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Stopping power for energetic ions in solids and plasmas

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The stopping power for heavy ions in several ionized materials is determined over a range of densities and temperatures using a generalized stopping-power formula and an effective charge chosen to give an agreement with stopping power in cold materials. The stopping power can be increased or decreased relative to cold material, depending on the atomic number, the density, temperature, and ion energy.

The stopping power for heavy ions in materials over a range of temperatures and densities is of importance for the determination of beam requirements for heavy-ion fusion targets. Semiempirical estimates have been given for cold materials by Northcliffe and Schilling,[1] extrapolated and interpolated from a large amount of experimental data. The predictions for heavy ions ($A \simeq 200$) are, however, based on low-energy data (less than 80 MeV in high-Z foils and less than 150 MeV in low-Z foils). The extrapolation to higher energy therefore is uncertain, although probably correct within a factor of order unity. Detailed reviews of experiment and theory of the energy loss and of the equilibrium charge state of fast heavy ions in solid and gaseous media have been given by Betz[2] and Ahlen.[3] For partially ionized media these results must be properly generalized to take into account the effects of finite temperature and dielectric polarization of medium. Estimates of the stopping power for the specialized cases of protons in gold at 1 keV,[4] or heavy ions at very high energies,[5,6] typically greater than 10 MeV/amu have been reported. These calculations are based on the approximation that the stopping electrons can be divided into two groups: those bound to the plasma ions and those which constitute the plasma free electrons. Then the contribution of each group of electrons is calculated separately. This approximation cannot be justified for arbitrary temperatures and projectile energies.

The stopping power in ionized media depends on the effective charge and energy of the heavy ion and on the dielectric polarization of the medium. To predict the stopping power, we have used a relatively simple model which has been checked against Northcliffe and Schilling's results and then used to predict the results for ionized media over a range of temperature and density. A novel feature of our work is that we have not made the approximation of calculating the contributions of the plasma electrons and bound electrons to the stopping power separately. We have used a simple model for the dielectric properties of the ionized medium to take into account the effect of both groups at the same time.

A charged particle passing through an ionized medium will induce an electric field by polarizing the medium. The induced electric field will then act back on the particle, resisting its motion, and cause it to lose energy. This field can be related to the dielectric function $\epsilon(\vec{k},\omega)$ of the medium through its Fourier transform. Then, assuming the fast ion moves in a straight line in the x direction, the energy loss per unit distance is given by[7]

$$\frac{dE}{dx}=2\left(\frac{Z^*e}{2\pi}\right)^2\int d\vec{k}\int_{-\infty}^{\infty}d\omega\,\delta(\omega-k_x v)\frac{k_x}{k^2}\,\mathrm{Im}\left(\frac{1}{\epsilon(k,\omega)}\right)\,, \qquad (1)$$

where $k^2=k_x^2+k_\perp^2$; Z^* is the effective charge and v is the velocity of the projectile ion.

A basic assumption in the model is that the dielectric function $\epsilon(\vec{k},\omega)$ of the ionized medium can be expressed as the sum of separate contributions of the plasma electrons and the bound electrons. Thus we have

$$\epsilon(\vec{k},\omega)=1-\frac{\omega_p^2}{\omega(\omega+i\nu_c)-(kv_e)^2}-\sum_j\frac{\omega_{\mathrm{pl}}^2(j)}{\omega(\omega+i\Gamma_j)-\omega_j^2}\ . \qquad (2)$$

In Eq. (2) the second term represents the contribution of the plasma electrons to the dielectric function. The effect of collisional damping is included through the collision frequency ν_c, while $(k v_e)$ (with $v_e^2=\theta/m$) is a frequency associated with thermal motions of the plasma electrons, and ω_p is the plasma frequency. The third term represents the dielectric function of the bound electrons, which is calculated by using a simple model[8] of the atom whose jth shell electrons experience a harmonic force of frequency ω_j, and a phenomenological damping force measured by Γ_j. The frequency $\omega_{\rm pl}^2(j) = 4\pi N_a e^2 n_j/m$, where N_a is the number density of atoms and n_j is the dipole oscillator strength.

The expression for $\epsilon(\vec{k},\omega)$ is complicated because of the discrete sum which requires a knowledge of atomic shell structure, the binding energies and the oscillator strengths. For simplicity we make a continuum approximation by using the Thomas-Fermi model of an atom to estimate bound electron energies and densities. First we consider the contribution to $\epsilon(\vec{k},\omega)$ from the bound electrons. Let Z_0 be the atomic number. In the continuum approximation we assume the jth shell to be of width dr and at a distance r from the respective nucleus. Then the oscillator strength n_j can be expressed in the form

$$n_j=4\pi r^2\rho(r)\,dr \quad , \tag{3}$$

where $\rho(r)$, the local electron density, is given by

$$\rho(r)=\frac{Z_0^2}{4\pi b^3}\frac{\chi(s^2)^{3/2}}{s^2} \quad . \tag{4}$$

In Eq. (4), $\chi(s^2)$ is the universal function appearing in the Thomas-Fermi equation; $b\equiv(3\pi/4)^{2/3}\hbar^2/2me^2$, and the dimensionless parameter s is defined by the expression

$$r=s^2bZ_0^{-1/3} \quad .$$

For the frequency ω_j we take $\hbar\omega_j \simeq I(s)$ with $I(s)$ representing the average over the momentum distribution of the local ionization energy of a bound electron. In the Thomas-Fermi approximation $I(s)$ is given by

$$I(s)=\hbar\omega_0\left(\frac{2}{5}\frac{\chi}{s^2}-\frac{1}{2s}\frac{d\chi}{ds}\bigg|_{s=s_0}\right) \quad , \tag{5}$$

where $\omega_0=(4Z_0^2/3\pi)^{2/3}1/\hbar(2me^4/\hbar^2)$, and s_0 denotes the atom boundary. Thus, using Eqs. (3)–(5), the sum over j in Eq. (2) can be replaced by an integral over s.

Next we consider the plasma contribution to the dielectric function. We assume the sole contributors of electrons to the plasma are the (Z_0-Z) times ionized atoms of the medium where Z is the average number of bound electrons in an ion. Then the plasma frequency can be written as

$$\omega_p^2=\frac{4\pi n_e e^2}{m}=\frac{4\pi N_a(Z_0-Z)e^2}{m} \quad . \tag{6}$$

Using these results, Eq. (2) now becomes

$$\epsilon(\vec{k},\omega)=1-\frac{4\pi N_a Z_0 e^2}{m}\left(\frac{1-Z/Z_0}{\omega(\omega+i\nu_c)-(k v_e)^2}+2\int_0^{s_0}\frac{s^2\chi^{3/2}_{(s)}\,ds}{\omega(\omega+i\Gamma_s)-[I(s)/\hbar]^2}\right) \quad . \tag{7}$$

The above model of $\epsilon(\vec{k},\omega)$ gives a complete description of the bound and continuum electrons, including collisional damping; collective and screening effects. To see the latter, let us suppose the projectile velocity $v \ll v_e$ where v_e is the thermal speed of the plasma electron. Then, since $\omega \simeq kv \ll kv_e$, the plasma contribution to $\epsilon(\vec{k},\omega)$ is just $(k_D/k)^2$, where $k_D=(4\pi n_e e^2/\theta)^{1/2}$ is the Debye wave vector that determines the screening.

The numerical calculation of Eq. (1) using Eq. (7) is still complicated as it involves three-dimensional integration. To proceed further we will consider cases with low enough plasma temperature and large enough v such that $v \gg v_e$. Then kv_e is generally very small compared to ω and the dielectric function can be approximated by an expression independent of k, i.e., $\epsilon(\vec{k},\omega)\simeq\epsilon(\omega)$, which means we have a cold plasma with damped nonpropagating plasma oscillations and no Debye screening. In this approximation the k integration in Eq. (1) is straightforward and we obtain

$$\frac{dE}{dx}=\frac{(Z^*e)^2}{\pi v^2}\int_0^\infty \omega\,d\omega\,\ln\left(\frac{(q_m v)^2+\omega^2}{\omega^2}\right)\left|\mathrm{Im}\left(\frac{1}{\epsilon(\omega)}\right)\right| \quad . \tag{8}$$

The cutoff wave vector q_m is necessary to prevent the logarithmic divergence when $q^2=k_y^2+k_z^2=\infty$. This divergence is a result of the breakdown of the classical dielectric function at small distances. Now, since most of the contribution to the integral in Eq. (8) is from regions with $\omega < q_m v$, the upper limit of integration is essentially $q_m v$.

It is easy to show that if we assume $\nu_c \ll 1$ and neglect the bound electron contribution, the present formula reduces to the formula for stopping power in an electron plasma.[8] Also if we assume $\Gamma_j \ll 1$ and neglect the effect of other electrons when considering the contribution from a bound electron in shell j, our formula for energy loss reduces to the Bohr or Bethe formula[8] depending on the choice of q_m. Thus since $(1/q_m)$ is essentially the minimum impact parameter we have used a q_m value corresponding to the Bohr formula if $\eta \equiv Z^* e^2/\hbar v > 1$, and a value corresponding to Bethe's formula if $\eta < 1$.

The evaluation of (dE/dx) is now straightforward if the effective ion charge Z^*, the plasma collision frequency ν_c, and the phenomenological damping constants Γ_j are known. The Γ_j are generally very small compared with the binding or resonant frequencies ω_j and the result is independent of Γ. So to make the numerical integration in the resonance regions converge with a choice of reasonable number of points, we will assume Γ_j to be a few percent of ω_j. The collision frequency ν_c which gives the collision rate of electrons with neutral atoms and ions in the medium is very small at low temperatures. At finite temperatures ν_c is essentially given by the collision rate of electrons with ions. We have tested this model for the well-known cases of protons on aluminum and gold. The results are within 10% of the corresponding semiempirical values: The stopping power from our model being smaller in the case of protons on aluminum and larger for protons on gold.

Northcliffe and Schilling[1] estimated the stopping power for heavy ions in various cold materials by interpolation and extrapolation from experimental data. For a given ion they made the basic assumption that the ratio of stopping powers for different materials was a function of ion velocity and independent of the ion. This assumption is valid providing that the effective charge Z^* is independent of material and the Coulomb logarithm is the same for different ions. Neither of these assumptions is literally correct. Experiments with gases and foils show an apparent difference in Z^* for a given ion. The Coulomb logarithm is a function of ion velocity only if the quantum distance of closest approach is used for q_m^{-1}. If the classical minimum distance is used, however, the logarithm is also a function of Z^* and therefore ion dependent.

Since Z^* has not been obtained from first principles semiempirical expressions have been used. A formula for the effective charge can be obtained by utilizing experimental stopping power data or by direct measurement of the charge state of ions passing through foils.

An effective charge was obtained by Brown and Moak[9] from stopping power measurements in foils. They also assumed that the Coulomb logarithm is a function of ion velocity only and hence the same as for protons. This is incorrect if the classical minimum impact parameter must be used. In addition to the Brown and Moak semiempirical formula for Z^*, which is identical to that given by Betz,[2] stopping power data have also been used to derive other effective charge formulas[10]; the Betz formula, however, has been commonly used.

To decide on which expression for Z^* to use in our model, we have calculated the cold stopping power of xenon and uranium ions on aluminum (Fig. 1) using both the Betz formula and the effective charge measured[11] directly from the charge state of ions after passing through low-Z foils. According to Ref. 11,

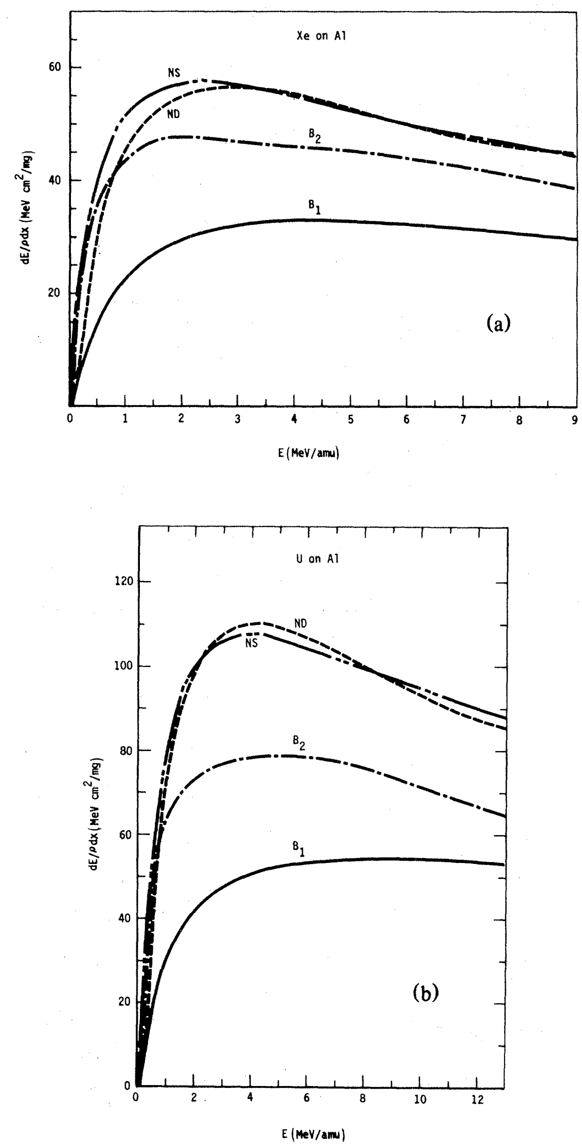

FIG. 1. The cold stopping powers for (a) xenon and (b) uranium on aluminum. (curve B_1) using Betz's Z^* and the classical q_m^{-1} if $\eta = Z^* e^2/\hbar v > 1$ and quantum q_m^{-1} if $\eta < 1$; (curve B_2) using Betz's Z^* and the quantum q_m^{-1}; (curve ND) using Nikolaev and Dmitriev's Z^*; (curve NS) Northcliffe and Schilling's result.

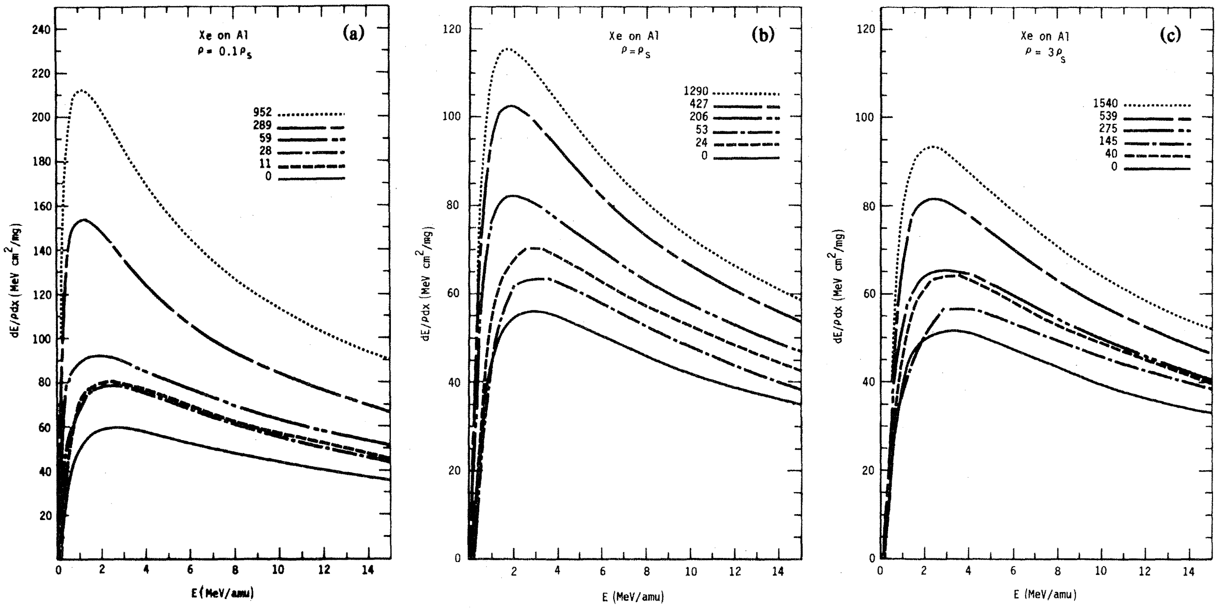

FIG. 2. The stopping power for xenon on aluminum at (a) 0.1 solid density, (b) solid density, and (c) three times solid density, as a function of energy per amu. The curves are labeled by the corresponding temperature in eV.

Z^* is given by the semiempirical formula

$$Z^* = Z\left[1+\left(\frac{Z^{\alpha}v'}{v}\right)^{1/k}\right]^{-k}, \qquad (9)$$
$$\alpha = 0.45, \quad v' = 3.6\times 10^{8}\ \text{cm/sec}, \quad k = 0.6 .$$

We could also have considered other values for Z^* but have found that this is unnecessary (see below). In addition we have used Thomas-Fermi ionization energies and the classical or quantum expression for q_m as appropriate. The Thomas-Fermi approximation gives zero ionization energy for the outermost electrons of a neutral atom. The presence of such weakly bound electrons somewhat increases the stopping power; we therefore have used the *ad hoc* correction of choosing an ionization state for which the

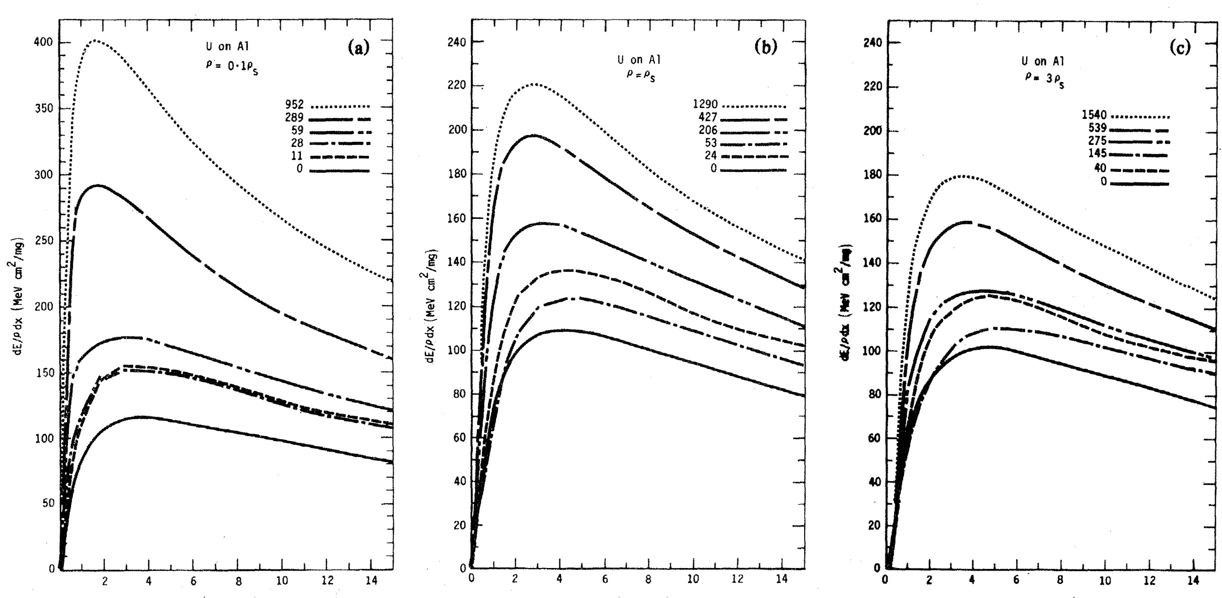

FIG. 3. The stopping power for uranium on aluminum at (a) 0.1 solid density, (b) solid density, and (c) three times solid density, as a function of energy per amu. The legend for each curve is the temperature in eV.

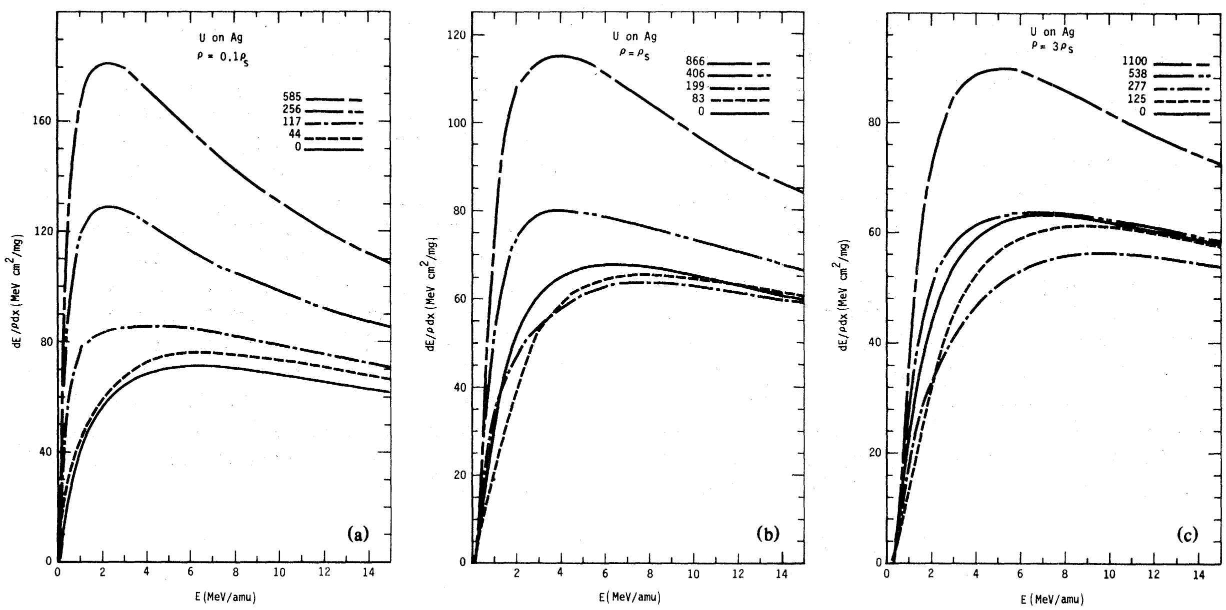

FIG. 4. The stopping power for uranium on silver at (a) 0.1 solid density, (b) solid density, and (c) three times solid density, as a function of energy per amu. The legend for each curve is the temperature in eV.

Thomas-Fermi ionization agrees with experiment. Northcliffe and Schilling's semiempirical results are also given in Fig. 1, in very good agreement with the results of the calculation using Eq. (9). Thus in our model we have chosen Eq. (9), since this formula contributes to a very good agreement between computed and experimental stopping powers when used in conjunction with our choice of excitation energies and appropriate q_m. We have not considered further the implication of the success of the effective charge given by Eq. (9); we choose instead to consider this result only as an *ad hoc* input (justified by agreement with experiment) to our calculations.

We next apply the model to obtain results for partially ionized media. The ionization and excitation energies have again been obtained from the

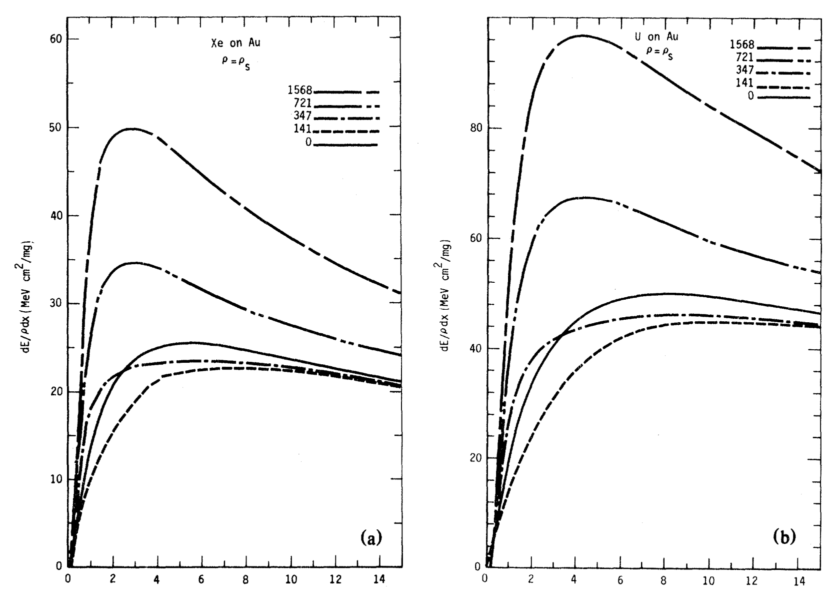

FIG. 5. The stopping power as a function of energy per amu for (a) xenon on gold and (b) uranium on gold. All cases are at solid densities and the legend for the curves is the temperature in eV.

Thomas-Fermi equation for ions, but with the approximation made of evaluating the Thomas-Fermi equation at zero temperature. This neglects excited states of the ions and hence introduces an error (which we believe to be small) in the stopping power. From the ionization energies so obtained, we have used the Saha equation to determine the equilibrium charge state of the target material as function of temperature and density.

The stopping power for xenon and uranium ions in aluminum, silver and gold is given in Figs. 2–5. The variation with temperature is in general nonmonotonic. As bound electrons are ionized, their contribution to the stopping power increases, provided that collisional damping is small. The damping is weaker in low-Z materials; hence the stopping power increases with temperature. For high-Z materials, the collisional damping is more important and the stopping power can be reduced (relative to the cold material) for temperatures up to several hundred electron volts. In order to test the effect of collisional damping we have recalculated the stopping power of xenon on gold, arbitrarily reducing the collision rate by an order of magnitude. We found that the variation of the stopping power with temperature becomes monotonic, similar to the case of the low-Z materials.

These results are valid only when the average electron thermal velocity is appreciably less than the ion velocity. For uranium ions, this requires $\theta < 2320E\,(\text{GeV})$ electron volts. The results also are approximately correct only when degeneracy corrections to the plasma electron equation-of-state can be ignored, i.e., $\theta >> \theta(\text{degeneracy})$ which is satisfied above 10 to 20 eV.

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