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Quantum-limit magnetoresistance in intrinsic semiconductors

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The magnetoresistance of intrinsic semiconductors is calculated theoretically in the quantum limit where only the lowest Landau level is thermally populated. The variation of the intrinsic carrier concentration with magnetic field is taken into account. We find that in this limit the magnetoresistance increases exponentially with magnetic field due to the field-induced freeze-out of the intrinsic carriers. This arises because of the increase in the effective band gap of the semiconductor with magnetic field.

I. INTRODUCTION

The study of electrical transport in semiconductors in the presence of a magnetic field yields useful information about the various electronic properties related to band structure and scattering mechanisms. At low magnetic fields, the semiclassical Boltzmann transport equation is adequate for the description of electronic transport,[1,2] both in intrinsic and extrinsic semiconductors. The magnetic field is treated as a perturbation in this semiclassical approach under the condition $\hbar\omega_c \lesssim k_B T$ (classical limit), where $\omega_c = eB/m^* c$ is the cyclotron frequency of the electrons of effective mass m^* in a magnetic field B. This semiclassical approach predicts a quadratic dependence of the magnetoresistance on magnetic field for intrinsic as well as extrinsic semiconductors.[2] In high magnetic fields, when $\hbar\omega_c \gtrsim k_B T$, the quantized nature of the electronic energy levels needs to be taken into account. A quantum theory for the extrinsic case was developed earlier by Arora and Peterson[3] and was found to give, in the quantum limit, a linear magnetoresistance for the case of the spherical parabolic band model.[4-6]

Recently, there has been renewed interest in the magnetoresistance of narrow band-gap semiconductors[7-11] which do have an appreciable concentration of intrinsic carriers even at low temperatures. Some experimental results on intrinsic magnetoresistance were reported earlier by Bate *et al.*[12] With the advent of higher magnetic fields[11,13] and the development of narrow band-gap semiconductors with small effective masses for the carriers,[14] it is of interest to investigate the quantum limit magnetoresistance of intrinsic semiconductors. Because of the narrow band gaps, relatively pure samples can remain intrinsic down to lower temperatures while with higher magnetic fields and smaller carrier effective masses, the condition $\hbar\omega_c > k_B T$, which is needed for the quantum limit, can be attained at higher temperatures.

In this paper, we apply the theory developed by Arora and Peterson[3] to intrinsic semiconductors. The application of this theory to intrinsic semiconductors requires two modifications: taking into account the conductivities of both the electrons and holes and considering the magnetic field dependence of the intrinsic carrier concentration due to the increase in the effective band gap with magnetic field. The latter effect, which leads to the magnetic freeze-out of the intrinsic carrier concentration with magnetic field, dominates the magnetic field dependence of the magnetoresistance, leading to an exponentially increasing magnetoresistance with magnetic field.

In Sec. II we obtain the components of the conductivity tensor for an intrinsic semiconductor with equal concentrations of electrons and holes. In Sec. III we obtain the intrinsic carrier concentration in a nondegenerate semiconductor as a function of magnetic field and use the results to obtain in Sec. IV the magnetoresistance for acoustic phonon and ionized impurity scattering in in-

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trinsic semiconductors. Finally, in Sec. V, we present a discussion of the results which we have obtained.

II. MAGNETOCONDUCTIVITY IN INTRINSIC SEMICONDUCTORS

In an intrinsic semiconductor the densities of negatively charged electrons and positively charged holes are equal, $n=p$. Since the total current in an intrinsic semiconductor is the sum of the electron and hole currents $\vec{J}=\vec{J}_e+\vec{J}_h$, the conductivity tensor is given by

$$\overleftrightarrow{\sigma}=\overleftrightarrow{\sigma}_e+\overleftrightarrow{\sigma}_h , \tag{1}$$

where $\overleftrightarrow{\sigma}_e$ and $\overleftrightarrow{\sigma}_h$ are the electron and hole magnetoconductivity tensors, respectively. From our previous results for the components of the magnetoconductivity tensor,[4] we have

$$\sigma_{1e}=\frac{nec\beta_e}{\pi^{1/2}B}(\ln\beta_e^{-2}-\gamma) ,$$
$$\sigma_{2e}=\frac{nec}{B}(1-\pi^{1/2}\beta_e) , \tag{2a}$$

$$\sigma_{1h}=\frac{nec\beta_h}{\pi^{1/2}B}(\ln\beta_h^{-2}-\gamma) ,$$
$$\sigma_{2h}=-\frac{nec}{B}(1-\pi^{1/2}\beta_h) , \tag{2b}$$

where β is a parameter which in general depends upon the magnetic field B and absolute temperature T and on the mechanism by which the electrons and holes are scattered. e and h denote quantities associated with the electrons and holes, respectively, $\gamma=0.57$ is Euler's constant, and the remaining notation is the same as that used in our previous work.[4] Here, the subscripts 1 and 2 denote the directions of the applied electric field and the Hall field, respectively. For deformation-potential scattering of the carriers by acoustic phonons[4]

$$\beta=\frac{2\hbar}{3\pi^{1/2}k_BT\tau_a} , \tag{3a}$$

while for the scattering of the carriers by ionized impurities[5]

$$\beta=\frac{8}{\pi^{1/2}}\left[\frac{k_BT}{\hbar\omega_c^2\tau_i}\right] , \tag{3b}$$

where τ_a and τ_i are the zero-field relaxation times for the scattering of the carriers by acoustic phonons and ionized impurities, respectively. Using the electron and hole conductivities from Eq. (2) in Eq. (1), we obtain for the components of the magnetoconductivity tensor transverse to the magnetic field

$$\sigma_1=\frac{nec}{\pi^{1/2}B}[\beta_e\ln\beta_e^{-2}+\beta_h\ln\beta_h^{-2}-\gamma(\beta_e+\beta_h)] , \tag{4a}$$

and

$$\sigma_2=\frac{\pi^{1/2}nec}{B}(\beta_h-\beta_e) . \tag{4b}$$

III. INTRINSIC CARRIER CONCENTRATION

The derivation of the intrinsic carrier concentration in semiconductors in the absence of a magnetic field can be found in many textbooks.[15] The result for the zero-field intrinsic carrier concentration is

$$n(0)=2\left[\frac{k_BT}{2\pi\hbar^2}\right]^{3/2}(m_em_h)^{3/4}\exp\left[-\frac{E_g}{2k_BT}\right] , \tag{5}$$

where m_e (m_h) is the electron (hole) effective mass and E_g is the band gap separating the conduction band from the valence band. In the presence of the magnetic field, the derivation of the intrinsic carrier concentration is the same except that the energy eigenvalues of the electrons and holes in a magnetic field are used. The electron and hole concentrations are given by

$$n=\frac{eB}{2\pi^2\hbar c}\sum_{n=0}^{\infty}\int_{-\infty}^{+\infty}dk_z\exp\left[\frac{E_F-E_e}{k_BT}\right] , \tag{6a}$$

and

$$p=\frac{eB}{2\pi^2\hbar c}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}dk_z\exp\left[\frac{E_h-E_F}{k_BT}\right] , \tag{6b}$$

where E_F is the Fermi energy of the carriers and the electron and hole energies measured from the top of the valence band are

$$E_e=E_g+(n+\tfrac{1}{2})\hbar\omega_{ce}+\frac{(\hbar k_z)^2}{2m_e} , \tag{7a}$$

and

$$E_h = -(n+\tfrac{1}{2})\hbar\omega_{ch} - \frac{(\hbar k_z)^2}{2m_h}\,, \tag{7b}$$

respectively. Here m_i is the effective mass of the ith carrier and $\omega_{ci} = eB/m_{ic}$ the carrier cyclotron frequency. Using the energy eigenvalues given by Eq. (7) in Eq. (6), we get the following for the electron and hole densities:

$$n = \frac{eB}{2\pi\hbar c}\left[\frac{m_e k_B T}{2\pi\hbar^2}\right]^{1/2} \frac{\exp\left[\dfrac{E_F - E_g}{k_B T}\right]}{\sinh\left[\dfrac{\hbar\omega_{ce}}{2k_B T}\right]} \tag{8a}$$

and

$$p = \frac{eB}{2\pi\hbar c}\left[\frac{m_h k_B T}{2\pi\hbar^2}\right]^{1/2} \frac{\exp\left[-\dfrac{E_F}{k_B T}\right]}{\sinh\left[\dfrac{\hbar\omega_{ch}}{2k_B T}\right]}\,. \tag{8b}$$

In an intrinsic semiconductor $n = p$ and using this result together with Eq. (8), we obtain the magnetic-field-dependent intrinsic carrier concentration

$$n(B) = \frac{eB}{2\pi\hbar c}\left[\frac{k_B T}{2\pi\hbar^2}\right]^{1/2} \times \frac{(m_e m_h)^{1/4}\exp\left[-\dfrac{E_g}{2k_B T}\right]}{\left[\sinh\dfrac{(\hbar\omega_{ce})}{2k_B T}\sinh\dfrac{(\hbar\omega_{ch})}{2k_B T}\right]^{1/2}}\,. \tag{9}$$

In the classical limit, $\hbar\omega_{ci} << k_B T$, Eq. (9) reduces to the zero-field result given by Eq. (5). In the quantum limit, where the condition $\hbar\omega_{ci} >> k_B T$ is satisfied for both types of carriers, the ratio of the intrinsic carrier concentration to its zero-field values reduces to

$$\frac{n(B)}{n(0)} = \frac{e\hbar B}{(m_e m_h)^{1/2} c k_B T}\exp\left[-\frac{\hbar}{4k_B T}(\omega_{ce} + \omega_{ch})\right]. \tag{10}$$

In this limit, the intrinsic carrier concentration decreases exponentially with increasing magnetic field. This magnetic-field-induced freeze out of the intrinsic carrier concentration is due to the increase in the effective band gap with magnetic field, i.e.,

$$E_g(B) = E_g(0) + \tfrac{1}{2}\hbar(\omega_{ce} + \omega_{ch})\,.$$

As a result of the increase of the effective band gap with field, the concentration of carriers thermally excited across the band gap to yield electron-hole pairs decreases.

Another case of interest occurs in semiconductors where one type of carrier has a much smaller effective mass than the other (for example, InSb, where the electron effective mass $m_e = 0.013m_0$ while the hole effective mass[16] $m_h = 0.52m_0$, where m_0 is the free-electron mass). In this case, at reasonable laboratory magnetic fields, the quantum limit applies for the lighter carrier while the classical limit is valid for the heavier carrier. Under these conditions, the intrinsic carrier concentration still decreases exponentially with increasing magnetic fields:

$$\frac{n(B)}{n(0)} = \left[\frac{\hbar\omega_{ce}}{k_B T}\right]^{1/2} \exp\left[-\frac{\hbar\omega_{ce}}{4k_B T}\right]. \tag{11}$$

Here we have assumed (as is frequently the case) that the electrons are the lighter carriers and the holes the heavier carriers.

IV. MAGNETORESISTANCE

The transverse magnetoresistance and the Hall coefficient can be related to the components of the magnetoconductivity tensor σ_1 and σ_2:

$$\rho_T = \frac{\vec{J}_T \cdot \vec{E}}{E} = \frac{\sigma_1}{\sigma_1^2 + \sigma_2^2} = \frac{\pi^{1/2} B}{nec}\frac{A}{A^2 + C^2}\,, \tag{12}$$

and

$$RB = \frac{E_2}{J_1} = \frac{\sigma_1}{\sigma_1^2 + \sigma_2^2} = \frac{\pi^{1/2} B}{nec}\frac{C}{A^2 + C^2}\,, \tag{13}$$

where

$$A = \beta_e \ln\beta_e^{-2} + \beta_h \ln\beta_h^{-2} - \gamma(\beta_e + \beta_h) \tag{14a}$$

and

$$C = \pi(\beta_h - \beta_e)\,. \tag{14b}$$

Here $\vec{J}_T$ is the component of the current flowing transverse to the magnetic field and E_2 is the Hall field induced under open circuit conditions ($J_2 = 0$). In terms of the zero-field intrinsic carrier concentration, the transverse magnetoresistance and the Hall coefficient are

$$\rho_T = \frac{(\pi m_e m_h)^{1/2} k_B T}{n(0) e^2 \hbar} \frac{A}{A^2 + C^2} \times \exp\left[\frac{\hbar}{4k_B T}(\omega_{ce} + \omega_{ch})\right] \quad (15)$$

and

$$RB = \frac{(\pi m_e m_h)^{1/2} k_B T}{n(0) e^2 \hbar} \frac{A}{A^2 + C^2} \times \exp\left[\frac{\hbar}{4k_B T}(\omega_{ce} + \omega_{ch})\right] . \quad (16)$$

For acoustic phonon scattering via deformation-potential coupling, β and therefore A and C are independent of magnetic field,[4] and the transverse magnetoresistance shows a purely exponential increase with magnetic field. For ionized impurity scattering, β and therefore A and C are monotonically decreasing functions of magnetic field,[5] and the preexponential factor in Eq. (15) increases as B^2. However, both the transverse magnetoresistance and the Hall coefficient are dominated by the exponential increase with magnetic field due to carrier freeze out.

Expressions similar to (12) and (15) can be obtained for the longitudinal magnetoresistance. Again, regardless of the detailed scattering mechanism, the longitudinal magnetoresistance will increase exponentially with magnetic field in the quantum limit due to the freeze out of the intrinsic carrier concentration.

V. DISCUSSION AND SUMMARY

In intrinsic semiconductors, in magnetic fields such that the quantum limit can be attained for at least one type of carrier, we expect that both the transverse and longitudinal magnetoresistance, as well as the Hall coefficient, will be exponentially increasing functions of the magnetic field. This exponential increase with field arises from the freeze out of the intrinsic carrier concentration due to the increase of the effective band gap with magnetic field. The exponential increase should dominate the magnetoresistance regardless of the detailed mechanism by which the carriers are scattered.

In cases of practical interest, intrinsic and extrinsic behavior can be simultaneously present in a particular semiconducting example. As is expected, in wide-band-gap doped semiconductors, the intrinsic carrier concentration is negligibly small at low temperatures, thereby giving rise to purely extrinsic conduction. The application of a magnetic field suppresses even further the intrinsic character due to the freeze out of the intrinsic carrier concentration. On the other hand, in narrow-band-gap semiconductors, the intrinsic carriers control the conductivity at sufficiently high temperature and this dominance may continue even in the low-temperature regime if the gap is small enough. At this stage, it may be useful to define a "critical temperature" T_c at which an intrinsic-extrinsic transition takes place. Obviously T_c so defined will depend upon the doped carrier concentration and the magnetic field. At temperatures much lower than T_c, the semiconductor may exhibit extrinsic character, and at temperatures much higher than T_c, it may exhibit an intrinsic character.

The presence of a magnetic field which induces the freeze out of the carriers will tend to increase T_c because the effective band gap is larger in a magnetic field. In the extrinsic regime, below T_c, a change in power law from quadratic (classical limit) to linear (quantum limit) behavior of the magnetoresistance with increasing magnetic field is predicted by us[6] and is confirmed by a number of experimental works.[10–12,17,18] In the intrinsic regime, above T_c, the magnetoresistance is expected to show the exponential behavior predicted by Eq. (15). Nimtz and Schlicht[10] have measured experimentally the longitudinal magnetoresistance in $Hg_{0.8}Cd_{0.2}Te$ at a magnetic field of $B=74$ kG and have observed the magnetoresistance as a function of temperature. The lattice temperature dependence observed by them can be fitted with an exponential function of the form $\exp(a/T)$, where $a=9K$ in the temperature range between 4 and 1.8 K. It would be interesting to observe the magnetic field dependence of a to see if it conforms to that predicted by our theory. They attribute this unusual behavior to electron localization in the form of a Wigner lattice[19] predicted by several theories.[20–23] They conclude their results by stating that the unusual behavior of the magnetoresistance suggests the breakdown of the independent particle model by the formation of a highly correlated electronic ground state (Wigner condensation).

Localization theories[20–25] predict a phase transition of an electron gas in a magnetic field, but the temperature at which this phase transition takes place is shown to be below 1 K. For example, in a

recent work of Gerhardt,[24] a critical temperature of 1 K is predicted at a magnetic field of 50 kG. This critical temperature is shown to be a function of carrier concentration as well as magnetic field. Although it is claimed that the predicted critical temperature is in qualitative agreement with experiments, no systematic experimental study of the dependence of the critical temperature on magnetic field or electron concentration is reported. At this stage, we are unable to make any useful comparison of the prediction of localization theories with the results presented in this paper.

The freeze out predicted by Eq. (10) becomes apparent in several reported experimental observations.[26,27] Raymond *et al.*[26] have observed a freeze-out effect in $Hg_{1-x}Cd_xTe$ ($0.18 \leq x \leq 0.34$). They have specifically indicated a transition between degenerate statistics in weak magnetic fields (Shubnikov—de Haas effect) and nondegenerate statistics in high magnetic field (freeze-out effect). The activation energy is found to increase with magnetic field approximately linearly at high magnetic fields. Although our simple parabolic model predicts a linear rise in activation energy with magnetic field, deviations from this behavior are expected in a nonparabolic model where $\hbar\omega_c$ is to be replaced by $E_{n+1}(k)-E_n(k)$, where $E_n(k)$ is the quantized energy of an electron in a nonparabolic model.[28] A similar freeze out has been reported by Sugihara[27] in his experiments on graphite. Our theoretical results above are consistent with these experimental observations.

As stated earlier, our model is strictly valid for parabolic semiconductors in which spin splitting is neglected. The inclusion of nonparabolicity should give results slightly different from those given by the simple parabolic model. Analytical expressions so obtained[28] are not so simple, but should be included for any quantitative study. It was recently shown by Arora[29] that in parabolic semiconductors, the spin splitting does not affect the magnetoresistance. But, in nonparabolic semiconductors, this may not be so. Of course spin splitting will be very important in magnetic semiconductors where spin-flip interaction may play an active role in relaxation phenomena.

Finally, several theoretical[30–32] and experimental[33] works have considered the magnetoresistance in this extrinsic hopping conductivity region. Mikoshiba[32] has predicted an exponentially increasing magnetoresistance in this region which has been experimentally observed in low carrier-concentration samples of transmutation doped Ge.[33] However, the argument of the exponential for extrinsic hopping conduction is quadratic in the magnetic field, while for the freeze-out effect we predict, this argument of the exponential is linear in the magnetic field. Therefore, these two mechanisms predicting an exponentially increasing magnetoresistance should be experimentally distinguishable. In conclusion, we predict an exponentially increasing magnetoresistance in narrow-band-gap intrinsic semiconductors due to carrier freeze out which is consistent with experimental observation.

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