
\epsilon_\tau&=\rho_\tau/2k\ ,\\
\eta_\tau&=\frac{\sigma_\tau(E-\epsilon_\tau)}{|\vec p_0-\vec\tau|}\ ,\\
\rho_\tau&=\left[(E-\eta_\tau)^2-\frac{[\tau^2p_0^2-(\vec\tau\cdot\vec p_0)^2]}{|p_0-\tau|^2}\times\left(1-\frac{(E-\epsilon_\tau)^2}{|\vec p_0-\vec\tau|^2}\right)\right]^{1/2}\ ,\\
\sigma_\tau&=\vec p_0\cdot(\vec p_0-\vec\tau)\ .
\end{aligned} \tag{7d}$$

Notice, in particular, that $p_\perp(\vec\tau)$ is the magnitude of the part of $\vec p$ which is perpendicular to $\hat k$ for a given p_0, $\vec\tau$. The prime in the sum in (7a) means that the sum runs over all those $\vec\tau$'s such that

$$|E_0-\sigma_\tau|<|\vec p_0-\vec\tau|\ , \tag{7e}$$

which is equivalent to requiring that $|\hat k\cdot(\vec p_0-\vec\tau)|\leq|\vec p_0-\vec\tau|$ for the $\vec p_0$, $\vec\tau$ considered, implying an inequality expressed approximately by (3d).

Although each term of the series in (7a) is a lengthy combination of elementary functions, the series can be easily computed numerically on a fast electronic computer. This is what was done in the present study. The results of these calculations will be discussed in the following Sec. IV.

Mozley and DeWire[31] have pointed out that an axial collimation of the emitted radiation has the effect of additionally monochromatizing the coherent bremsstrahlung peaks. This effect was verified experimentally at 1-GeV energies by Tsuru *et al.*[32]; it rests on the fact that the lower-energy photons emitted farther from the forward direction get eliminated by the collimation, so that the originally relatively wide coherent bremsstrahlung peaks have their low-energy tails removed, resulting in narrower spikes.

Calling θ_c the collimation angle for photons around the incident electron direction, and introducing

$$\psi_c\equiv\theta_c/(m/E_0)\ , \tag{8a}$$

then for given values of τ_L and θ_c, only photons with energies k satisfying the inequality

$$k_\tau-\Delta k_\tau\leq k\leq k_\tau \tag{8b}$$

will pass through the collimator, where

$$\Delta k_\tau\cong\frac{k_\tau(1-k_\tau/E_0)\psi_c^2}{1+(1-k_\tau/E_0)\psi_c^2}\ . \tag{8c}$$

Equation (8c) is an approximation valid for $m/E_0\ll 1$. The numerical results on axially collimated coherent bremsstrahlung reported in Sec. IV are based on exact kinematical formulas and on a formula for the spectrum of the resultant radiation which is an exact consequence of (5). The latter formula is analogous to (7a) but much more complicated, and hence will be omitted. Again, the photon spectrum predicted by this formula was readily computed on a fast electronic computer.

IV. NUMERICAL RESULTS

In this section, a series of numerical results will be discussed that were obtained from the foregoing theory. They all refer to thin Si single-crystal targets at room temperature. We only show the coherent part of the bremsstrahlung cross section, and plot the quantity

$$(x/\sigma_0)\frac{d\sigma_{\rm coh}}{dx} \tag{9}$$

vs $x=k/T_0$, where T_0 is the kinetic energy of the incident electron as before. The values of $F(\vec\tau)$ used in these numerical calculations were obtained from the analytical expression for the atomic scattering factor of Si given by Bonham and Strand.[33] The values of $W(\tau)$ used in our numerical work correspond to the room-temperature value $B\equiv(8\pi^2/3)\langle u^2\rangle$

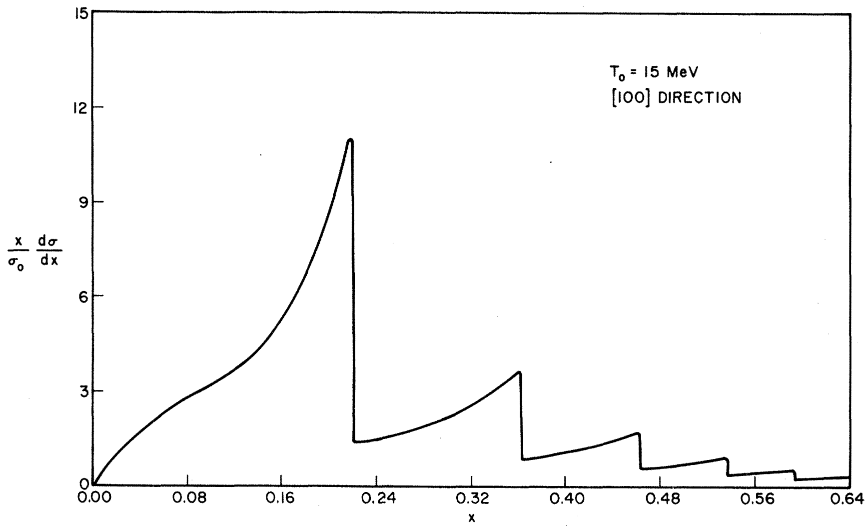

FIG. 1. Coherent bremsstrahlung spectrum of $T_0=15$ MeV electrons incident along the [100] direction of a Si crystal at room temperature.

$=0.45$ Å^2 derived from x-ray measurements,[34] where $\langle u^2\rangle$ is the mean-square vibrational displacement of the atoms from their equilibrium positions.

Coulomb multiple-scattering effects have not been included, but are known to have no overwhelming effect. We have estimated that for $T_0=\frac{1}{2}$ MeV electrons, Si crystals of thickness $d=1$ μm, and for $T_0=10$ MeV electrons, crystals with $d=5-10$ μm will be adequate to keep the half-angle of divergence of the electron beam caused by multiple scattering below 1–2°, which will be seen in the following to be sufficient to preserve the appearance of the coherent bremsstrahlung peaks.

The first series of figures to be shown in the following refer to case B where the most prominent peaks originate from the successive reciprocal-lattice planes that are approximately normal to the direction of incidence, except from the plane that contains the

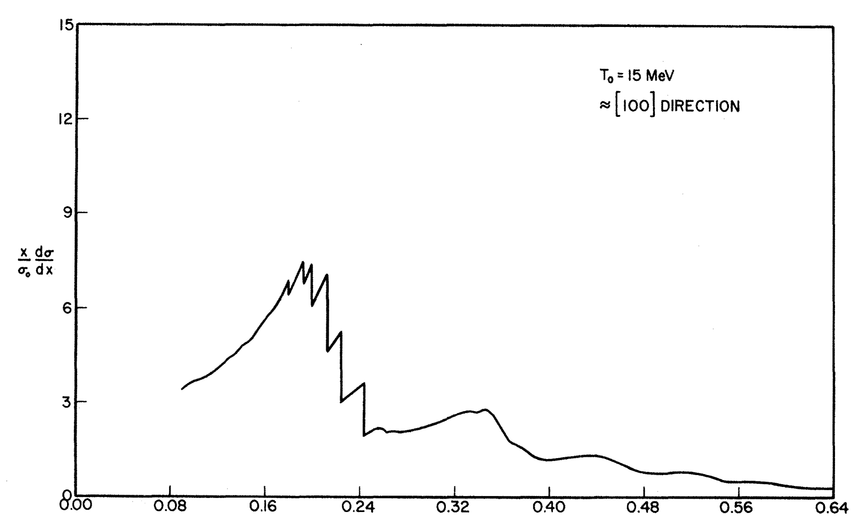

FIG. 2. As in Fig. 1, but with the electrons incident in the (001) plane at an angle of $\Theta=2°$ with the [100] direction.

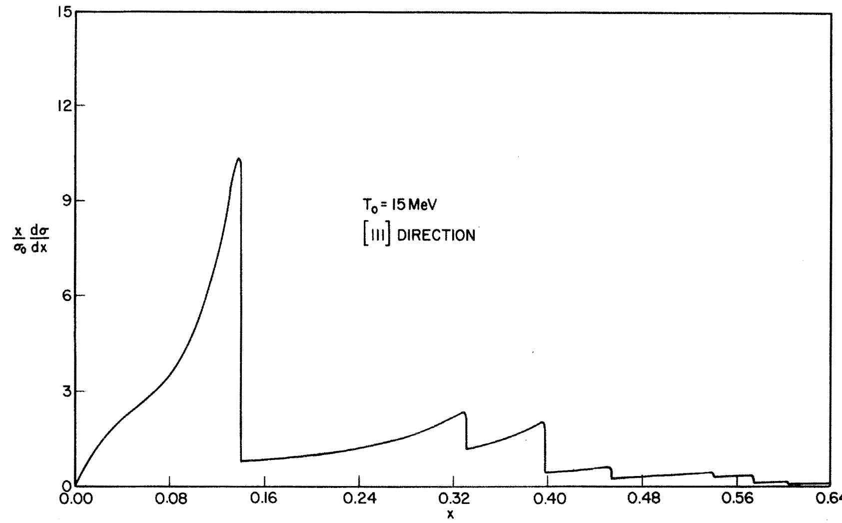

FIG. 3. As in Fig. 1, but with the electrons incident along the [111] direction.

origin. Figure 1 shows the coherent bremsstrahlung spectrum for electrons of $T_0 = 15$ MeV, incident exactly parallel to the [100] direction. The two lowest and most prominent peaks are seen to occur at $x = 0.22$ and 0.36, i.e., at $k = 3.3$ and 5.4 MeV, respectively. The equally spaced peaks end abruptly since at the cutoff energy, the contribution of an entire reciprocal lattice plane cuts off in the present case.

If now the beam is misset somewhat from the [100] direction, namely, if it is taken to be incident in the (001) plane and makes an angle of $\Theta = 2°$ with

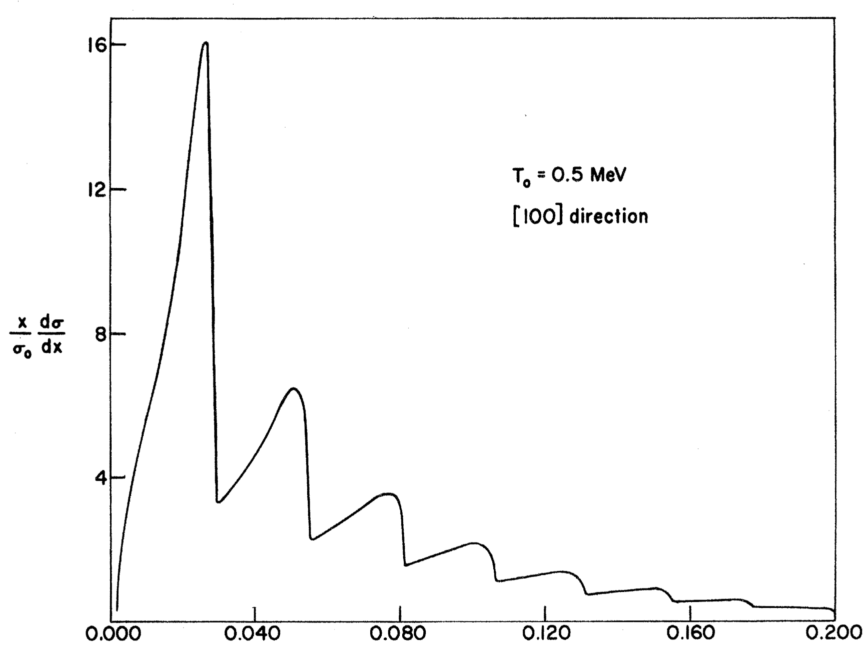

FIG. 4. As in Fig. 1, but with $T_0 = 0.5$ MeV electrons.

the latter direction, then as shown in Fig. 2 the peaks "melt," i.e., spread out, showing a fine structure. This structure comes about since now the cutoff frequencies, corresponding to the reciprocal lattice points in each contributing reciprocal plane, are slightly different. The numerical calculations depicted in this figure, as well as similar calculations not shown here, demonstrate that at the low electron energies considered in this section a half-angle of divergence of $\lesssim 2°$, e.g., as it results from multiple scattering, will not substantially affect the overall peak structure of the coherent bremsstrahlung spectrum at these energies.

Figure 3 shows the coherent peaks for $T_0 = 15$ MeV electrons incident along the [111] direction. The results are as in Fig. 1 but the peaks are no longer equidistant due to the more irregular spacing of the contributing reciprocal-lattice planes in this case. Nevertheless, each reciprocal plane takes part in the cutoff as a whole, so that the peaks terminate abruptly. In all cases the peak positions can be predicted easily by using, in particular, Eq. (3d) and by excluding the values of (n_1, n_2, n_3) for which $S(\vec{\tau})$ vanishes.

The next two figures refer to $T_0 = 0.5$ MeV electrons [at which energy Eq. (7a) may be somewhat inaccurate], incident along the [100] direction (Fig. 4) and the [111] direction (Fig. 5). The peaks in these figures, shown as rounded, actually consist of a series of small spikes, since at these low electron energies the contributions of the pertinent $\vec{\tau}$'s no longer cut off quite simultaneously.

The following discussion will include coherent peaks of type A, of which the most prominent ones are generated by contributions of reciprocal planes that are almost orthogonal to the direction of incidence and contain the origin, and whose normal makes a small but nonzero angle Θ with the direction of incidence. These peaks are located in the low-energy portion of the spectrum. We first consider electrons of $T_0 = 15$ MeV incident on the Si crystal in the (001) plane and making an angle of 1° with the [110] direction. Figure 6(a), which depicts the high-energy portion of the spectrum, shows a series of "melted" type-B peaks (again consisting of small spikes that have been averaged in the figure) and a large rise of the spectral intensity at low energies. The latter consists of a series of much more closely spaced type-A peaks, the first of which occurs at about 100 keV, as shown in Fig. 6(b). From our previous discussion, all the latter peaks disappear for $\Theta \to 0$, causing the well-known dip in the coherent bremsstrahlung intensity at the origin when plotted versus Θ. The observed minimum of this dip is, of course, furnished by the incoherent bremsstrahlung contribution which is independent of Θ.

Figure 7 corresponds closely to the geometry of the experiment of Walker, Berman, and Bloom.[8] We assume a beam of $T_0 = 28$ MeV electrons (or positrons) to be incident in the (110) plane of a Si crystal, misset from the [111] direction by $\Theta = 1°$. The figure exhibits the type-A peaks of the coherent spectrum, in which the lowest and most prominent peak is located at $k = 560$ keV, close to the experimental

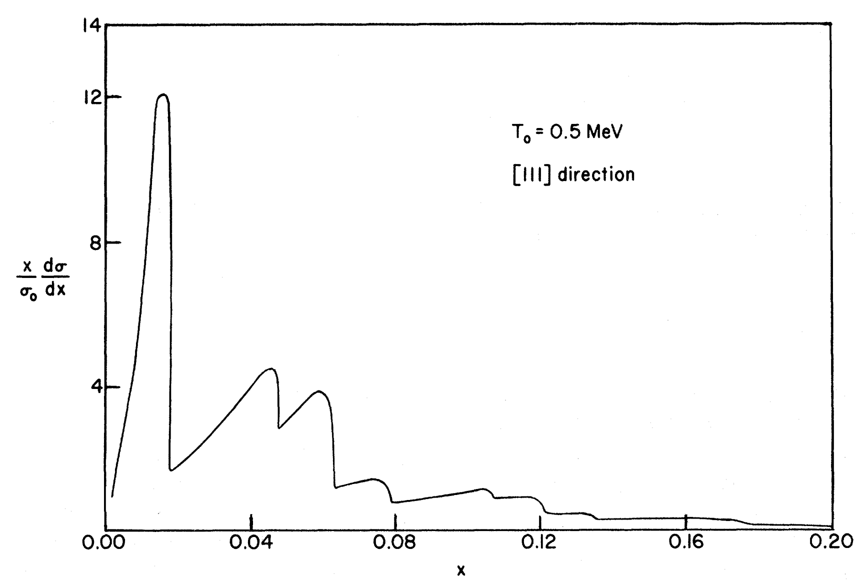

FIG. 5. As in Fig. 3, but with $T_0 = 0.5$ MeV electrons.

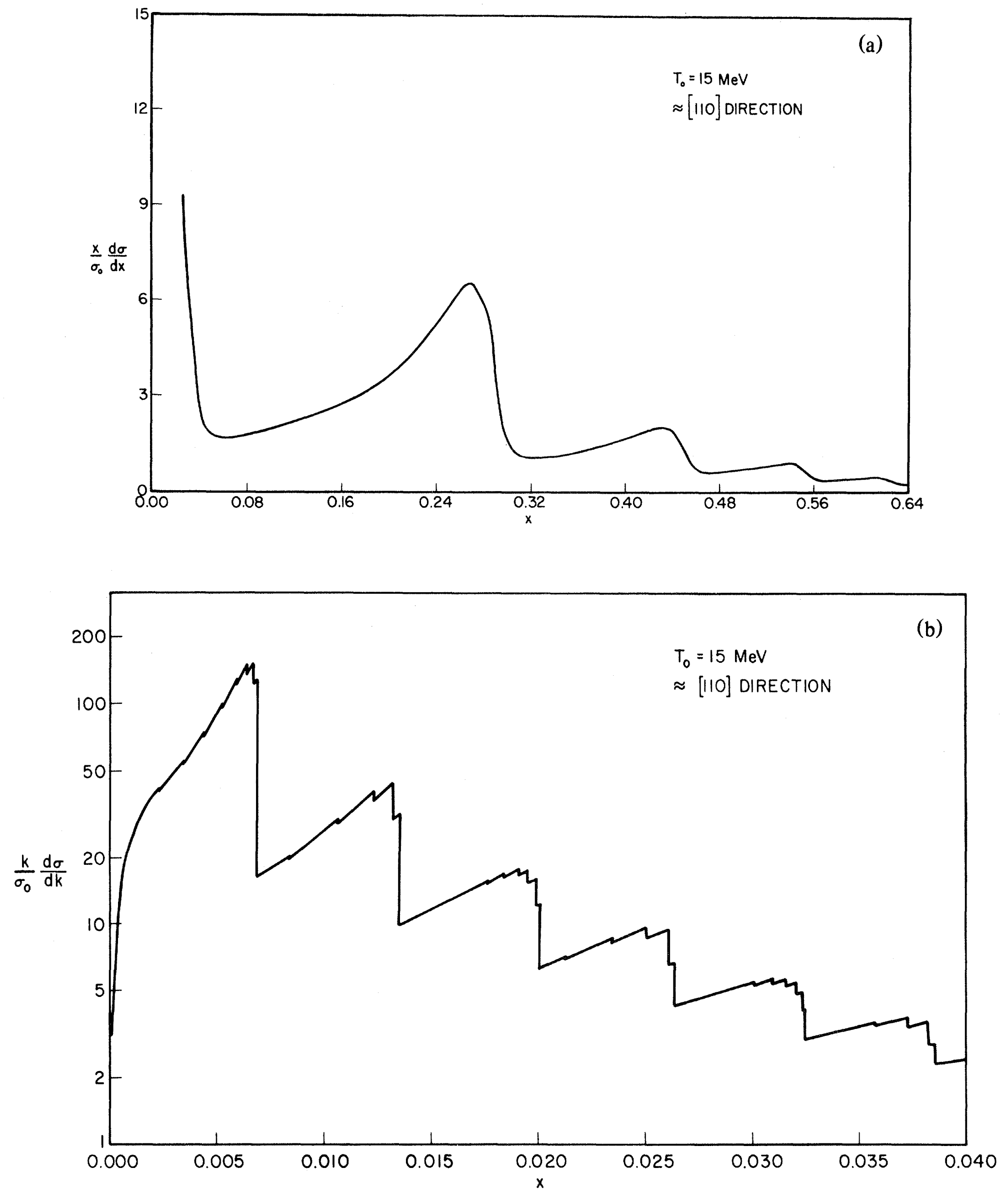

FIG. 6. Coherent bremsstrahlung spectrum of $T_0=15$ MeV electrons incident in the (001) plane of a Si crystal at room temperature, making an angle of $\Theta=1°$ with the [110] direction: (a) high-energy portion, and (b) low-energy portion of the spectrum.

value of ~ 0.5 MeV.[8] Note that this coherent structure lies on top of a much larger, (presumably) incoherent bremsstrahlung contribution in these experiments.

Finally, we illustrate the effect of a Mozley–De Wire-type collimation on the type-B coherent bremsstrahlung peaks. Figure 8 shows the peaks of Fig. 1 ($T_0=15$-MeV electrons incident along the [100] direction) as a dashed line; when the observed bremsstrahlung is collimated so that only photons making an angle of $\theta_c=0.5°$ or less with the incident electron direction are counted, the solid curve results which indicates a very substantial sharpening of the peaks due to this effect. In Fig. 9(a), the same is shown for $T_0=1.5$ MeV electrons and a collimation angle of $\theta_c=4°$. Finally, Fig. 9(b) presents details of the collimation of Fig. 9(a) for the first two peaks and a succession $\theta_c=3°$, 4°, 5°, and 6° of collimation angles.

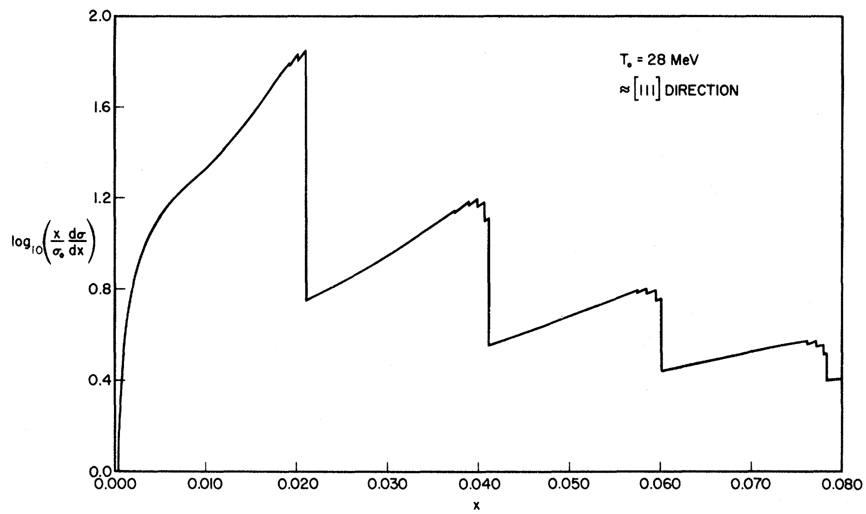

FIG. 7. Low-energy portion of the coherent bremsstrahlung spectrum of $T_0 = 28$ MeV electrons incident in the (110) plane of a Si crystal at room temperature, making an angle of $\Theta = 1°$ with the [111] direction.

V. DISCUSSION AND CONCLUSIONS

A calculation of coherent bremsstrahlung has been carried out in lowest Born approximation which, unlike the previous theories[1,12,13,26] that were designed to describe high-energy ($\gtrsim 100$ MeV) experiments, does not make use of the high-energy or small-angle approximations. It can be applied to predict coherent bremsstrahlung spectra from electrons (or positrons) of energies in the range $1 \lesssim T_0 \lesssim 60$ MeV in which a number of relevant experiments have been carried out in recent years,[4–8,20] and of course at higher energies. For electrons incident at small enough angles with respect to a set of channeling planes, a lowest-order Born calculation of analogous type has been shown[20] to constitute a valid approximation to a more exact treatment in which the electronic states in the crystal are described in terms of Bloch functions, pro-

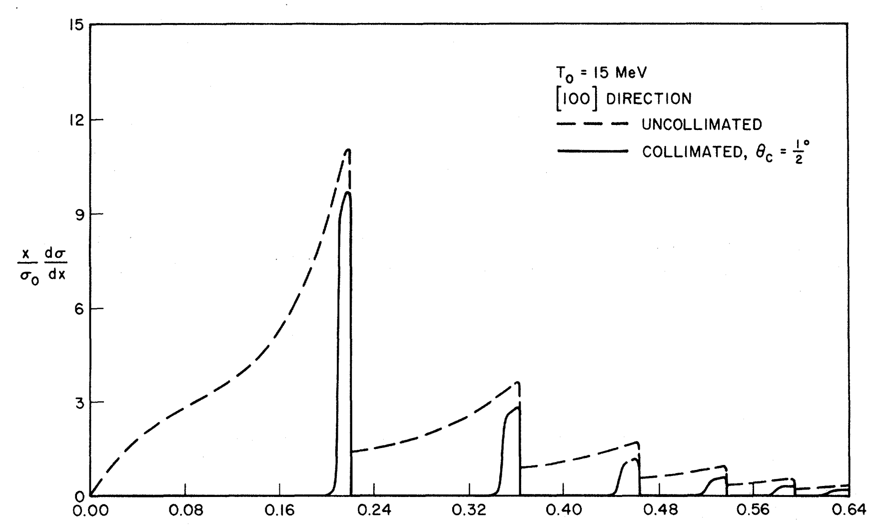

FIG. 8. As in Fig. 1 (dashed curve), but with the emitted bremsstrahlung collimated by $\theta_c = 0.5°$.

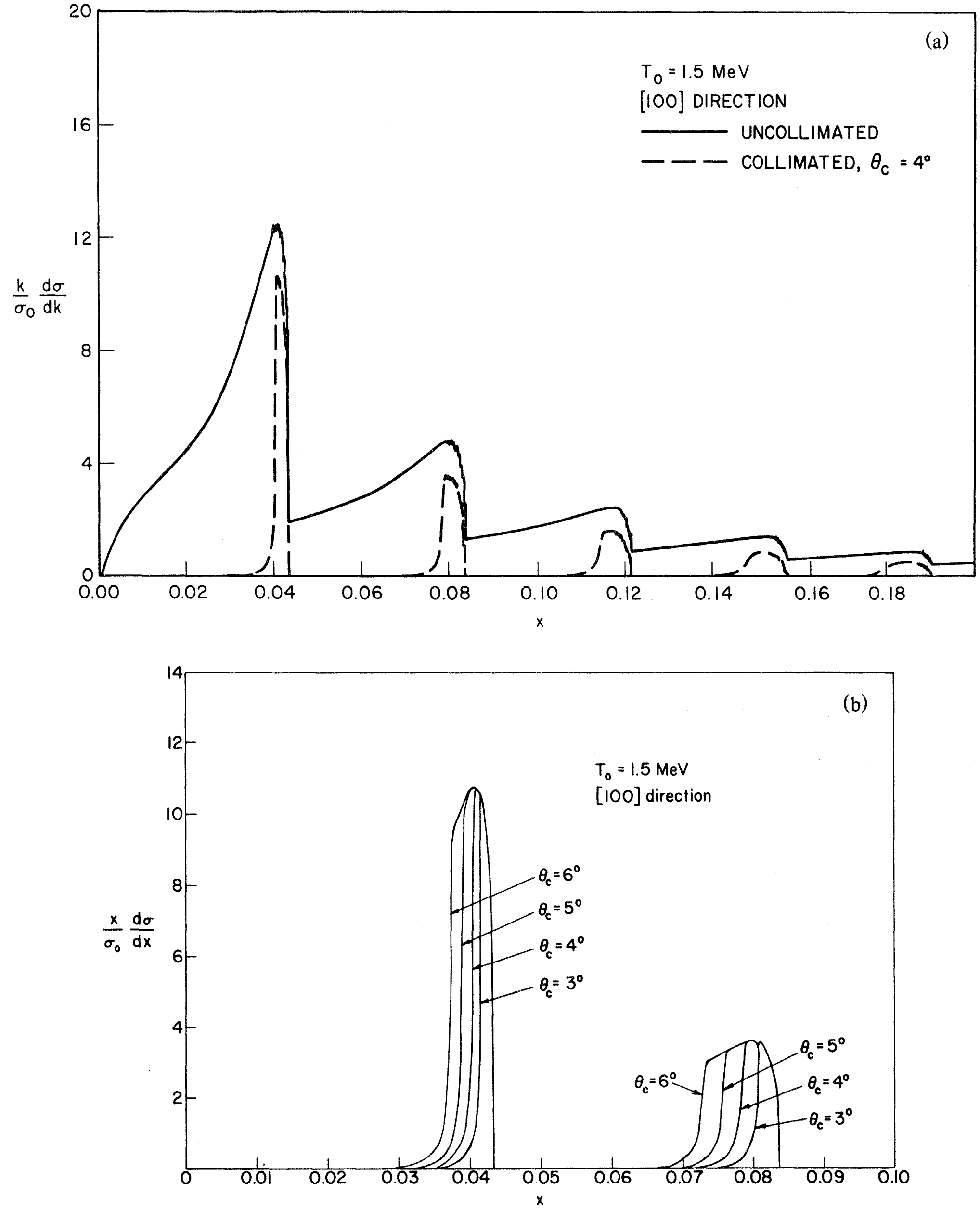

FIG. 9. Coherent bremsstrahlung spectrum of $T_0 = 1.5$ MeV electrons incident along the [100] direction of a Si crystal at room temperature: (a) uncollimated (solid curve), and collimated by $\theta_c = 4°$ (dashed curve); (b) the two lowest peaks collimated by $\theta_c = 3°$, 4°, 5°, and 6°.

vided that only transitions between free (continuum) states are considered. These latter transitions, however, correspond precisely to coherent bremsstrahlung emission,[20] while bound-bound transitions lead to channeling radiation.[19,20] Due to the smaller energy differences between bound-bound as compared to free-free transitions, the channeling radiation peaks often lie at much smaller energies than the coherent bremsstrahlung peaks, and can thus be distinguished from the latter on such grounds. For example, in a recent axial-channeling experiment[35] with electrons incident in the [111] direction of a silicon crystal at 1.5 MeV, channeling radiation peaks up to 4 keV were found. Scaling up our results of

Fig. 5 from $T_0=0.5$ to 1.5 MeV, the lowest coherent bremsstrahlung peak is predicted at 27 keV, thus lying above the energy region studied in the mentioned experiment.[35]

In contrast to calculations of channeling radiation in crystals,[19,20] our calculation takes into account the full lattice structure, rather than employing the potentials of continuously smoothed-out lattice planes or strings. The resulting cross-section sum over the full reciprocal lattice then leads to a distinction between type-*A* and type-*B* coherent bremsstrahlung spectral peaks (Sec. III), completely borne out by the detailed calculation. The former peaks lie in the very-low-energy region of the spectrum, stem from the reciprocal-lattice plane that contains the origin, and show a minimum in the coherent bremsstrahlung if the electrons are incident at an angle $\Theta=0$ with a lattice axis. The latter peaks lie at higher energies, stem from higher-order reciprocal-lattice planes, and show no such minimum at $\Theta=0$. In addition to a discussion of these effects, we also presented a numerical study of the sharpening of coherent bremsstrahlung peaks by photon collimation.

It is clear from some of the foregoing remarks that a comprehensive theoretical study would be desirable in which both channeling radiation and coherent bremsstrahlung are treated on a unified basis, taking into account the full three dimensionality of the crystal lattice. This might explain the interference effects between coherent bremsstrahlung and channeling radiation ("sidebands") which were found in a calculation of the radiation from electrons which move along classical trajectories through a three-dimensional crystal.[24]

ACKNOWLEDGMENTS

We wish to thank, in particular, Mr. S. Podgor for his valuable aid with the numerical calculations. We are also indebted to Mr. L. S. Birks and Dr. D. B. Brown for useful information on atomic scattering factors and Debye-Waller factors, and to Dr. E. F. Skelton for a useful conversation on the latter factors. We are especially grateful to Dr. S. Datz for having informed us of recent studies of channeling radiation and coherent bremsstrahlung at Aarhus. One of us (H.Ü.) was supported by a grant from the National Science Foundation.

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*Also at the Department of Physics, Catholic University of America, Washington, D.C. 20064.
[1]H. Überall, Phys. Rev. 103, 1055 (1956).
[2]G. Barbiellini, G. Bologna, G. Diambrini Palazzi, and G. P. Murtas, Phys. Rev. Lett. 8, 454 (1962).
[3]Yu. S. Korobochko, V. F. Kosmach, and V. I. Mineev, Zh. Eksp. Teor. Fiz. 48, 1248 (1965) [Sov. Phys. JETP 21, 834 (1965)].
[4]B. D. Grachyov, A. P. Komar, Yu. S. Korobochko, V. I. Mineyev, and A. E. Vinnichenko, Phys. Lett. 22, 498 (1966).
[5]T. F. Godlove and M. E. Toms, U.S. Naval Research Laboratory, Nuclear Physics Division Annual Report 1969 (unpublished), p. 96.
[6]R. L. Walker, B. L. Berman, R. C. Der, T. M. Kavanagh, and J. M. Khan, Phys. Rev. Lett. 25, 5 (1970).
[7]A. P. Komar, Yu. S. Korobochko, V. I. Mineev, and A. F. Petrochenko, Zh. Tekh. Fiz. 41, 807 (1971) [Sov. Phys.-Tech. Phys. 16, 631 (1971)].
[8]R. L. Walker, B. L. Berman, and S. D. Bloom, Phys. Rev. A 11, 736 (1975).
[9]H. Überall, Z. Naturforsch. Teil A 17, 332 (1962).
[10]A. I. Akhiezer, P. I. Fomin, and N. F. Shul'ga, Pis'ma Zh. Eksp. Teor. Fiz. 13, 713 (1971) [JETP Lett. 13, 506 (1971)].
[11]A. I. Akhiezer, V. F. Boldyshev, and N. F. Shul'ga, Fiz. Elem. Chastits At. Yadra 10, 51 (1979) [Sov. J. Part. Nucl. 10, 19 (1979)].
[12]G. Diambrini Palazzi, Rev. Mod. Phys. 40, 611 (1968).
[13]U. Timm, Fortschr. Phys. 17, 769 (1969).
[14]H. A. Bethe and L. C. Maximon, Phys. Rev. 93, 768 (1954).
[15]L. I. Schiff, Phys. Rev. 117, 1394 (1960). Note, however, that the actual fashion in which Bloch functions are being employed in this reference is incorrect.
[16]J. Lindhard, K. Dan. Vidensk. Selsk. Mat.-Fys. Medd. 34, No. 14 (1965).
[17]J. U. Andersen, S. K. Andersen, and W. M. Augustyniak, K. Dan. Vidensk. Selsk. Mat.-Fys. Medd. 39, No. 10 (1977).
[18]A. A. Vorobiev, V. V. Kaplin, and S. A. Vorobiev, Nucl. Instrum. Methods 127, 265 (1975).
[19]M. A. Kumakhov, Phys. Lett. 57, 17 (1976).
[20]J. U. Andersen, K. R. Eriksen, and E. Laegsgaard (unpublished).
[21]A. W. Sáenz, H. Überall, and A. Nagl, Nucl. Phys. A 372, 90 (1981).
[22]O. Greschner and R. Wedell, Phys. Status Solidi B 95, 137 (1979).
[23]J. U. Anderson, Nucl. Instrum. Methods 170, 1 (1980).
[24]M. J. Alguard, R. L. Swent, R. H. Pantell, S. Datz, J. H. Barrett, B. L. Berman, and S. D. Bloom, Nucl. Instrum. Methods 170, 7 (1980).
[25]S. Kato *et al.*, J. Phys. Soc. Jpn. 20, 303 (1965).
[26]M. L. Ter-Mikaelian, *High-Energy Electromagnetic Processes in Condensed Media* (Wiley-Interscience, New York, 1972).
[27]D. H. Rester and W. E. Dance, Phys. Rev. 161, 85 (1967). See also the paper by H. K. Tseng and R. H. Pratt, Phys. Rev. 95, 249 (1971), where detailed comparisons are made between exact and approximate calculations for ordinary bremsstrahlung.
[28]L. Van Hove, Phys. Rev. 95, 249 (1954).
[29]In particular, Eq. (2.7) in A. W. Sáenz, Phys. Rev. 119, 1542 (1960), was used [a factor *i* is missing in the second

exponential in (2.7)].

[30]The simplicity of computing $d\sigma_{coh}/dk$ exactly from Eq. (5) by the method mentioned in this paragraph of the text is to be contrasted with the complicated procedure used in the literature (see, e.g., Refs. 1 and 12) to calculate $d\sigma_{coh}/dk$ from the Bethe-Heitler formula using the high-energy and small-angle approximations.

[31]R. F. Mozley and J. De Wire, Nuovo Cimento 27, 1281 (1963).

[32]T. Tsuru *et al.*, Phys. Rev. Lett. 27, 609 (1971).

[33]R. A. Bonham and T. G. Strand, J. Chem. Phys. 39, 2200 (1963).

[34]*International Tables for X-Ray Crystallography*, edited by K. Lonsdale (Kynoch Press, Birmingham, England, 1968), Vol. III, p. 237.

[35]J. U. Anderson and E. Laegsgaard, Phys. Rev. Lett. 44, 1079 (1980).