Phase coherence in a weakly coupled array of 20 000 Nb Josephson junctions

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The properties of a very weakly coupled two-dimensional array of 20 000 1-μm^2 Nb Josephson junctions have been studied to 3 mK. Not only are many of the characteristics similar to those of granular films, but new effects, particularly resistance periodicities with magnetic field, provide direct evidence for the increase of two-dimensional phase coherence as the temperature is lowered through the resistive transition.

The application of vortex-unbinding theories, based on the Kosterlitz-Thouless[1] (KT) model of a two-dimensional (2D) phase transition, to the resistive transition in thin, often granular, superconductors has generated considerable interest.[2–14] Although many of the experimental characteristics agree with those expected from theory,[5,6,12] discrepancies remain.[7,8] In particular, the dc $R(T)$ dependence is rarely of the expected form and the effect of inhomogeneities on the transition width is controversial. Such systems are often modeled as isolated grains connected via Josephson junctions.[15–17] Sufficiently below the grain T_c the amplitude of the order parameter is fixed, the coupling energy E_J is determined by the phase difference between the grains, and the absence of resistance corresponds to a long-range phase locking.[17] In "real" films E_J as well as the charging energy varies widely, neither is independently measurable, and the effects of randomness or clustering cannot be separated from the desired physics.[14,15] It is now possible, however, using modern lithographic techniques, to construct large arrays of well-characterized Josephson tunnel junctions in which all the relevant parameters can be measured. Such macroscopically constructed "atomic" systems offer a useful new experimental system for understanding physics in lower dimensions. In this Communication, we describe the characteristics of a very weakly coupled array. In many ways the behavior is similar to that of granular films indicating that the array is, indeed, a viable model. However, it is the spatial periodicity and homogeneous coupling of the array relative to granular films, the wide temperature range probed, and the ability to compare a single junction with a two-dimensional collection that allow us to uniquely probe the phase-coupling transition. Other studies on junction arrays are examining the effect of artificially introduced inhomogeneities.[9] Although ease of fabrication has lead many groups to study proximity-coupled arrays,[10] the strongly temperature-dependent coupling and the lack of detailed information about a single "junction" make interpretation more difficult.

The all-Nb junctions were patterned on oxidized Si chips using electron beam lithography in a crossed-strip geometry with an overlap area of 1 μm^2. Similar junctions have been entensively studied[18] and the capacitance C is estimated to be 0.1 ± 0.02 pF. Each chip contained a 2D square array of 20 000 junctions, as well as 1D arrays and isolated junctions. An electron micrograph of one corner of the array is shown in Fig. 1(a). Opposite sides were shorted by large superconducting pads for four-terminal measurements. Two types of "grains" were used, squares and crosses, corresponding to a 100×100 lattice where each grain is coupled to its four nearest neighbors

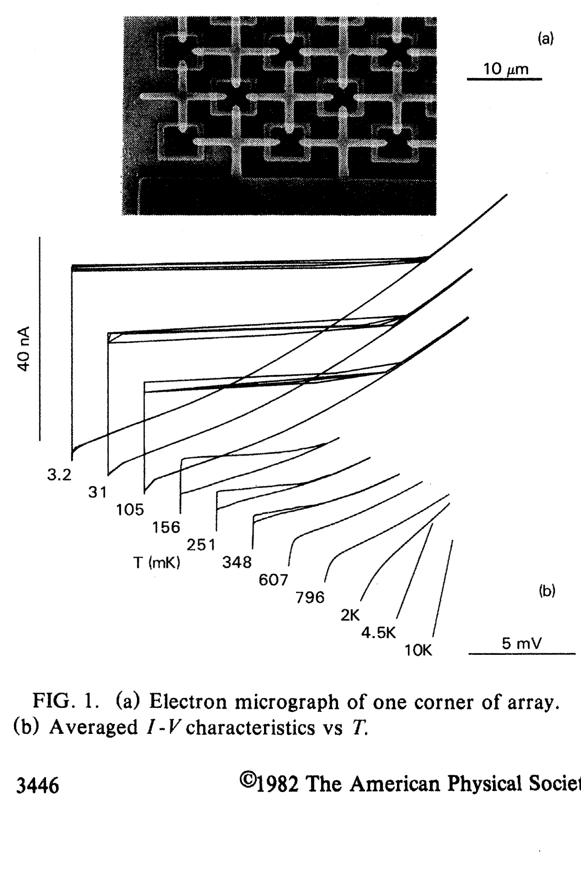

FIG. 1. (a) Electron micrograph of one corner of array. (b) Averaged I-V characteristics vs T.

through the overlap junctions. The samples were placed inside the copper mixing chamber of a dilution refrigerator. A small magnetic field could be applied to the sample inside its cooled mu-metal shield. As with previous experiments,[18] the samples were carefully isolated from external noise.

Figure 1(b) shows a series of I-V characteristics of the array. Above the Nb $T_c \approx 9$ K, the array has a resistance $R \approx 50$ kΩ. No significant change is observed on cooling through T_c, indicating that the conduction properties are dominated by the junctions. By 2 K a thermal-noise-rounded critical current appears. I_c continues to grow and sharpen until about 400 mK where the characteristic becomes hysteretic. In this range the transition to the normal state is gradual with successive traces at fixed T giving about the same characteristic. The sequence above 150 mK mimics the characteristics reported for granular films.[11] At some point below 150 mK, depending on sample history, there is a sudden increase in I_c, the transition from "supercurrent" to normal resistance is now abrupt, and the behavior of the array approaches that of a single hysteretic junction.[18] Similar coherent behavior has not been reported for granular films. As shown by the superposition of four traces at low T in Fig. 1(b), successive traces now give a wide distribution of currents for the transition to finite voltage.

Figure 1(b) shows that the array strongly reflects the behavior of the individual junctions. Similar hysteresis has not been reported (and would not be expected) for proximity coupled arrays[10] in which individual elements are nonhysteretic. This striking dependence on junction characteristics, which is obvious in an array, has not been considered in recent interpretations of film I-V nonlinearities[11-13] in terms of vortex unbinding. This dependence may also explain the differences in I-V characteristics between granular Al systems[11,13] (and junction arrays) and amorphous Hg-Xe systems[12] (and proximity effect arrays).

Figure 2(a) shows the measured I_c vs T as determined by the maximum $|\partial V/\partial I|$. The maximum I_c was $\approx$40 nA corresponding to a critical current per junction $i_c \approx 0.4$ nA for a homogeneous array, consistent with measurements of the 1D arrays on the same chip. Measurements on higher current-density chips, where each i_c could be accurately determined, showed i_c variations of about $\pm 50\%$ over the chip and, since i_c varies exponentially with oxide thickness, we expect comparable relative variations at other current densities. In the range 10–100 mK either the abrupt or gradual transitions discussed above could be observed. Both, however, showed the same surprising T dependence, $I_c \propto -\ln(T/T_c)$. In this range ($T \lesssim T_c/2$) the individual junction characteristics are expected to be independent of T so the $\ln T$ behavior may be characteristic of the 2D coupling.

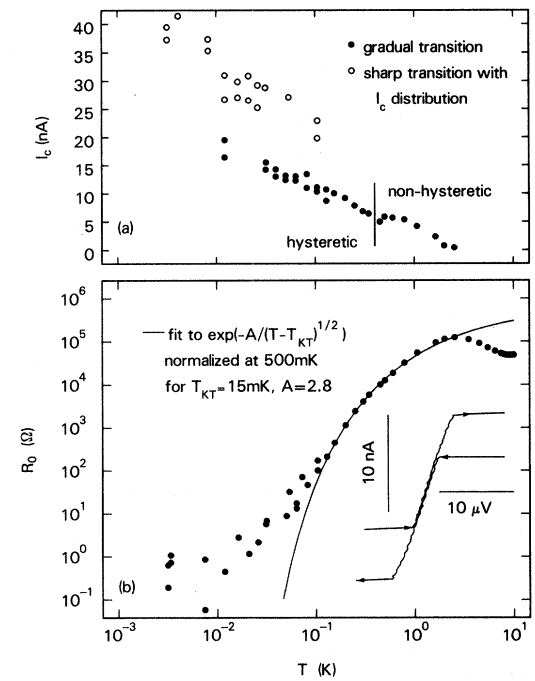

FIG. 2. (a) I_c vs T. (b) Zero-bias resistance R_0 vs T with expected form for a 2D resistive transition. Inset shows I-V characteristic at 156 mK where R_0 is clearly visible.

Even though the I-V characteristics at low T show a well-defined I_c, the "supercurrents" in Fig. 1(b) do not correspond to a true zero-resistance state. This coexistence of superconductivity (phase coherence on fast time scales) with resistance (eventual phase disruption due to flux quantum or vortex motion) on longer time scales is well known in flux-flow experiments and is precisely the mechanism proposed[2] for the broad transitions in thin films. Similarly, there is always a finite probability for flux crossing an individual junction[18] and a nonzero resistance R_0 around zero bias at all T. Figure 2(b) shows $R_0(T)$ measured from computer fits of I-V characteristics with zero average magnetic field. The inset shows the characteristic at 156 mK where R_0 is clearly visible at the magnified voltage scale. As T is lowered from 10 K the resistance initially rises due to formation of a superconducting gap in the grains (an effect that may be confused with localization) and then begins to fall as the grains couple.

Figure 3 illustrates the oscillatory behavior of R_0 at small magnetic fields H. At higher T a single period (0.17 Oe) is observed that corresponds roughly to a flux quantum ϕ_0 within the smallest path in the array containing junctions. This fundamental modulation, which vanishes for $T \gtrsim 700$ mK, indicates the presence of phase coherence over the lattice spacing. At fields larger than those shown in Fig. 3, the modula-

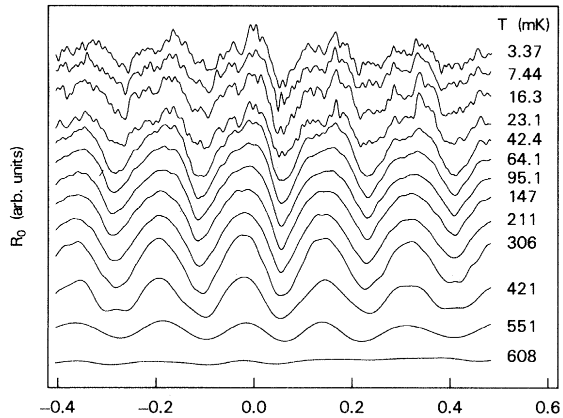

FIG. 3. Dependence of R_0 on perpendicular magnetic field H at various T.

tion amplitude decreases and a positive magnetoresistance is observed. As T is lowered the fundamental period remains dominant until ≈80 mK where a large number of reproducible higher periodicities begin to appear. Fourier analysis of these curves clearly show the development of higher harmonics as T decreases.

This $R_0(H)$ provides direct evidence that a broad resistive transition need not be due to inhomogeneities. The periodic modulation of resistance is a quantum interference effect that requires some degree of phase coherence over the area of ϕ_0. Higher periodicities require coherence over larger areas. As T is lowered, the number of free vortices that destroy coherence decrease, R decreases, and the coherence length ξ [as demonstrated by $R_0(H)$] increases. In an inhomogeneous system, portions would be either strongly superconducting ($R=0$) or normal, depending on temperature, but neither type could produce the periodic modulation. The multiple periodicities can also be understood in terms of the mobility of an array of flux quanta of varying commensurability relative to the grain lattice.[19] The cusplike resistance minimum near $H=0$ in Fig. 3 is probably characteristic of the small field effects discussed by Doniach *et al.*[3]

Beasley *et al.*[2] proposed a simple relation between the transition (phase locking) temperature T_{KT} and the sheet resistance $R_\square$. For $T_{KT} << T_c$,

$$T_{KT} \approx 2.18\, T_c(\hbar/e^2)/R_\square \quad . \tag{1}$$

Here, $R_\square \approx 50$ k$\Omega/\square$, $T_c \approx 9$ K, and $T_{KT} \approx 1$ K. It must be remembered, however, that Eq. (1) assumes that the same mechanism is responsible for both the resistance and the superconductivity. In cases with a significant parallel normal conduction or for nonideal junctions, Eq. (1) overestimates T_{KT}. On a more fundamental level, T_{KT} is directly related to the coupling strength[1] which in superconductors is measured by the kinetic sheet inductance $L_\square$. For a square array, $L_\square = \phi_0/2\pi i_c$ and

$$k_B T_{KT} \approx \frac{\pi}{2}\left(\frac{\hbar}{2e}\right)^2 \frac{1}{L_\square} = \frac{i_c\phi_0}{4} = \frac{\pi}{2} E_J \quad . \tag{2}$$

For our array, $E_J/k_B \approx 9.5$ mK (comparable to the charging energy $E_c/k_B = e^2/2Ck_B \approx 9.3$ mK). In an ideal 2D superconductor such weak coupling would correspond to $R_\square \approx 5.4 \times 10^6\ \Omega/\square$. Equation (2) gives $T_{KT} \approx 15$ mK in much better agreement with the data than Eq. (1).

For $T \gtrsim T_{KT}$, R_0 is expected to be dominated by the density of free vortices[4] and to take the form $R_0 \propto \exp[-A/(T-T_{KT})^{1/2}]$. The solid line in Fig. 2(b) shows this form adjusted to fit at 500 mK for $T_{KT} = 15$ mK. Similar agreement is possible for T_{KT} in the range 10–20 mK. Although the fit is quite good over three orders of magnitude, R_0 departs from the fit for $T \lesssim 80$ mK. This is the same range where the I-V characteristic begins to mimic a single junction and the higher periodicities in $R_0(H)$ become obvious in Fig. 3 and suggests that $\xi(T)$ has grown to the array size. As T is further lowered the system acts zero dimensional and $R_0(H)$ shows little additional change. We estimate that ξ increases[4] to the array size when $T \approx 28$–50 mK depending on the choice of $T=0$ Ginsburg-Landau coherence length. In addition to this finite-size effect, thermal activation and quantum tunneling,[18] magnetic fields,[3] and the competition between E_c (favoring localization)[20,21] and E_J (favoring phase coherence) may influence R_0 at the lowest T. Thus, a unique interpretation in this range is impossible at present.

In conclusion, the array behavior can be understood as the approach to a 2D phase-coupling transition with $T_{KT} \approx 15$ mK as estimated from E_J and not $R_\square$. The $R_0(H)$ periodicities provide compelling evidence that this 2D transition is due to increasing phase coherence and not inhomogeneities. The I-V characteristics demonstrate the influence of the individual coupling in determining overall behavior and suggest caution in ascribing film I-V nonlinearities to "universal" behavior. In addition, the measurements raise some intriguing questions as to the origin of $I_c \propto -\log T$ and the details of $R_0(T)$ at low T when ξ exceeds the array size. Finally, the observed phase coherence demonstrates that superconductivity can dominate over localization even when $R_\square > 30$ k$\Omega/\square$. This limit[14,15] is seemingly only the beginning of a gradual transition[20] from phase coupling to localization as $R_\square$ increases.

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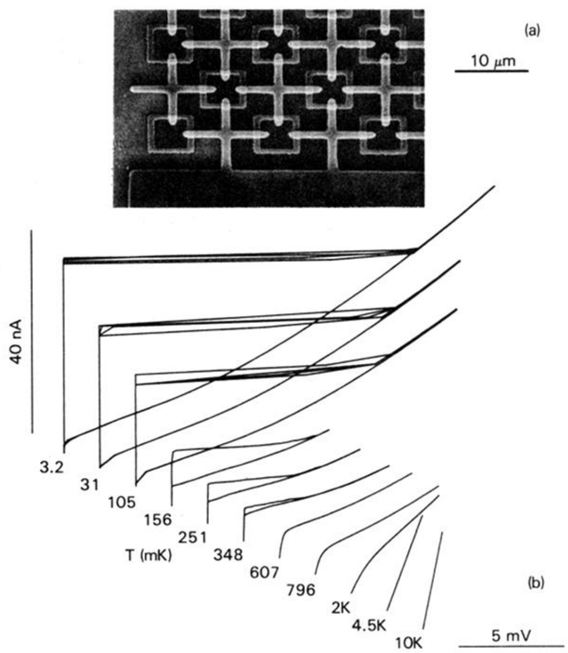
[1]J. M. Kosterlitz and D. J. Thouless, J. Phys. C 6, 1181 (1973).
[2]M. Beasley, J. Mooij, and T. Orlando, Phys. Rev. Lett. 42, 1165 (1979).
[3]S. Doniach and B. A. Huberman, Phys. Rev. Lett. 42, 1169 (1979).
[4]B. I. Halperin and D. R. Nelson, J. Low Temp. Phys. 36, 599 (1979).
[5]A. F. Hebard and A. T. Fiory, Phys. Rev. Lett. 44, 291 (1980).
[6]R. F. Voss, C. M. Knoedler, and P. M. Horn, Phys. Rev. Lett. 45, 1523 (1980).
[7]K. A. Muller, M. Pomerantz, C. M. Knoedler, and D. Abraham, Phys. Rev. Lett. 45, 832 (1980).
[8]P. A. Bancel and K. E. Gray, Phys. Rev. Lett. 46, 148 (1981).
[9]A. Davidson and C. C. Tsuei, Physica (Utrecht) 108B+C, 1243 (1981).
[10]See, for example, B. Giovannini, D. H. Sanchez, and J. L. Berchier, in *Inhomogeneous Superconductors—1979,* edited by T. L. Francavilla, D. V. Gubser, J. R. Leibowitz, and S. A. Wolf, AIP Conf. Proc. No. 58 (AIP, New York, 1979), p. 152; D. H. Sanchez and J. L. Berchier, *ibid.,* p. 239; D. J. Resnick *et al.,* Phys. Rev. Lett. 47, 1542 (1981).
[11]Z. Ovadyahu, Phys. Rev. Lett. 45, 375 (1980).
[12]K. Epstein, A. M. Goldman, and A. M. Kadin, Phys. Rev. Lett. 47, 534 (1981).
[13]S. A. Wolf *et al.,* Phys. Rev. Lett. 47, 1071 (1981).
[14]A. F. Hebard and J. M. Vandenberg, Phys. Rev. Lett. 44, 50 (1980).
[15]B. Abeles, Phys. Rev. B 15, 2828 (1974).
[16]P. K. Hansma and J. R. Kirtley, J. Appl. Phys. 45, 4016 (1974).
[17]J. Rosenblatt, Rev. Phys. Appl. 9, 217 (1974).
[18]R. F. Voss and R. A. Webb, Phys. Rev. Lett. 47, 265 (1981).
[19]P. Martinoli and J. R. Clem, in *Inhomogeneous Superconductors—1979,* edited by T. L. Francavilla, D. V. Gubser, J. R. Leibowitz, and S. A. Wolf, AIP Conf. Proc. No. 58 (AIP, New York, 1979), p. 176.
[20]Y. Imry and M. Strongin, Phys. Rev. B 24, 6353 (1981).
[21]E. Simanek, Phys. Rev. Lett. 45, 1442 (1980).



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