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Influence of the cosϕ conductance on fluxons propagating in long Josephson junctions

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The influence of a cosϕ conductance on the motion of fluxons in long and narrow Josephson junctions is investigated by numerical computations and by a perturbation analysis. It turns out that the presence of the cosϕ term will have opposite effects on the motion of a fluxon and on plasma waves or breathers. If the fluxon motion is damped, the plasma waves are enhanced by the cosϕ term and vice versa. The presence of loss and bias results, in any event, in stabilization of the fluxon in a stationary motion. Good agreement between the numerical result for fluxon motion and the perturbation analysis is found.

I. INTRODUCTION

Nonlinear solitary waves are currently being used in a remarkable variety of contexts in almost every area of physics. In particular the sine-Gordon 2π-kink solution is ubiquitous in its application as a model for dislocations in crystals, domain walls in ferromagnets, and propagation of flux quanta, fluxons, in Josephson transmission lines.

The dynamics of fluxons on Josephson-junctions transmission lines has been studied extensively. In particular we mention the papers by McLaughlin and Scott[1] and Kaup and Newell[2] which contain many references. In Refs. 1 and 2 a perturbation theory for fluxons propagating on Josephson lines with bias, impurities, and losses is formulated. Some of the results from this theory were compared with numerical solutions by Christiansen and Olsen.[3] A numerical investigation of the reflection of a single fluxon on a semi-infinite Josephson line at a passive or an active boundary has been performed by Christiansen and Olsen.[4] They found a single reflected fluxon, an antifluxon, absorption of the incident fluxon, or fission into an arbitrary number of fluxons depending on the boundary conditions. In Ref. 5 the reflection of a single fluxon on a boundary condition modeling an external magnetic field has been performed numerically. The outcome of the reflection can be an arbitrary number of fluxons or antifluxons, the number depending on the magnetic field and the velocity of the incoming fluxon.

Besides the 2π-kink solution, the sine-Gordon equation possesses the breather solution which recently has attracted considerable interest.[6–9] The breather can be considered as a bound state of a fluxon and an antifluxon. Furthermore, fluxons, antifluxons, and breathers possess the remarkable soliton property. Also, breathers have an oscillatory degree of freedom, which increases their physical potential. In Ref. 10 the effect of a boundary on a breather was examined numerically. Depending on the boundary condition (modeling either a passive or an active termination) the breather was reflected into a breather of decreased or increased energy.

Fluxon propagation in long Josephson-line cavities has been used to explain the so-called zero-field steps (without external magnetic field) in the current-voltage characteristics.[11,12] The nth zero-field step corresponds to a situation in which n fluxons are propagating back and forth along the junction, being reflected at the ends. Each reflection creates a voltage pulse. Thus a single fluxon on a Josephson line of length l produces a microwave signal of frequency $u/2l$, where u is the velocity of the fluxon. Recently microwave emission from long Josephson tunnel junctions dc-current biased on zero-field steps and Fiske steps has been measured[13] (the Fiske steps appear when an external magnetic field is applied). It is remarkable that the even Fiske steps coincide with the zero-field steps. The measurements of the radiation emitted from the long junctions biased on Fiske steps show exactly the same features as the radiation from zero-field steps. Therefore it was concluded that Fiske steps in long junctions are due to propagating fluxons.[14]

In the present paper we examine the influence of the cosϕ conductance[15] on a propagating fluxon. It is easily seen that a positive cosϕ loss term will enhance the damping of small-amplitude plasma waves while a negative $\cos\phi$ term will reduce the damping. The result of the present investigation is that the effect of the $\cos\phi$ term on fluxon motion is the opposite: A negative $\cos\phi$ term enhances the resistance against the motion while a positive reduces the resistance.

The outline of the paper is as follows. In Sec. II we apply a well-known perturbation analysis for fluxon propagation to derive the trajectory for single fluxons. The results are compared to numerical solutions of the corresponding initial value problem for the perturbed sine-Gordon equation in Sec. III. In Sec. IV we comment on the results.

II. PERTURBATION ANALYSIS BASED ON ENERGY ARGUMENT

In this section we consider a perturbed normalized sine-Gordon equation describing the motion of fluxons in a long and narrow Josephson junction[1,2] including a $\cos\phi$ term.[15] Further the perturbation theory from Ref. 1 is applied to develop an ordinary differential equation for the velocity of a single fluxon. This equation is integrated to obtain the trajectory for a fluxon.

We consider the perturbed sine-Gordon equation

$$\phi_{xx}-\phi_{tt}=\sin\phi+\alpha\phi_t(1+\epsilon\cos\phi)+\eta \ , \tag{2.1}$$

where $\phi(x,t)$ is the space- and time-dependent phase difference between the two superconducting films. The spatial variable x is measured in units of the Josephson penetration depth $\lambda_J=(\hbar/\mu_0 2edj_J)^{1/2}$ and the time t in units of the reciprocal plasma frequency ω_p^{-1}, $\omega_p=(2ej_J/\hbar C)^{1/2}$. Here $-e$ is the electronic charge, $2\pi\hbar$ is Planck's constant, μ_0 is the permeability of free space, j_J is the maximal Josephson current density through the barrier, d is the magnetic thickness of the barrier $(2\lambda+t_0)$, where λ is the London penetration depth, and t_0 the thickness of the barrier, and C is the capacitance per unit area of the junction $(\epsilon_0\epsilon_r/t_0)$. The second term on the right side of (2.1) represents dissipative effects. Thus $\alpha=G_0(\hbar/2edj_JC)^{1/2}$ and $\epsilon=G_1/G_0$, where G_0 is the conductance due to the quasiparticle tunneling current and G_1 the conductance due to the quasiparticle interference current. The terms G_0 and G_1 are fairly complicated functions of voltage and temperature but in the following we regard them as independent of voltage. Furthermore physical reality requires that $-1\leq\epsilon\leq 1$.[15]

The presence of dissipative terms require a mechanism of energy input in order to obtain physically interesting solutions. Such a mechanism is represented by the η term which models a uniformly distributed bias current (only valid when the width of the junction is much less than the Josephson penetration depth).

The development of small amplitude plasma waves are determined by the equation

$$\phi_{xx}-\phi_{tt}=(1-\eta^2)^{1/2}\cdot\phi+\alpha\phi_t[1+\epsilon(1-\eta^2)^{1/2}] \ , \tag{2.2}$$

which can be derived by replacing ϕ by $-\sin^{-1}\eta+\phi$ in (2.1) and linearizing in ϕ.

Equation (2.2) shows that the $\cos\phi$ term changes the damping parameter for the plasma waves from α to α_{plasma} given by

$$\alpha_{\text{plasma}}=\alpha[1+\epsilon(1-\eta^2)^{1/2}] \ , \tag{2.2a}$$

i.e., a negative $\cos\phi$ term (negative ϵ) will reduce the damping of the plasma waves while a positive ϵ will enhance the damping.

Now we examine fluxon solutions to Eq. (2.1). The sine-Gordon equation [i.e., Eq. (2.1) with $\alpha=\eta=0$] has the Hamiltonian or total energy

$$H=\int_{-\infty}^{\infty}(\tfrac{1}{2}\phi_x^2+\tfrac{1}{2}\phi_t^2+1-\cos\phi)\,dx \ . \tag{2.3}$$

The fluxon solution to the sine-Gordon equation is given by

$$\phi_0(x,t)=4\tan^{-1}\{\exp[\gamma(u)(x-ut-x_0)]\} \ . \tag{2.4}$$

Here x_0 is the initial position of the fluxon with the velocity u and $\gamma(u)$ is the Lorentz factor

$$\gamma(u)=(1-u^2)^{-1/2} \ . \tag{2.5}$$

Insertion of (2.4) into (2.3) yields the energy of a single fluxon

$$H_f=8\gamma(u) \ . \tag{2.6}$$

The parameters α and η are small. First we assume the effect of the perturbation on a fluxon is to modulate its velocity. Thus the corresponding fluxon solution to (2.1) can be written as

$$\phi(x,t)=4\tan^{-1}\exp\{\gamma(u(t))[x-X(t)]\} \ , \tag{2.7}$$

where the location of the fluxon is given by

$$X(t)=\int_0^t u(t)\,dt+x_0 \ . \tag{2.8}$$

Second, we derive the time derivative of H_f

$$\frac{d}{dt}H_f=8\gamma(u)^3u\frac{du}{dt} \ . \tag{2.9}$$

Differentiating the Hamiltonian (2.3), integrating by parts, and inserting the perturbed Eq. (2.1) yields

$$\frac{d}{dt}H_f=-\int_{-\infty}^{\infty}\phi_t[\alpha\phi_t(1+\epsilon\cos\phi)+\eta]\,dx \ . \tag{2.10}$$

Finally, insertion of (2.7)–(2.9) in (2.10), integration, and rearranging give an ordinary diffential equation for the velocity of the fluxon

$$\gamma(u)^3\frac{du}{dt}=\frac{d}{dt}[u\gamma(u)]=-\alpha\left(1-\frac{\epsilon}{3}\right)u\gamma(u)+\frac{\pi\eta}{4} \ . \tag{2.11}$$

Equation (2.11) shows that the cosϕ term changes the damping parameter for the fluxon motion from α to $\alpha_{\rm fl}$ given by

$$\alpha_{\rm fl}=\alpha\left[1-\frac{\epsilon}{3}\right] . \qquad (2.11a)$$

Comparison of Eqs. (2.2a) and (2.11a) shows the opposite effect of the cosϕ term on plasma waves and fluxons. For instance we would, with a small bias η and an ϵ close to -1, have a situation where plasma waves or breathers experience practically no damping while the fluxon will meet an effective damping parameter of $\frac{4}{3}\alpha$.

We now introduce the quantity z, which is $\frac{1}{8}$ of the momentum of the fluxon, by

$$z=u\cdot\gamma(u) , \qquad (2.12)$$

and the inverse

$$u=\left[\frac{z^2}{1+z^2}\right]^{1/2} . \qquad (2.13)$$

Note that the dissipative terms cause the fluxon to slow down while the bias term accelerates the fluxon in positive direction. The stationary velocity found from (2.11), representing a balance between energy input and energy dissipation, is

$$u_\infty=\left[\frac{z_\infty^2}{1+z_\infty^2}\right]^{1/2} , \qquad (2.14)$$

where

$$z_\infty=\frac{\pi\eta}{4\alpha_{\rm fl}} . \qquad (2.15)$$

This result is of course independent of the initial conditions.

In order to determine the trajectory of a fluxon we integrate equation (2.11) to obtain

$$\begin{aligned} z&=z_\infty+(z_0-z_\infty)e^{-\alpha_{\rm fl}\cdot t}\\ &=z_0+\frac{\pi\eta}{4}t, \quad \text{for } \alpha=0 , \end{aligned} \qquad (2.16)$$

where $z_0=u_0\gamma(u_0)$, u_0 being the initial velocity of the fluxon. Insertion of (2.16) via (2.13) into (2.8) yields[3]

$$\begin{aligned} x&=x_0+u_\infty t-\frac{1}{\alpha_{\rm fl}}\ln\frac{z+(z^2+1)^{1/2}}{z_0+(z_0^2+1)^{1/2}}\\ &\quad-\frac{u_\infty}{\alpha_{\rm fl}}\ln\frac{1+z_\infty z_0+(z_\infty^2+1)^{1/2}(z_0^2+1)^{1/2}}{1+z_\infty z+(z_\infty^2+1)^{1/2}(z^2+1)^{1/2}}\\ &=x_0+\frac{4}{\pi\eta}\left\{\left[1+\left(\frac{\pi\eta}{4}t+z_0\right)^2\right]^{1/2}-(1+z_0^2)^{1/2}\right\}\\ &\qquad\qquad \text{for } \alpha=0 . \end{aligned} \qquad (2.17)$$

In Sec. III we solve the corresponding initial value problem numerically and compare the results with the results from the analysis in this section.

III. NUMERICAL RESULTS

In this section we consider a single fluxon propagating under the influence of loss and bias. The results are then compared to the results obtained in Sec. II.

We solve the initial value problem (2.1) with

$$\begin{aligned} &\phi(x,0)=F(x,0) ,\\ &\phi_t(x,0)=F_t(x,0) , \\ &\phi_x(0,t)=\phi_x(l,t)=0 , \end{aligned} \qquad (3.1)$$

where

$$F(x,t)=4\tan^{-1}\{\exp[\gamma(u_0)(x-u_0t-x_0)]\}-\sin^{-1}\eta . \qquad (3.2)$$

Here the expression for $F(x,t)$ represents a fluxon with initial location x_0 and initial velocity u_0. The fluxon is Lorentz contracted. The boundary conditions at $x=0$ and $x=l$ model an open-ended junction.

The numerical results are obtained by means of a computer program based on the method of characteristics and are displayed in terms of $\phi_x(x,t)$.

A. Influence of loss

In this section we present numerical solutions of the problem (2.1) and (3.1) and (3.2) with $\eta=0$. Figure 1 shows the propagation of fluxon in a junction with loss. The parameters are chosen to be

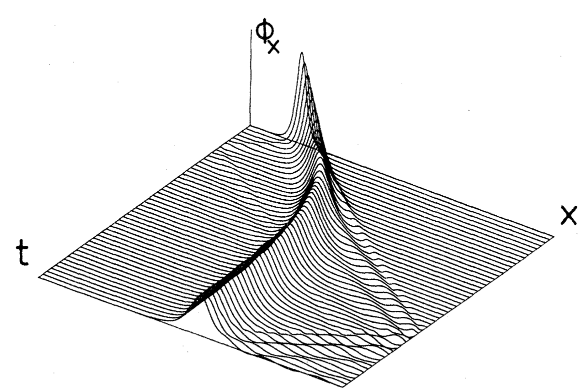

FIG. 1. Propagation of a fluxon in a junction with loss. The parameter values are $\alpha=0.1$, $\eta=0$, $\epsilon=-1$, $x_0=5$, and $u_0=0.9$. The results are displayed in terms of ϕ_x for $0\leq x\leq 30$ and $0\leq t\leq 40$. For $t\to\infty$ the fluxon converges towards a static location at $x=16.04$. The radiation is undamped.

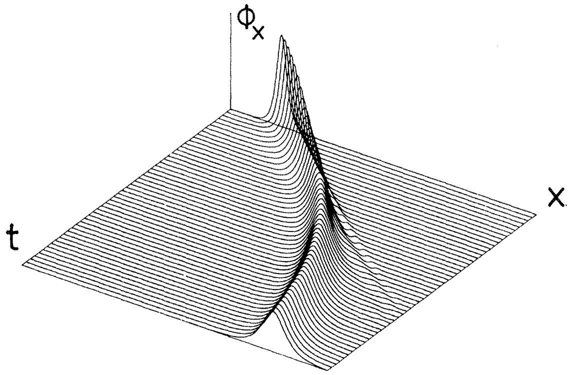

FIG. 2. Propagation of a fluxon in a junction with loss. The parameter values are as in Fig. 1 except for $\epsilon=1$. The results are displayed in terms of ϕ_x for $0\leq x\leq 30$ and $0\leq t\leq 40$. For $t\to\infty$ the fluxon converges towards a static location at 27.08. Compared to Fig. 1 the radiation dies rapidly out.

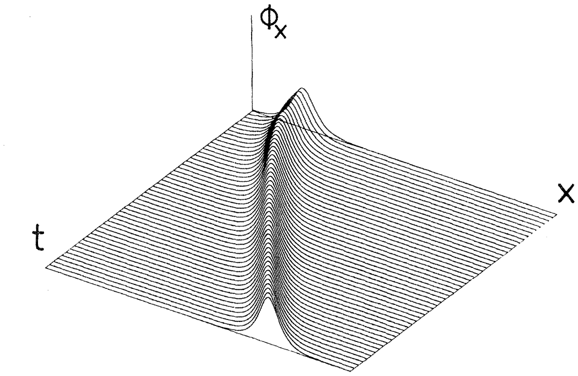

FIG. 3. Propagation of a fluxon in a junction with loss and bias. The parameter values are $\alpha=0.1$, $\eta=0.1$, $\epsilon=-1$, $x_0=5$, and $u_0=0$. The results are displayed in terms of ϕ_x for $0\leq x\leq 30$ and $0\leq t\leq 40$. The initially static fluxon stabilizes at a velocity determined by the balance between loss and bias.

$\alpha=0.1$, $\epsilon=-1$, $x_0=5$, $u_0=0.9$, and $l=30$. The fluxon is seen to converge towards a static location. For $t=40$ we find numerically $x=16.1$. For $u_\infty=0$, Eq. (2.17) becomes[3]

$$x=x_0+\frac{1}{2\alpha_{\text{fl}}}\left[\ln\frac{1+u_0}{1-u_0}-\ln\frac{1+u}{1-u}\right] . \tag{3.3}$$

For the parameter values given above, (3.3), (2.13), and (2.16) give, at $t=40$, $x=15.97$ which agrees well with the numerical result. As a result of the deceleration of the fluxon there is a small amount of radiation propagating at the characteristic velocity. The radiation is reflected at the boundary at $x=30$ and is not damped in accordance with the analysis in Sec. II [Eq. (2.2a)].

In Fig. 2 we have changed the sign of the coefficient of the $\cos\phi$ term. Thus the parameter values in (2.1) (3.1), and (3.2) are $\alpha=0.1$, $\eta=0$, $\epsilon=1$, $x_0=5$, and $u_0=0.9$. In this case the fluxon travels a longer distance (compared to Fig. 1) before it stops. This indicates that the fluxon is less damped for $\epsilon=1$ than for $\epsilon=-1$. The radiation resulting from the deceleration of the fluxon is seen to die rapidly out in accordance with the results in Sec. II. At $t=40$ the position of the fluxon in Fig. 2 is $x=24.7$, while Eq. (3.3) yields $x=24.94$. Thus a good agreement between the perturbation theory and the numerical results is observed.

The results in this section illustrate how the $\cos\phi$ term effects fluxons and plasma waves differently [Eqs. (2.2a) and (2.11a)].

B. Influence of loss and bias

In this section the combined effect of loss and bias is investigated. Thus we solve the initial value problem in (2.1), (3.1), and (3.2) with $\alpha=0.1$, $\eta=0.1$, $\epsilon=-1$, $x_0=5$, $u_0=0$, and $l=30$. In Fig. 3 the results are displayed in terms of $\phi_x(x,t)$. An initially static fluxon is accelerated into a stationary motion, with the velocity u_∞ given by Eqs. (2.14) and (2.15), $u_\infty=0.508$. The numerical results show that the velocity at $t=40$ is $u=0.507$. Insertion of the parameter values in the expressions (2.16) and (2.13) for u yields $u=0.506$. At $t=40$ the numerical computations yield the location of the fluxon $x=22.0$ while Eq. (2.17) yields $x=21.96$. For other parameter values, e.g., $\alpha=\eta=0.2$ and $\epsilon=-1$, the same good agreement between numerical results and the analysis is found.

We note that if the fluxon starts out with a negative velocity it will stop, return, and enter a stationary movement (from the analysis the time and location

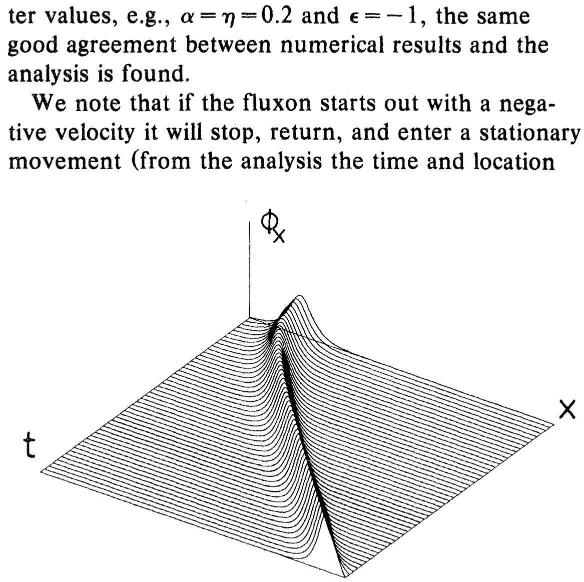

FIG. 4. Propagation of a fluxon in a junction with loss and bias. The only change in parameter values compared to Fig. 3 is a change of ϵ from -1 to 1. The results are displayed in terms of ϕ_x for $0\leq x\leq 30$ and $0\leq t\leq 40$. The velocity of the stationary fluxon is increased in accordance with the analysis.

of the return event can easily be found).

In Fig. 4 the only change in parameter values compared with Fig. 3 is a change in ϵ from -1 to 1. Again an initially static fluxon is accelerated into a stationary motion. The stationary velocity, however, is increased. From the numerical results we find at $t=40$, $u=0.75$ while the analysis yields $u=0.74$ ($u_\infty=0.76$). The location of the fluxon at $t=40$ in Fig. 4 is $x=28.2$, from Eq. (2.17) the analysis gives $x=28.01$. Again a good agreement is observed for larger values of α and η (e.g., $\alpha=\eta=0.2$). For lower values of α and η the numerical and the analytical results differ less than 1%.

IV. CONCLUSION

In the present paper we have examined the influence of the $\cos\phi$ conductance on propagating fluxons and plasma waves. It turns out that the $\cos\phi$ term effects fluxons and plasma waves differently: If the fluxon motion is damped the plasma waves are enhanced and vice versa [Eqs. (2.2a) and (2.11a)]. Furthermore, the presence of loss and bias results in any event in stabilization of the fluxon in a stationary motion. A perturbation method based on energy argument has been applied and the results from this perturbation theory have been compared to results obtained by numerical solution of the corresponding initial value problem. A good agreement between the analytical and the numerical results has been found. Actually the physically relevant α values are of the order $10^{-2}-10^{-4}$ for which the perturbation theory works very well.

Finally we remark that the results obtained for a single fluxon also hold for an antifluxon if the sign of η is changed.

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