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Anomalous magnetoresistance of thin metallic films

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Magnetoresistance observed recently in such thin metallic films as Cu, Pt, Pd, and Pd-Au alloys are analyzed in terms of interplay between spin-orbit interactions and Coulomb interactions in two-dimensional localization theories.

After the scaling proposal by Abrahams *et al.*[1,2] there has been a great deal of interest in the transport properties in two-dimensional systems. There are several experiments to date for the metallic films,[3–10] which observed the logarithmic temperature dependence characteristic of the weakly localized regime.[11] Irrespective of this common feature in the temperature dependence, the magnetoresistance (MR) behaves very differently from system to system, and both positive and negative MR have been observed.

In this Communication we will discuss these experimental results by considering the effects of spin-orbit scattering and Coulomb interactions. The theoretical results of the MR for a case with only spin-orbit scattering are already given elsewhere.[12] The effects of Coulomb interactions in the absence of spin-orbit interactions have been examined[13–15] and shown to play important roles in the weakly localized regime. In the presence of strong spin-orbit scattering, i.e., $\tau_{so} << \tau_\epsilon$, where τ_{so} and τ_ϵ are the lifttimes due to spin-orbit scattering and the inelastic scattering, respectively, the Coulomb interactions play different roles[16] and the results for the conductivity are as follows:

$$\sigma'(T) = -\frac{e^2}{2\pi^2\hbar}\left[-\frac{1}{2}\ln\frac{\tau_\epsilon}{\tau} + \left(g_1 - \frac{g_3}{2}\right)\ln\frac{\hbar}{4\pi\tau k_B T} - \frac{g_2+g_4}{2}\ln\frac{\hbar}{4\pi\tau k_B T_M}\right] , \quad (1)$$

where $g_1 \sim 1$, $g_2 = g_3 = g_4 + \frac{1}{2}F$, $T_M = \mathrm{Max}(T, 1/2\pi\tau_\epsilon)$, and F is the average matrix element of the Coulomb interaction across the Fermi sphere. The first term of the right-hand side of Eq. (1) is due to usual localization but with a different sign from that of the normal impurity scattering.[17] On the other hand, the second and the third terms are because of interactions, the former from particle-hole diffusion processes and the latter from particle-particle diffusion processes. It is noteworthy that in the particle-hole diffusion only parallel spin electrons give singular contributions, while electrons with antiparallel spins contribute to the particle-particle diffusion. By these facts the MR has the following characteristic features. The MR in a perpendicular field and for a field strength $a \equiv 4\epsilon_F\tau eH/mc >> 1/\tau_\epsilon$ and T, where ϵ_F and τ are the Fermi energy and the relaxation time, respectively, is given by

$$\Delta\sigma'(H) = \sigma'(H) - \sigma'(0) = -\frac{e^2}{2\pi^2\hbar}\frac{1+F}{2}\ln H . \quad (2)$$

The MR is positive. The saturation at $a \gtrsim 1/\tau$ is given by

$$\Delta\sigma'(\infty) = -\frac{e^2}{4\pi^2\hbar}\left(\ln\frac{\tau_\epsilon}{\tau} + F\ln\frac{\hbar}{4\pi\tau k_B T_M}\right) . \quad (3)$$

In a parallel field the MR is also positive and is given by Eq. (3) once $\mu_B H >> k_B T$ and $\hbar/\tau_\epsilon$. If $\epsilon_F\tau >> 1$ and $T < \hbar/\tau_\epsilon$ we can expect anisotropy in the MR for the field region $(4\epsilon_F\tau\tau_\epsilon e/mc)^{-1} \lesssim H \lesssim \hbar/\mu_B\tau_\epsilon$ with a larger MR in the perpendicular field. It is interesting to note that even a parallel field suppresses the loalization term, since it is due to the particle-particle diffusion processes with antiparallel spins.[12,18]

We now analyze the following experimental data a–d.

a. Very metallic Cu with $R_\square$ *ranging approximately between* $1\,\Omega/\square$ and $10^2\,\Omega/\square$.[4–6] The prefactor α_T of the logarithmic temperature dependence defined by

$$\sigma'(T) = \frac{e^2}{2\pi^2\hbar}\alpha_T \ln T , \quad (4)$$

takes a value $\alpha_T = 2$.[4] The MR is observable only if the field is applied perpendicularly, in which case it is negative and its field dependence is nicely represented by that of localization theory[15,17]

$$\Delta\sigma'(H) = \frac{e^2}{2\pi^2\hbar}\alpha_H\left[\ln a\tau_\epsilon + \psi\left(\frac{1}{2} + \frac{1}{a\tau_\epsilon}\right)\right] , \quad (5)$$

with empirical parameter τ_ϵ and $\alpha_H \sim 0.75 \pm 0.05$.[6] Here $\psi(Z)$ is the digamma function. This is a good example of a system in the weakly localized regime where both effects of localization and interactions are seen.[11] The theoretical value[14] for $\alpha_H = 1 + g_2 - 2g_4 \sim 1 - \frac{1}{2}F$ is in perfect agreement with the experimental result since F is estimated to be $F \sim 0.6$.[4] Spin-orbit interactions hardly play any role, since Cu is not a heavy metal and the films are clean.

b. Granular Cu film with $R_\square$ ranging approximately between $10^2\,\Omega/\square$ and $10^3\,\Omega/\square$. In a perpendicular field the MR is positive for very weak fields and turns negative at higher fields, $H \gtrsim 1$ T. The field dependence at $H \gtrsim 1$ T is close to Eq. (4). In a parallel field the MR behaves similarly to that in the perpendicular orientation. However, the field at which the MR changes sign is larger; i.e., $H \sim 7$ T. A moderate spin-orbit interaction due to the granularity of these films is sufficient to explain the anisotropy of the MR in the weak-field region.[12] In the parallel field the saturation value of the MR is essentially proportional to $\tau_\epsilon/\tau_{\rm so}$ as far as $\tau_\epsilon/\tau_{\rm so} \lesssim 1$.[12]

One noticeable fact in this system is that the temperature dependences, i.e., α_T of Eq. (4), are almost the same in $H = 0$ and 7 T applied perpendicularly.[8] This behavior can be understood if we assume that the value of F in such granular films is reduced compared to that in genuine metallic films. The $\ln T$ dependence, then, comes from g_1 in Eq. (1). Moreover we argue that τ_ϵ is weakly temperature dependent, may be, because of a small amount of paramagnetic impurities.

c. Pt film with $R = 7.6$ k$\Omega/\square$.[9] The experimental example of this system is not so metallic: $\epsilon_F\tau = 1.7$. The prefactor of the temperature dependence, α_T, is $\alpha_T = 1.50 \pm 0.30$. The MR in a perpendicular field is essentially negative with an unexplained positive background. The maximum change of the conductivity is because of the small field, i.e., 1.4×10^{-6} mho/□ compared to $e^2/2\pi^2\hbar = 1.2 \times 10^{-5}$ mho/□. We think that the spin-orbit interaction in this case is stronger than in b, but not strong enough to overwhelm the inelastic scatterings completely.

d. Pd film with $R_\square$ asymptotically equal to several k$\Omega/\square$.[10] The slope of the temperature dependence is $\alpha_T = 0.63$ for a sample $R(3\text{ K}) = 2043\,\Omega/\square$. The MR is positive for both orientations of the field and $|\alpha_H|$ is around unity. Films of alloys with Au (Pd, 42 wt.%; Au, 58 wt.%) show essentially the same features; $\alpha_T \sim 0.8$ and $|\alpha_H|$ around unity. This is the behavior expected when $\tau_{\rm so} \ll \tau_\epsilon$, as is seen by Eq. (3).

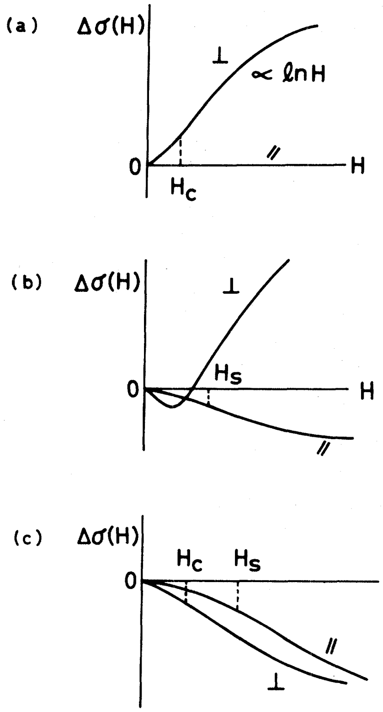

FIG. 1. Schematic representations of the field dependence and the anisotropy of the magnetoresistance when $k_BT < \hbar/\tau_\epsilon$. (a) Very weak spin-orbit scattering, $\tau_\epsilon \ll \tau_{\rm so}$. Here τ_ϵ and $\tau_{\rm so}$ are lifetimes due to the inelastic and spin-orbit scatterings, respectively. The characteristic field, H_c, above which $\ln H$ behavior is expected is $H_c = mc/\epsilon_F\tau\tau_\epsilon e$. (b) Moderate strength of $\tau_{\rm so}$, $\tau_\epsilon \sim \tau_{\rm so}$. The characteristic field, H_s, in the parallel configuration is $H_s \sim \hbar/\mu_B\tau_\epsilon$. (c) Very strong spin-orbit scattering, $\tau_\epsilon \gg \tau_{\rm so}$.

In summary we have analyzed the experimental data on thin metallic films in terms of the strength of spin-orbit scattering in the presence of Coulomb interactions. Typical field dependences and anisotropy of the MR are schematically shown in Fig. 1.

As regards the temperature dependence one may remark that there is a systematic decrease of α_T as one goes from light clean metal films (pure Cu films, $\alpha_T \sim 2$) to heavy granular ones (Pd and Pd-Au films, $\alpha_T < 1$). This change in α_T correlates well with the change in the behavior of the MR and confirms our interpretations.

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