Quantitative analysis of weak localization in thin Mg films by magnetoresistance measurements

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The field dependence of the resistance of thin Mg films ($50 < R < 200\ \Omega/\square$) is measured. The field dependence allows a quantitative analysis of the weak localization. The absolute value of the magnetoconductance agrees well with the theory. The temperature dependence of the inelastic scattering time is determined.

Thin metallic films as an example of a quasi-two-dimensional system show fascinating electrical properties at low temperatures. The resistance diverges logarithmically with decreasing temperature[1–4] and is very sensitive to small magnetic fields.[5–7] At the present time there are two mechanisms which predict a logarithmic temperature dependence of the resistance; weak localization[1] and the impurity-induced electron-electron interaction[2] in disordered electronic systems. Weak localization yields the following dependence of the conductance:

$$\Delta L = -L_{00}\ln(T^{-p}) \ , \qquad L_{00} = \frac{1}{2\pi^2}\frac{e^2}{\hbar} \ . \tag{1}$$

The exponent p arises from the temperature dependence of the inelastic scattering time of the conduction electrons. (The inelastic scattering time τ_i is assumed to depend on T as T^{-p}.) Besides the exponent p there is no adjustable parameter in the theory. The electron interaction yields a similar logarithmic divergence of the conductance,

$$\Delta L = (1-F)L_{00}\ln(T) \ , \tag{2}$$

where $(1-F)$ describes the shielding factor.

The total prefactor of $L_{00}\ln(T)$ is $A = [p + (1-F)]$ if one assumes that both phenomena add independently (which is a severe simplification). Therefore, a simple resistance measurement does not allow the determination of p. However, the strong field dependence of the weak localization[5–7] is particularly favorable to independent determination of the contribution of weak localization since its contribution to the conductance depends on the field in the following manner:

$$L(H) - L(0) = -L_{00}\left[\ln\frac{H_i}{H} - \psi\left(\frac{1}{2}+\frac{H_i}{H}\right)\right] \ . \tag{3}$$

Here $H_i(T) = \hbar/(4eD\tau_i)$ is a field which is characteristic for the inelastic scattering time τ_i, $D = 1/RdNe^2$ is the diffusion constant, R is the resistance, d is the thickness of the film, and N is the density of states at the Fermi energy. $H_i(T)$ is inversely proportional to the product of elastic and inelastic mean free path $ll_i(T)$. Obviously, the magnetoconductance is a universal function of $H/H_i(T)$ and depends only implicitly on the temperature [via $H_i(T)$]. Therefore a measurement of the magnetoconductance allows a check of the universal behavior, an absolute determination of the inelastic scattering time τ_i, its temperature dependence (in particular the exponent p); in other words, the contribution of the weak localization to the divergence of the resistance is determined completely. The magnetoconductance in the interaction picture will be negligible in the interesting field range (see below).

Since the method of quenched condensation is a very favorable method to prepare homogeneous and continuous films with high resistance, it is particularly suitable for the present task. According to the results by Hikami *et al.*[6] the spin-orbit coupling changes the resistance behavior of the weakly localized state completely. Therefore it is absolutely essential to keep the spin-orbit coupling as small as possible. For this reason, Mg films are investigated since the spin-orbit coupling varies with the fourth power of the nuclear charge Z.

Mg films with a resistance (per square) between 50 and 200 $\Omega/\square$ are condensed on a substrate of crystalline quartz which is at 5 K and afterwards annealed up to 40 K. The film thickness varies between 70 and 120 Å. The resistance, plotted as a function of the logarithm of the temperature in the range $4.2 < T < 30$ K, follows, for the thinner films, a straight line for $T < 12$ K.

In Fig. 1 the field dependence of the conductance is plotted as a function of H/H_i^*, according to Eq. (3), for five temperatures between 4.5 and 19.4 K. The full curve is calculated with the theoretical expression (3). Here $H_i^*(T)$ is a fit parameter which is adjusted so that the experimental points follow the theoretical curve. With the exception of the highest temperature of 19.4 K the agreement between the experimental points and the theory is good. The values

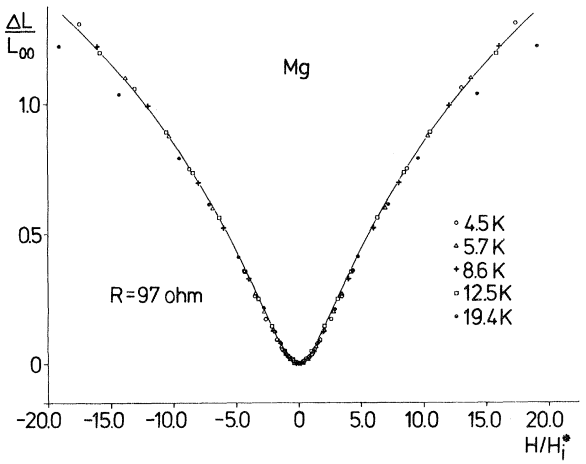

FIG. 1. Normalized conductance $\Delta L/L_{00}$ of the Mg film as a function of the (normalized) magnetic field $H/H_i^*(T)$ for different temperatures compared with the theory (full curve), $L_{00}=\hbar/(2\pi^2e^2)$.

$H_i^*(T)$ are plotted in Fig. 2 as a function of T for three film resistances. The values of $H_i^*(T)$ do not follow a straight line in the log-log plot but approach a slope of about 2 at high temperatures.

At this point it is helpful to discuss the dimensionality of the film. One has to distinguish between the "dimensionality" of different physical situations. (i) With respect to the normal conduction process the film is three dimensional because the thickness d is larger then the mean free path l, which varies between 5 and 10 Å.[8] (ii) The thin film is two

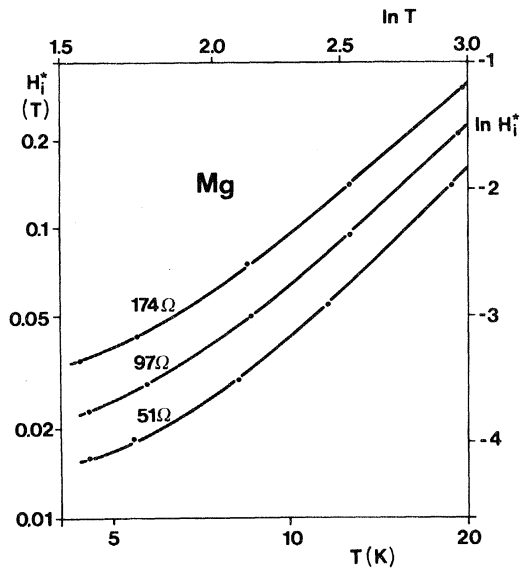

FIG. 2. Adjusted characteristic field $H_i^*(T)$ as a function of temperature in a log-log plot for three Mg films.

dimensional with respect to the weak localization when the diffusion time from the top of the film to the bottom is small compared to the lifetime of inelastic scattering which means that $D\tau_i/d^2 >> 1$ (d = film thickness). The ratio is always larger than 100 for the Mg film. (iii) With respect to the electron interaction the thin film is two dimensional when $(D\hbar/kT)/d^2 >> 1$. At 4 K this ratio is about 20.[8] Since the film is three dimensional with respect to the normal conduction process, one has to use the three-dimensional diffusion constant $D = lv_F/3$ in the equations.

Recent calculations within the electron interaction model[9] yield a magnetoconductance with opposite (negative) sign on a field scale of $H_e = kT/2\mu_B$ which is of the order of 3 T at 4 K. The characteristic field for the weak localization at 4 K is of the order of 0.02 T, and Fig. 1 shows the experimental points in the field range from −0.5 to 0.5 T where no contribution from electron interaction is expected.

Since the experiment yields the characteristic field $H_i^*(T)$ and the effective resistivity $\rho = Rd$, one needs only the density of states to calculate the inelastic relaxation time τ_i^* according to the relation.

$$\tau_i = (\hbar eNRd/4)/H_i \ .$$

We take the density of states from the free-electon model and obtain for τ_i^* at 4.5 K about 1.5×10^{-11} sec. Meservey *et al.*[10] measured for Al—the neighbor of Mg in the periodic system of elements—the spin-orbit coupling time for similar disordered films and obtained $\tau_{so} = 2 \times 10^{-11}$ sec. One has to conclude that even in the Mg film the influence of the spin-orbit coupling is already important and interferes with the measurement of $H_i(T)$ or $\tau_i(T)$, respectively. It is probable that the nonlinear behavior of $\ln[H_i^*(T)]$ as a function of $\ln(T)$ is due to the influence of the spin-orbit coupling. Only at 20 K, where τ_i is reduced by a factor of 10, the spin-orbit coupling has a minor influence [because it yields corrections of the order of $(\tau_i/\tau_{so})^2$ (Ref. 16)]. From the asymptotic behavior at high temperatures one may assume a T^2 dependence of $H_i(T)$ or $(1/\tau_i)$. But since at present there is no reliable method to determine the spin-orbit coupling time independently in these films this is only a suggestion. Van Haesendonck *et al.*[11] measured the magnetoconductance of thin Cu films. They could show that the observed magnetoconductance was due to weak localization because it vanished for the field parallel to the Cu film. But they observed almost no temperature dependence of the magnetoconductance. Cu has a higher nuclear charge than Mg, and its spin-orbit coupling should be larger by a factor of 50 than in Mg. Therefore it is quite probable that the influence of the spin-orbit coupling is responsible for the peculiar temperature independence of the magnetoconductance in Cu.

The experimentally determined $H_i^*(T)$ does not

depend via a simple power law on temperature. This means that the resistance should not vary as $\ln T$ in the experimental temperature range since the spin-orbit coupling modifies the temperature dependence. If the inelastic scattering time increases proportional to T^{-2} with decreasing temperature, then the contribution of weak localization to the resistance should even reverse its sign at low temperatures.[6] In the intermediate temperature range the resistance should pass through a maximum. Experimentally, however, the resistance follows a $\ln(T)$ law nicely as long as the phonon contribution is negligible. This problem requires a detailed study of the spin-orbit coupling. Its influence should be much larger in systems generally studied such as Cu, Pd, etc.

The theory of weak localization considers, as usual, only homogeneous films. Experimentally one is, however, in a regime where the resistance is neither proportional to l/d nor obeys a Fuchs-Sondheimer formula. In several experiments the films are even granular or show tunneling conductivity between islands. Although the quenched condensation yields high-resistance films with a rather homogeneous structure, the effective resistivity Rd still changes by a factor of 2 in the range $50\ \Omega < R < 200\ \Omega$. If one ignores this experimental complication for a moment one may study the dependence of H_l on the resistivity according to the relation $H_l = [(\hbar Ne/4)/\tau_l]\rho$. A plot of $H_l(20\ \mathrm{K})$ (at 20 K the spin-orbit coupling is negligible) versus ρ for the three films yields a straight line through the origin. This means that τ_l is independent of the disorder in these films and Rd gives the adequate value for the resistivity. If further studies confirm that τ_l is structure independent then the weak localization provides a new physical quantity which can be used for example to study the size effect.[12]

Finally one should emphasize that the prediction of the weak-coupling theory yields a quantitative agreement with the experimental magnetoconductance and allows a determination of the inelastic relaxation time. Measurements to investigate the influence of the spin-orbit coupling on weak localization are in progress.

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[12]In particular it allows a separate study of the mean free path of the conduction electrons traveling parallel and perpendicular to the surface of the film.