Magnetoresistance and Hall effect of a disordered interacting two-dimensional electron gas

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We calculate the corrections to the resistance R and Hall resistance R_H of a two-dimensional disordered electronic system due to interactions in the strong-field limit $\omega_c < \epsilon_F$, $\epsilon_F\tau > 1$ where localization effects are suppressed. We find that $\Delta\sigma_{xy}=0$ for both $\omega_c\tau \gtrless 1$. With the result that $(\delta R_H/R_H)/(\delta_R/R)=2\,/[1-(\omega_c\tau)^2]$ oscillating with field because of the field dependence of τ and eventually diverging when $(\omega_c\tau)=1$. $\delta R/R$ decreases with increasing field going through zero when $\omega_c\tau=1$.

I. INTRODUCTION

Two aspects of electronic transport in disordered systems have been the subject of much discussion recently. First, the effect of localization[1,2]; we use the term to describe the properties of a single electron in a random potential. Second, the effect of electron-electron interaction,[3–6] which has only been worked out in the limit of weak disorder. The theories based on localization or interaction predict similar behavior for the conductivity.

In the localization theory, perturbation theory based on the summation of maximally crossed diagrams[1,7–9] and field-theoretic methods[2,10–15] predict a logarithmic correction to the conductivity in two dimensions

$$\delta\sigma_L=\frac{e^2}{\hbar}\frac{1}{2\pi^2}\ln(\tau/\tau_{\rm in})\ . \tag{1}$$

$\tau_{\rm in}$ is the inelastic scattering time and τ is the elastic scattering time.

In the interaction theory, the dynamically screened Coulomb interaction was treated to first order, and the correction to the conductivity $\delta\sigma(\omega)$, where ω is the frequency or temperature, is given by[4,5]

$$\frac{\delta\sigma}{\sigma}=(2-2F)\frac{1}{2\pi smD}\ln\omega\tau\ , \tag{2}$$

where F is the angular average of the statically screened Coulomb interaction $V_s(\vec{q})$ and D is the diffusion constant.

A systematic calculation, however, should investigate the effect of localization on the interaction theory. It can, in fact, be shown that the two effects are additive (see Appendix A for details). The effect of localization on electronic screening is simply to renormalize the diffusion constant

$$\tilde{D}=D\left[1+\frac{1}{2\pi mD}\ln\omega\tau\right]\ , \tag{3}$$

where $\omega=\max(\omega,kT,\tau_{\rm in}^{-1})$ which with Eq. (2) clearly shows that the effects of localization on the interaction theory are higher order in $1/\epsilon_F\tau$. ϵ_F is the Fermi energy.

In view of the similarity of the corrections to the conductivity from each source it is important to develop techniques which distinguish between the two phenomena. It has been suggested[5] that this could be accomplished by carrying out experiments in weak magnetic fields. First, as the maximally crossed diagrams responsible for Eq. (1) are sensitive to the time-reversal symmetry breaking due to the magnetic field,[5] localization phenomena are suppressed by very weak magnetic fields, $\omega_c\tau > (1/\epsilon_F\tau)(\tau/\tau_{\rm in})$, and a negative magnetoresistance should be observed. Further,[5] the change in the Hall constant $\delta R_H/R_H$, predicted by the interaction theory in extremely weak fields when the magnetic field may be considered a perturbation, should be $\delta R_H/R_H=2$, whereas the localization theory would predict no change. Measurement of this effect should provide a direct check of the theory. However, as noted, in extremely weak magnetic fields both effects contribute to the electronic conductivity of the two-dimensional disordered system. It is also known that in the weak-field limit the predictions of the interaction theory are complicated by spin splitting[16] and Hartree[5,16] contributions to the conductivity. Because of these difficulties we focus in this work on the effects of strong magnetic fields ($\omega_c\tau \gtrless 1$ but $\omega_c/\epsilon_F < 1$) on the conductivity and Hall conductivity of a disordered two-dimensional electronic system. We may therefore concentrate on interaction effects only, as localization is suppressed under these conditions.

Following Ref. 5 the interaction is taken to be the dynamically screened Coulomb interaction

$$V_S(\vec{q},\omega)=V_B(\vec{q})/[1+V_B(\vec{q})\Pi(\vec{q},\omega)]\ , \tag{4}$$

where $V_B(\vec{q})=2\pi e^2/q$ is the bare interaction in two dimensions. As we show in Appendix A the polarization in the zero magnetic field limit reduces to

$$\Pi(\vec{q},\omega)=sN_1\tilde{D}q^2/(-i\omega+\tilde{D}q^2)\ , \tag{5}$$

and hence in the small q,ω limit and subject to the condition $D\kappa q >> |\omega|$ where $\kappa=e^2sN_1$ is the screening constant,

$$V_s^{H=0}(\vec{q},\omega)=\frac{1}{sN_1}\frac{-i\omega+\tilde{D}q^2}{\tilde{D}q^2}\ , \tag{6}$$

where s is the spin degeneracy, $N_1=m/2\pi$ is the single-particle density of states for a single spin, and $\tilde{D}$ is the renormalized diffusion constant [Eq. (3)]. The form of the screened interaction given in Eq. (6), which is important in subsequent discussion, arises because electron-electron scattering in the small q,ω limit is dominated by the diffusion pole in the particle-hole propagator. Thus our first task is to find the residue of this pole in the strong-field limit. The diffusion propagator will be derived in Sec. II, the magnetoconductivity tensor is given in Sec. III, and a summary of this work and its bearing on experiment is given in Sec. IV.

II. THE POLARIZABILITY AND PARTICLE-HOLE DIFFUSION PROPAGATOR

The polarizability of the noninteracting electronic system in the presence of disorder is given by

$$\Pi(\vec{x},\vec{x}\,';\omega)=\frac{1}{2\pi i}\int_{-\infty}^{\infty}d\epsilon[f(\epsilon+\omega)-f(\epsilon)]\Pi(\vec{x},\vec{x}\,';\epsilon+\omega,\epsilon)+N_1(\epsilon_F)\ , \tag{7}$$

where $\Pi(\vec{x},\vec{x}\,';\ \epsilon+\omega,\epsilon)$ is the propagator of a particle with energy $\epsilon+\omega$ and a hole with energy ϵ in the presence of disorder:

$$\Pi(\vec{x},\vec{x}\,';\epsilon+\omega,\epsilon)=\langle G^+(\vec{x},\vec{x}\,';\epsilon+\omega)G^-(\vec{x}\,',\vec{x};\epsilon)\rangle\ . \tag{8}$$

Here, G^+ and G^- are the advanced and retarded single-particle Green's functions, respectively, and $\langle\ \rangle$ implies impurity averaging. In the weak-scattering limit the particle-hole propagator Π satisfies the Dyson equation

$$\Pi(\vec{x},\vec{x}\,';\omega)=\Pi^0(\vec{x},\vec{x}\,';\omega)+u^2\sum_{\vec{x}_1}\Pi^0(\vec{x},\vec{x}_1;\omega)\Pi(\vec{x}_1,\vec{x}\,';\omega)\ . \tag{9}$$

Here, u^2 is the mean-square impurity potential.

The Feynman diagrams corresponding to this equation are shown in Fig. 1. The particle-hole diffusion propagator, on the other hand, is defined by the Dyson equation

$$D(\vec{x},\vec{x}';\omega)=u^2\delta_{\vec{x},\vec{x}\,'}+u^2\sum_{x_1}\Pi^0(\vec{x},\vec{x}_1;\omega)D(\vec{x}_1\vec{x}\,';\omega)\ , \tag{10}$$

which is illustrated by the Feynaman diagrams shown in Fig. 2. The basic building block of both quantities is the "bare bubble," $\Pi^0(\vec{x},\vec{x}';\omega)$, the labeling of which is shown in Fig. 3.

To evaluate Π^0 we work in the Landau gauge choosing a vector potential $A(x)$ such that

$$A(x)=(0,Hx,0)\ , \tag{11}$$

where H is a uniform magnetic field in the z direction, chosen to be perpendicular to the two-dimensional electronic system; the states are labeled by Landau level index n and a wave vector k in the

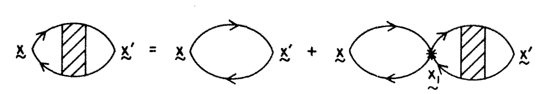

FIG. 1. Diagrams corresponding to the Dyson equation for the polarizability. The star represents impurity scattering.

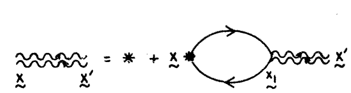

FIG. 2. Diagrams corresponding to the Dyson equation for the particle-hole diffusion propagator.

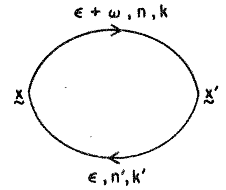

FIG. 3. Diagram for the bare polarizability. n and k are the Landau level labels.

y direction:

$$\Pi^0(\vec{x},\vec{x}\,';\omega)=\sum_{kk'} G^+_{\epsilon+\omega}(x,x';k')G^-_\epsilon(x',x;k)\times e^{(k'-k)(y-y')} \,. \tag{12}$$

The Green's function $G^\pm_\epsilon(x_1,x_2;k)$ is given by

$$G^\pm_\epsilon(x_1,x_2;k)=\sum_n \frac{\phi_n(x_1+k\alpha)\phi_n^*(x_2+k\alpha)}{\epsilon-\epsilon_n\pm i/2\tau} \,. \tag{13}$$

Here, the ϕ_n are harmonic oscillator functions, $\epsilon_n=(n+\frac{1}{2})\hbar\omega_c$, the cyclotron frequency $\omega_c = eH/mc$, $\alpha=1/m\omega_c$, and for simplicity the area of the sample has been set equal to 1. The effect of impurity scattering is included via a relaxation time which is energy dependent,

$$1/\tau(\epsilon)=2\pi N_1(\epsilon)u^2 \,, \tag{14}$$

and inversely proportional to the density of states which oscillates as a function of magnetic field. As all quantities are evaluated at the Fermi energy the field dependence of the theory is retained if this relationship is kept in mind; for example, Ando's[17] self-consistent calculation of the conductivity is reproduced. Accordingly, from now on we set $\tau=\tau(\epsilon_F)$. The detailed field dependence of τ may be obtained from Ando's work[17]; for example, when the magnetic field is rather weak, $\omega\tau_f \lesssim 1$, τ_f is the relaxation time obtained assuming the same scatterers but no magnetic field,

$$\frac{1}{\tau}=\frac{t}{\tau_f}\left[1+2\cos\left[\frac{2\pi\epsilon_f}{h\omega_c}\right]\exp\left[-\frac{\pi}{\omega_c\tau_f}\right]+\cdots\right] . \tag{15}$$

On the other hand, for strong magnetic fields, $\omega_c\tau_f >> 1$,

$$\frac{1}{\tau}=\frac{1}{\tau_f}\left[\frac{2\omega_c\tau_f}{\pi}\right]^{1/2}\left[1+\frac{\pi}{12}\left[\frac{1}{\omega_c\tau_f}\right]+\cdots\right] . \tag{16}$$

As $\Pi^0(x,x';\omega)$ is translationally invariant it is convenient to work with its Fourier transform. After a lengthy calculation, the details of which are given in Appendix B, we find

$$\Pi^0(\vec{q},\omega)=p\sum_{nn'} |F_{nn'}(\vec{q})|^2 D_{nn'} \,, \tag{17}$$

where

$$F_{nn'}(\vec{q})=\int_{-\infty}^{\infty} dx\,\phi^*_{n'}(x)e^{iqx}\phi_n(x) \,, \tag{18}$$

$$D_{nn'}=\frac{1}{(\epsilon+\omega-\epsilon_n+i/2\tau)(\omega-\epsilon_{n'}-i/2\tau)} \,, \tag{19}$$

and

$$p=\frac{1}{2\pi\alpha}=\frac{1}{2\pi}\left|\frac{eH}{\hbar c}\right| \tag{20}$$

is the degeneracy or the number of states per Landau level and $q=(q_x^2+q_y^2)^{1/2}$.

It is sufficient to evaluate the matrix elements $F_{nn'}(q)$ at small momentum transfer

$$F_{nn'}(\vec{q})\cong\int dx\,\phi^*_{n'}(x)\left[1+iqx-\frac{qx^2}{2}+\cdots\right]\phi_n(x) \,. \tag{21}$$

Representing x in terms of harmonic-oscillator annihilation and creation operators

$$x=\left[\frac{\alpha}{2}\right]^{1/2}(a+a^\dagger) \,, \tag{22}$$

the matrix elements between harmonic oscillator functions are

$$\langle n\,|\,x'\,|\,n'\rangle=\sqrt{\alpha}\left[\left[\frac{n'}{2}\right]^{1/2}\delta_{n,n'-1}+\left[\frac{n'+1}{2}\right]^{1/2}\delta_{n,n'+1}\right], \tag{23}$$

$$\langle n\,|\,x^2\,|\,n\rangle=\alpha(n+\tfrac{1}{2}) \,, \tag{24}$$

and we quote for future reference

$$\left\langle n\left|\frac{\partial}{\partial x}\right|n'\right\rangle=\frac{1}{\sqrt{\alpha}}\left[\left[\frac{n'}{2}\right]^{1/2}\delta_{n,n'-1}-\left[\frac{n'+1}{2}\right]^{1/2}\delta_{n,n'+1}\right]. \tag{25}$$

Given this information we find

$$\Pi^0(\vec{q},\omega)=p\sum_{nn'}D_{nn'}\left[1-q^2\alpha(n+\tfrac{1}{2})\delta_{nn'}+q^2\alpha^2\left[\frac{n+1}{2}\delta_{n',n+1}+\frac{n}{2}\delta_{n',n-1}\right]\right]$$

$$=p[Y_A-2q^2\alpha Y_B+q^2\alpha(Y_C+Y_D)]\,, \tag{26}$$

where

$$Y_A=\sum_n D_{nn}\,, \tag{27}$$

$$Y_B=\sum_n(n+\tfrac{1}{2})D_{nn}/2\,, \tag{28}$$

$$Y_C=\sum_n\left[\frac{n+1}{2}\right]D_{n,n+1}\,, \tag{29}$$

$$Y_D=\sum_n\left[\frac{n-1}{2}\right]D_{n,n-1}\,, \tag{30}$$

and the $D_{n,m}$ are defined by Eq. (17).

Each of the sums over the Landau levels [Eqs. (27)–(30)] has the form

$$Y=\sum_{n=0}^{\infty}\frac{f(n)}{(\epsilon_n-\epsilon_1+i/2\tau)(\epsilon_n-\epsilon_2-i/2\tau)}\,. \tag{31}$$

To evaluate the sum consider the contour integral

$$I=\frac{1}{2\pi i}\sum_n\int_c N(z)\frac{1}{(z-\epsilon_1+i/2\tau)(z-\epsilon_2-i/2\tau)}\,, \tag{32}$$

where

$$N(z)=\frac{1}{z-\epsilon_n-i\delta}\,. \tag{33}$$

δ is an infinitesimal. The contour integral may be evaluated by closing the contour in either the upper or the lower half-plane. If we close in the lower half-plane we find

$$I=-\sum_{n=0}^{\infty}\frac{f(\epsilon_1-i/2\tau)}{(\epsilon_1-\epsilon_2-i/\tau)(\epsilon_1-\epsilon_n-i/2\tau)}\,. \tag{34}$$

On the other hand, closing the contour in the upper half-plane

$$I=-\sum_{n=0}^{\infty}\frac{f(\epsilon_2+i/2\tau)}{(\epsilon_2-\epsilon_1+i/\tau)(\epsilon_2-\epsilon_n+i/2\tau)}+Y\,. \tag{35}$$

Equating the two expressions we find

$$y=\frac{1}{\epsilon_1-\epsilon_2-i/\tau}\left[f(\epsilon_2+i/2\tau)\sum_{n=0}^{\infty}\frac{1}{\epsilon_2-\epsilon_n+i/2\tau}-f(\epsilon_1-i/2\tau)\sum_{n=0}^{\infty}\frac{1}{\epsilon_1-\epsilon_n-i/2\tau}\right]. \tag{36}$$

As an example we examine the term Y_A. In this case $\epsilon_1=\epsilon_F$, $\epsilon_2=\epsilon_F+\omega$, $f=1$:

$$Y_A=\frac{1}{\omega+i/\tau}\sum_{n=0}^{\infty}\left[\frac{1}{\epsilon_n-(\epsilon_F+\omega)-i/2\tau}-\frac{1}{\epsilon_n-\epsilon_F+i/2\tau}\right]. \tag{37}$$

Breaking the sum into real and imaginary parts, we recognize that the imaginary part is proportional to the density of states at the Fermi surface. In fact,

$$Y_A=\frac{1}{\omega+i/\tau}\left[\frac{\pi i}{p}2N(\epsilon_F)+\sum_{n=0}^{\infty}\left[\frac{\epsilon_n-(\epsilon_F+\omega)}{\epsilon_n-(\epsilon_F+\omega)^2+(1/2\tau)^2}-\frac{\epsilon_n-\epsilon_F}{(\epsilon_n-\epsilon_F)^2+(1/2\tau)^2}\right]\right]. \tag{38}$$

The second term is identically zero when $\omega\to 0$, and hence

$$Y_A=\frac{1}{\omega+i/\tau}\frac{2\pi i N(\epsilon_F)}{p}+O(\omega)\,. \tag{39}$$

Similarly,

$$Y_B=\frac{1}{\omega+i/\tau}\frac{\epsilon_F}{2\omega_c}\frac{2\pi i N(\epsilon_F)}{p}+O(\omega)\,, \tag{40}$$

$$Y_C=\frac{1}{\omega_c+i/\tau}\frac{\epsilon_F}{2\omega_c}\frac{2\pi i N(\epsilon_F)}{p}+O\left[\frac{\omega_c}{\epsilon_F}\right], \tag{41}$$

$$Y_D=\frac{-1}{\omega_c-i/\tau}\frac{\epsilon_F}{2\omega_c}\frac{2\pi i N(\epsilon_F)}{p}+O\left[\frac{\omega_c}{\epsilon_F}\right]. \tag{42}$$

In summary, from Eqs. (26) and (39)—(42) we find that

$$\Pi^0(\vec{q},\omega)\simeq 2\pi N(\epsilon_F)\tau[(1+i\omega\tau)-q^2D_H\tau] \tag{43}$$

in the small q,ω limit. The diffusion constant

$$D_H=\frac{\epsilon_F\tau/m}{1+(\omega_c\tau)^2}\,, \tag{44}$$

equal to $v_F^2\tau/2$ when $H=0$, is in general an oscillatory function of the magnetic field H via τ [Eqs. (14)—(16)] and ϵ_F. Knowing $\Pi^0(\vec{q},\omega)$ we may now construct the particle-hole diffusion propagator $D(\vec{q})$ [Eq. (10)]:

$$D(\vec{q},\omega)=\frac{u^2\tau^{-1}}{D_Hq^2-i\omega}\,. \tag{45}$$

The screened interaction in the small-q limit [Eq. (6)] is

$$V_s(\vec{q},\omega)=\frac{1}{2N_1}\frac{-i\omega+D_Hq^2}{D_Hq^2}\,, \tag{46}$$

and the impurity renormalization of the screened interaction vertices,

$$\Gamma(\vec{q},\omega,\epsilon)=\begin{cases}(-i\omega+D_Hq^2)^{-1}\tau^{-1} & \text{if } \epsilon(\epsilon-\omega)<0\\ 1 & \text{otherwise}\,.\end{cases} \tag{47}$$

With these functions we proceed to the calculation of the dynamical conductivity tensor in the next section.

III. THE MAGNETOCONDUCTIVITY TENSOR

In this section we will determine the contributions to the magnetoconductivity tensor from

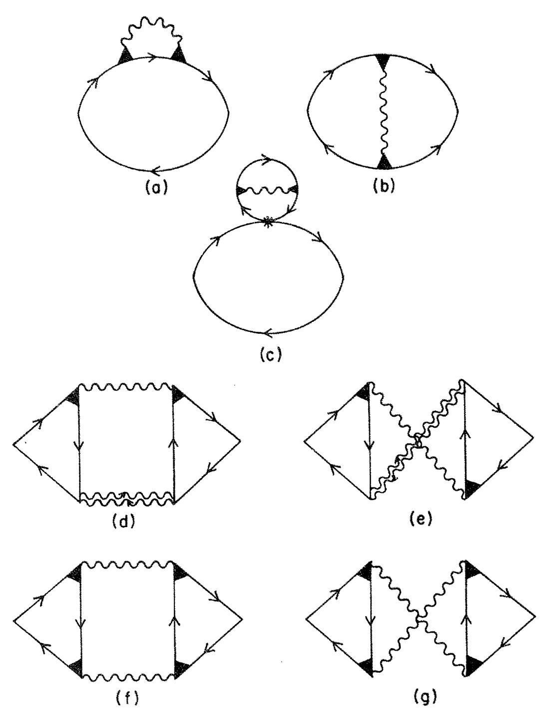

FIG. 4. Diagrams contributing to the conductivity tensor to first order in the Coulomb interaction (wavy line). The double wavy line represents the particle-hole diffusion propagator of Fig. 2.

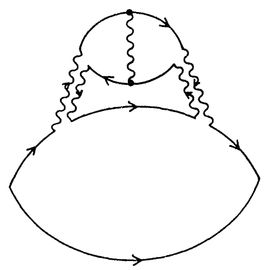

FIG. 5. Conductivity diagram generated from Hartree correction to self-energy.

electron-electron interaction. With the present technique, in addition to extending previous calculations[5] to higher magnetic fields, the longitudinal σ_{xx} and transverse σ_{xy} conductivities may be obtained in parallel. The effect of electron-electron interaction is considered at lowest order, and impurity scattering is treated by the conventional diagrammatic technique in the $k_F l >> 1$ limit.

The diagrams that contribute to the magnetoconductivity tensor are shown in Fig. 4. These diagrams are generated in a conserving approximation from the exchange contribution to the electron self-energy. There are also contributions to the conductivity from diagrams generated from the Hartree contribution to the electronic self-energy.[5,16] A typical diagram is shown in Fig. 5. However, as these terms play a role in our calculation complementray to the weak-field case[5] we will not discuss them further. We will show in detail in Appendix C that just as in the weak-field limit the contributions to the conductivity of diagrams 4(a)–4(c) to both σ_{xx} and σ_{xy} exactly cancel to

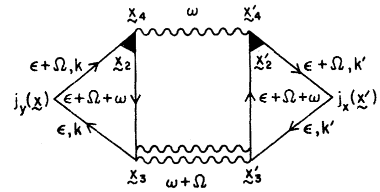

FIG. 6. Labeling of the diagram of Fig. 4(d).

$O(\omega_c/\epsilon_F)$. Further, the "Azlamazov-Larkin" diagrams 4(f) and 4(g) exactly cancel. It remains to determine the contributions from diagrams 4(d) and 4(e).

The components of the conductivity tensor are given in terms of the current-current correlation function. For example,

$$\sigma_{yx} = \lim_{\Omega\to 0} \frac{1}{i\Omega} \int \langle [j_y(\vec{x}), j_x(\vec{x}\,')] \rangle(\Omega) d^2x\, d^2x' , \quad (48)$$

where the current operators

$$j_y(\vec{x}) = -\frac{e}{m}\left[\frac{1}{i}\frac{\partial}{\partial y} + \frac{ex}{\alpha}\right] , \quad (49)$$

$$j_x(x) = -\frac{e}{m}\frac{1}{i}\frac{\partial}{\partial x} , \quad (50)$$

and Ω is the external frequency.

We will now evaluate the contribution of the diagram in Fig. 4(d) to the current-current correlation function. The detailed labeling of this diagram is given in Fig. 6. We note that in order to obtain a divergence in the vertex corrections Γ we must have $\epsilon, \epsilon+\Omega$ postitive and $\epsilon+\Omega+\omega$ negative, or vice versa. We consider in detail the first possibility. We have

$$d_{-}^{++} = M_y^L(\epsilon,\Omega,\omega;q)\Gamma_\omega(\vec{q})V_\omega(\vec{q})\Gamma_\omega(\vec{q})D_{\omega+\Omega}(q)M_x^R(\epsilon,\Omega,\omega;q) , \quad (51)$$

where

$$\begin{aligned} M_y^L(\epsilon,\Omega,\omega;\vec{q}) = -\frac{e}{m} \sum_{\substack{k\\ n,n',n''}} G(n,n',n'') &\int dx_1 \phi_{n'}^*(x_1+k\alpha^2)\left[k+\frac{x_1}{\alpha}\right]\phi_n(x_1+k\alpha^2) \\ &\times \int dx_2 \phi_{n''}^*(x_2+(k+q_y)\alpha) e^{iq_x x_2} \phi_{n'}(x_2+k\alpha) \\ &\times \int dx_3 \phi_n^*(x_3+k\alpha) e^{-iq_x' x_3} \phi_{n''}(x_3+(k+q_y)\alpha) , \end{aligned} \quad (52)$$

$$\begin{aligned} M_x^R(\epsilon,\Omega,\omega;q) = \frac{ie}{m} \sum_{\substack{k\\ n,n',n''}} G(n,n',n'') &\int dx_1 \phi_n^*(x_1+k\alpha)\frac{\partial}{\partial x_1}\phi_{n'}(x_1+k\alpha) \\ &\times \int dx_2 \phi_{n'}^*(x_2+k\alpha) e^{-iq_x x_2} \phi_{n''}(x_2+(k+q_y)\alpha) \\ &\times \int dx_3 \phi_{n''}^*(x_3+(k+q_y)\alpha) e^{iq_x' x_3} \phi_n(x_3+k\alpha) , \end{aligned} \quad (53)$$

and

$$G(n,n',n'')=\frac{1}{(\epsilon-\epsilon_n+i/2\tau)(\epsilon+\Omega-\epsilon_{n'}+i/2\tau)(\epsilon+\Omega+\omega-\epsilon_{n''}-i/2\tau)} . \tag{54}$$

Expanding the current vertices M for small q we find

$$M_y^L(\vec{q})=\frac{e}{m}p\sum_n\frac{n+1}{2}\{(q_y-iq_x)[G(n,n+1,n)-G(n,n+1,n+1)] + (q_y+iq_x)[G(n+1,n,n)-G(n+1,n,n+1)]\} \tag{55}$$

and

$$M_x^R(\vec{q})=\frac{ie}{m}p\sum_n\frac{n+1}{2}\{(q_y-iq_x)[G(n+1,n,n)-G(n+1,n,n+1)] + (q_y+iq_x)[G(n,n+1,n+1)-G(n,n+1,n)]\} . \tag{56}$$

The sum over Landau levels may be evaluated by contour integral as before. The results in the low-frequency limit are

$$\sum_n\frac{n+1}{2}G(n+1,n,n+1)=\sum_n\frac{n+1}{2}G(n,n+1,n+1)=\frac{-\epsilon_F}{2\omega_c^2}\frac{2\pi iN(\epsilon_F)}{p}\left[i\tau+\frac{1}{\omega_c+i/\tau}\right] \tag{57}$$

and

$$\sum_n\frac{n+1}{2}G(n+1,n,n)=\sum_n\frac{n+1}{2}G(n,n+1,n)=\frac{\epsilon_F}{2\omega_c^2}\frac{2\pi iN(\epsilon_F)}{p}\left[i\tau-\frac{1}{\omega_c-i/\tau}\right] . \tag{58}$$

Hence,

$$M_y^L(\vec{q})=-2q_y\frac{e}{m}\epsilon_F\tau^3 2\pi N(\epsilon_F)/[1+(\omega_c\tau)^2] \tag{59}$$

and

$$M_x(\vec{q})=-2q_x\frac{e}{m}\epsilon_F\tau^3 2\pi N(\epsilon_F)/[1+(\omega_c\tau)^2] . \tag{60}$$

The first thing to note is that these expressions reduce to the correct zero-field limit $[2\pi N(\epsilon_F)](H=0)=m$, in the strong-field case, in addition to the factor $1/[1+(\omega_c\tau)^2]$, the density of states, $1/\tau$ and the Fermi energy oscillate as a function of H. Perhaps the most significant result is that because of the q dependence of the M's the contribution of these diagrams, and hence of interactions, to the transverse conductivity is identically zero.

The calculation of the contribution to the longitudinal current now proceeds as in Ref. 5. We find

$$\delta\sigma_{xx}=\frac{2i(2e^2)}{i\Omega}\int_\Omega^\infty\frac{d\epsilon}{2\pi}\int_{\epsilon-\Omega}^{\epsilon}\frac{d\omega}{2\pi}\frac{M_xM_xu^2}{-i(\omega+\Omega)+D_Hq^2}\frac{iV_s(q,\omega)}{(-i\omega+D_Hq^2)^2\tau^3} , \tag{61}$$

where the first diffusion pole is due to the impurity ladder and the $(-i\omega+D_Hq^2)^{-2}$ comes from the vertex correction V_s. Since the integrand is independent of ϵ it can be arranged as follows:

$$\delta\sigma_{xx}=\frac{i}{\pi}2e^2\frac{(2\epsilon_F\tau^3)^2}{m}\frac{[2\pi N(\epsilon_F)]^2}{[1+(\omega_c\tau)^2]^2} \times u^2\int_\Omega^{1/\tau}\frac{d\omega}{2\pi}\int\frac{d^2q}{(2\pi)^2}\frac{q_x^2}{q^2}\frac{1}{[-i(\omega+\Omega)+D_Hq^2](-i\omega+D_Hq^2)2N_1D_Hq^2\tau^3} . \tag{62}$$

Changing variables $D_Hq^2\tau\to q^2$, $\omega\tau\to\omega$, $\tilde{\Omega}=\Omega\tau$ and evaluating the angular integral, we have

$$\delta\sigma_{xx}=\frac{ie^2}{2\pi}\frac{(2\epsilon_F\tau^3)^2[2\pi N(\epsilon_F)]^2u^2}{[1+(\omega_c\tau)^2]^2}\frac{1}{2N_1D_H^2\tau^3}\int_{\tilde{\Omega}}^{1}\frac{d\omega}{2\pi}\int_0^{\infty}\frac{dq^2}{2\pi}\frac{1}{[-i(\omega+\tilde{\Omega})+q^2](-i\omega+q^2)} . \tag{63}$$

Recalling that $D_H=(\epsilon_F\tau/m)/[1+(\omega_c\tau)^2]$ and $1/\tau=2\pi N_1u^2$, $\delta\sigma_{xx}$ is found to have the same structure as the weak-field result:

$$\delta\sigma_{xx}=\frac{e^2}{2\pi^2\hbar}\ln(\Omega\tau) . \tag{64}$$

When Hartree[5] effects are included,

$$\delta\sigma_{xx}=\frac{e^2}{2\pi^2\hbar}(1-F)\ln(\Omega\tau) \tag{65}$$

with spin splitting F becomes a function of magnetic field.

IV. MAGNETORESISTANCE AND HALL CONSTANT

In this section we explore what bearing the results derived in the previous sections might have for experimental measurements. The central quantity is the resistivity tensor

$$\underline{\rho}=\begin{bmatrix}\rho_{xx} & \rho_{xy}\\ \rho_{yx} & \rho_{yy}\end{bmatrix} . \tag{66}$$

The components of $\underline{\rho}$ are related to the components of the conductivity tensor by

$$\rho_{xx}=\frac{\sigma_{xx}}{\sigma_{xx}^2+\sigma_{xy}^2} , \tag{67}$$

$$\rho_{xy}=-\rho_{yx}=\frac{\sigma_{xy}}{\sigma_{xx}^2+\sigma_{xy}^2} . \tag{68}$$

The quantities directly measured by experiment are the magnetoresistance $R=\rho_{xx}$ and the Hall constant $R_H=\rho_{xy}/H$. In zeroth order, when only the simple bubble diagram with impurity-renormalized single-particle Green's function is used to calculate σ_{xx} and σ_{xy}, they have the following values:

$$R^0=\frac{m}{ne^2\tau} , \tag{69}$$

$$R_H^0=\frac{1}{nec} , \tag{70}$$

where the carrier density $n=N(\epsilon_F)\epsilon_F$. It is important to note that at this level R^0 is field independent, whereas R_H^0 oscillates inversely with the density of states. It should be pointed out that if the field dependence of τ and $N(\epsilon_F)$ are retained as we have indicated, the self-consistent results for the conductivity obtained by Ando[17] are reproduced.

Now we consider the additional contributions to σ_{xx} and σ_{xy} discussed in the previous sections. If we define

$$\Delta_1=\delta\sigma_{xx}/\sigma_{xx}^0 \tag{71}$$

and

$$\Delta_2=\delta\sigma_{xy}/\sigma_{xy}^0 , \tag{72}$$

it follows from Eqs. (67)–(72) that

$$\frac{\delta R}{R_0}=\frac{\Delta_1[(\omega_c\tau)^2-1]-2\Delta_2(\omega_c\tau)^2}{1+(\omega_c\tau)^2} \tag{73}$$

and

$$\frac{\delta R_H}{R_H^0}=-\frac{2\Delta_1+\Delta_2[(\omega_c\tau)^2-1]}{1+(\omega_c\tau)^2} . \tag{74}$$

We note that even when $\delta\sigma_{xx}$ and $\delta\sigma_{xy}$ are not explicitly dependent on the magnetic field, in general there is a field dependence of the quantities δR and δR_H via zeroth order quantities σ_{xx}^0 and σ_{xy}^0.

In the strong-field limit, localization effects are completely suppressed and we need only to keep the leading logarithmic contributions to the conductivity stemming from interaction, derived in Sec. III. The most important point is to note that Coulomb interaction effects alone lead to $\delta\sigma_{xy}=0$ and as a result,

$$\frac{\delta R}{R_0}=\frac{-m}{2\pi^2\hbar n\tau}(1-F)[1-(\omega_c\tau)^2]\ln\tilde{\Omega} , \tag{75}$$

$$\frac{\delta R_H}{R_H^0}=\frac{-2m}{2\pi^2\hbar n\tau}(1-F)\ln\tilde{\Omega} . \tag{76}$$

The implications of Eqs. (75) and (76) are quite unambiguous. First, there is a logarithmic correction to the resistivity which decreases steadily as a function of magnetic field through the factor $1-(\omega_c\tau)^2$, changing sign at $\omega_c\tau=1$. Second, in contrast to the localization effects, there is a finite correction to the Hall constant $\delta R_H/R_H^0$. The ratio of these two quantities

$$\frac{\delta R_H/R_H^0}{\delta R/R^0}=\frac{2}{1-(\omega_c\tau)^2} , \tag{77}$$

which is close to 2 for $\omega_c\tau<<1$. However, it diverges as $\omega_c\tau$ approaches 1 and then changes sign. It also oscillates as a function of field because of the intrinsic field dependence of τ. Although present experimental data[18,19] are still limited to the range $\omega_c\tau << 1$, these results demonstrate that a detailed measurement of the magnetic field dependence of $\delta R/R_0$ and $(\delta_{R_H}/R_H^0)/(\delta r/R_0)$ in the strong-field limit would be an important test of the Coulomb interaction plus weak scattering theory.

At very weak magnetic fields where localization effects are still present and both effects must be included, the situation is considerably more complicated. From Eqs. (73) and (74) it can be seen that the ratio

$$\frac{\delta R_H/R_H^0}{\delta R/R}\frac{2-\gamma+\gamma(\omega_c\tau)^2}{1+(2\gamma-1)(\omega_c\tau)^2}\ , \tag{78}$$

where

$$\gamma=\Delta_2/\Delta_1\ . \tag{79}$$

The zero-field limit of this ratio is

$$2-\gamma=(2-2F)/(2-F)\ , \tag{80}$$

which varies from the value 1 when $F=0$ (in the limit $k_F/\kappa\to\infty$, κ is the inverse Thomas-Fermi screening length) to the value zero for $F=1$ ($k_F/\kappa\to 0$). For small but infinite fields, the localization, and spin-splitting contribution to the Hartree term do not have a pure $\ln\tilde{\Omega}$ or $\ln T$ form. A formula such as Eq. (80) is less useful in this case than a direct fit of δR or δR_H with the known dependence on both temperature and magnetic field. We should note that recent experiments[18,19] appear to give $(\delta R_H/R_H)/(\delta R/R)$ close to 2 in the weak-field limit. However, it would appear to us that these experiments, given the disorder of the samples, are in the region where both localization and interaction contribute.

In summary, we have considered both localization and Coulomb interaction effects on the magnetoresistance and Hall constant of a two-dimensional electron gas. At weak magnetic fields both effects are present and a complicated temperature and magnetic field dependence results. However, for strong fields, because of the result that there are no logarithmic corrections to the Hall conductivity σ_{xy}, the present theory provides strong predictions for both the magnetoresistance and Hall constant. It also poses the intriguing question as to how to construct a scaling theory of the resistance and Hall resistance in view of the completely different behavior of $\delta\sigma_{xx}$ and $\delta\sigma_{xy}$.

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APPENDIX A: THE EFFECT OF DISORDER ON THE POLARIZABILITY OF AN ELECTRON GAS IN ZERO FIELD

A basic component of the theortical work in this paper is the dynamically screened Coulomb interaction in two dimensions, which may be written when $H=0$

$$V^s(\vec{q},\omega)=2\pi e^2/[q+\Pi(\vec{q},\omega)2\pi e^2]=\frac{2\pi e^2}{q[\epsilon(q,\omega)]}\ , \tag{A1}$$

where the dielectric function ϵ is given in terms of the polarizability Π as

$$\epsilon(\vec{q},\omega)=[1-\Pi(\vec{q},\omega)2\pi e^2/q]\ . \tag{A2}$$

In the presence of impurities we must average over all possible distributions of the disorder, which if we neglect impurity correlations between different polarization bubbles amounts to replacing $\Pi(\vec{q},\omega_0)$ by its impurity average

$$\langle\Pi(\vec{q},\omega)\rangle=\left\langle\int d^dx\,dt\,e^{i\vec{q}x-i\omega t}G^+(\vec{x},t)G^-(0,0)\right\rangle\ , \tag{A3}$$

where $\langle \Pi(\vec{q},\omega)\rangle$ indicates the impurity average. Our goal is to calculate the polarization bubble in the presence of disorder.

The Dyson equation for the single-particle Green's function is written as

$$G_\epsilon^{-1}(\vec{p})=[G_\epsilon^0(p)]^{-1}-\Sigma(\epsilon,\vec{p}) , \quad (A4)$$

where at lowest order in the impurity scattering

$$\Sigma_\epsilon(\vec{p})=-\frac{i}{2}\tau\,\mathrm{sgn}\epsilon \quad (A5)$$

and

$$1/\tau=2\pi N_1 u^2 . \quad (A6)$$

The polarizability $\langle \Pi(\vec{q},\omega)\rangle$ can be written in general

$$\langle \Pi(\vec{q},\omega)\rangle=2i\int_{-\omega}^{0}\frac{d\epsilon}{2\pi}\frac{\phi_i(\vec{q},\epsilon,\omega)}{1-u^2\phi_i(\vec{q},\epsilon,\omega)} , \quad \omega>0 . \quad (A7)$$

Here, ϕ_i is the irreducible part of the electron-hole bubble, that is, that part of the particle-hole propogator that cannot be broken into two by taking out a single impurity line. At the level of approximation of Eqs. (A5) and (A6), ϕ_i is simply the "bare bubble"

$$\phi_i^0(\vec{q},\epsilon,\omega)=\int\frac{d^2p}{(2\pi)^2}G_{\epsilon+\omega_0}^{+}(\vec{p}+\vec{q})G_\epsilon^{-}(\vec{p}) , \quad (A8)$$

where

$$G_\epsilon(\vec{p})=\left[\epsilon-\xi_p-\frac{i}{2}\tau\right]^{-1} , \quad (A9)$$

$$\xi_p=p^2/2m-\mu . \quad (A10)$$

This expression is easily computed in the small-q and -ω limit,

$$\phi_i^0(\vec{q},\epsilon,\omega)\cong 2\pi N_1\tau[1-(ql)^2/2+i\omega\tau] , \quad (A11)$$

in agreement with Ref. 5. Thus

$$\Pi(\vec{q},\omega)=-i\int_{-\omega}^{0}\frac{d\epsilon}{2\pi}\frac{2\pi N_1\tau}{(ql)^2/2-i\omega\tau}-iN_1 . \quad (A12)$$

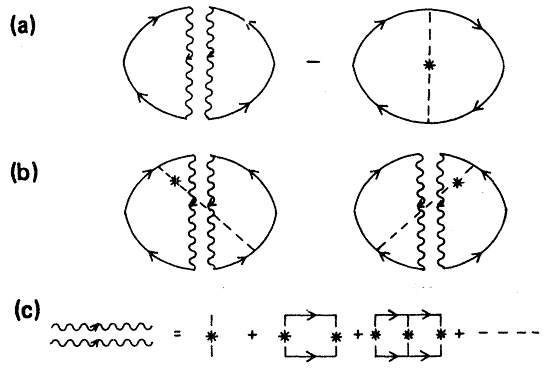

FIG. 7. Corrections to polarizability (above) arising from the particle-particle diffusion propagator (below). The first diagram has a subtraction to prevent overcounting.

The constant N_1 arises because the integral over ξ_p in the simple bubble ϕ_i^0 is not convergent in the region assumed. This is corrected by doing the integral over ϵ first for the simple bubble. All integrals involved in extensions of this approximation are convergent.

This simple approximation is easily generalized to include localization effects. In a conserving approximation the corrections to the bare irreducible polarizability in a consistent expansion is $1/k_F l$ are given by the three diagrams shown in Fig. 7. In these diagrams the double wiggly line is the particle-particle diffusion propogator in zero field

$$D(\vec{q})=\frac{u^2\tau^{-1}}{D_0q^2-i\omega} , \quad (A13)$$

where $D_0=v_F^2\tau/2$. The first diagram, Fig. 7(a), has a subtraction term; otherwise, overcounting of terms would result. Expanding each term for small momentum transfer and frequency we find

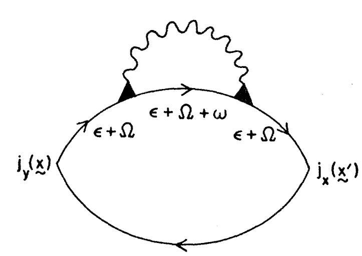

FIG. 8. Labeling of the diagram of Fig. 4(a).

$$\phi_a(\vec{q},\omega)=4\pi N_1\tau^3\int\frac{d^2q'}{(2\pi)^2}[D(q')-u^2]\left[1-\frac{(ql)^2}{2}-\frac{(q'l)^2}{2}+3i\omega\tau\right], \tag{A14}$$

$$\phi_{b+c}(\vec{q},\omega)=-4\pi N_1\tau^3\int\frac{d^2q'}{(2\pi)^2}D(q')[1-(ql)^2-(q'l)^2+4i\omega\tau]\,. \tag{A15}$$

Thus to this order the irreducible bubble is given by

$$\phi_i=\phi_i^0+\Delta\phi_i\,, \tag{A16}$$

$$\Delta\phi_i=\phi_i^a+\phi_i^b+\phi_i^c=4\pi N_1\tau^3\left[\int\frac{d^2q'}{(2\pi)^2}[D(\vec{q}\,')-u^2]\left[1-\frac{(ql)^2}{2}-\frac{(q'l)^2}{2}+3i\omega\tau\right]\right.$$
$$\left.-D(q')[1-(ql)^2-(q'l)^2+4i\omega\tau\right]\,. \tag{A17}$$

The first point to notice is that the leading terms cancel exactly. Now consider

$$I=\tfrac{1}{2}\int\frac{d^2q'}{(2\pi)^2}(q'l)^2D(q')=u^2\int\frac{d^2q'}{(2\pi)^2}\left[1+\frac{i\omega}{Dq'^2-i\omega}\right]. \tag{A18}$$

We notice that the leading term is canceled by the subtraction term from ϕ_a and the frequency-dependent part is canceled by the explicit frequency dependence in $\Delta\phi$. Thus we are left at this order with

$$\phi_i=\phi^0+\Delta\phi=\phi^0+4\pi N_1\tau^3\frac{(ql)^2}{2}\int\frac{d^2q'}{(2\pi)^2}D(q')=2\pi N_1\tau\left[1-\frac{(ql)^2}{2}\left[1-2\tau^2\int\frac{d^2q'}{(2\pi)^2}D(q')\right]+i\omega\tau\right] \tag{A19}$$

and therefore

$$\Pi(\vec{q},\omega)=i\left\{\int_{-\omega}^{0}\frac{d\epsilon}{2\pi}2\pi N_1\tau\left[\frac{(ql)^2}{2}\left[1-2\tau^2\int\frac{d^2q'}{(2\pi)^2}D(q')\right]-i\omega\tau\right]^{-1}-iN_1\right\}$$
$$=N_1\tilde{D}q^2/(\tilde{D}q^2-i\omega)\,, \tag{A20}$$

where

$$\tilde{D}=(v_F^2\tau/2)\left[1-2\tau^2\int\frac{d^2q'}{(2\pi)^2}D(q')\right]. \tag{A21}$$

Integration gives Eq. (3).

APPENDIX B: DERIVATION OF EQUATION (17)

It is clear from Eq. (12) that Π^0 is translationally invariant in the y direction. Taking the Fourier transform with respect to the y variable integrating over y, y' and summing over k' we find

$$\Pi^0(\vec{x},\vec{x}\,';q_yq_y';\omega)=\delta(q_y+q_y')\sum_k G^+_{\epsilon+\omega}(x,x';k+q_y)G^-_\epsilon(x',x;k)\,. \tag{B1}$$

It is evident from this equation and the form of the Green's function Eq. (13) that a shift of variable

$$k\alpha+x_1\rightarrow k\alpha$$

makes the translational invariance of Π^0 manifest. Thus

$$\Pi^0(\vec{q},\omega)=\delta(q_y+q_y')\int dx\int dx'e^{-iq_xx-iq_x'x'}\sum_{\substack{k\\nn'}}D_{nn'}\phi_n^*(x+(k+q_y)\alpha)\phi_n(x'+(k+q_y)\alpha)$$
$$\times\phi_{n'}^*(x+k\alpha)\phi_{n'}(x+k\alpha)\,, \tag{B2}$$

where $D_{nn'}$ is defined in the text [Eq. (19)]. Carrying out the shift of variable $x+k\alpha\rightarrow k\alpha$ we have

$$\Pi^0(\vec{q},\omega)=\delta(q_y+q_y')\int dx\int dx' e^{-iq_x x-iq_x' x'}\sum_{\substack{k\\nn'}} D_{nn'}\phi_n^*((k+q_y)\alpha)\phi_{n'}(k\alpha)$$
$$\times\phi_n(x'-x+(k+q_y)\alpha)\phi_n^*(x'-x+k\alpha)\ . \tag{B3}$$

A further shift $x'-x+k\alpha\rightarrow x'$ gives

$$\Pi^0(\vec{q},\omega)=\delta(q_y+q_y')\int dx\, dx' e^{-i(q_x+q_x')x-iq_x' x'+ik\alpha q_x'}$$
$$\times\sum_{\substack{k\\nn'}} D_{nn'}\phi_n^*((k+qy)\alpha)\phi_{n'}(k\alpha)\phi_{n'}^*(x')\phi_n(x'+q_y\alpha)\ . \tag{B4}$$

Integrating over x we find

$$\Pi^0(\vec{q},\vec{q}',\omega)=\delta(\vec{q}+\vec{q}')\Pi^0(\vec{q},\omega)\ , \tag{B5}$$

where

$$\Pi^0(\vec{q},\omega)=p\sum_{nn'}(\vec{q})\,|^2 D_{nn'}\ , \tag{B6}$$

and

$$F_{nn'}(\vec{q})=\int dx\, e^{iq_x x}\phi_{n'}^*(x)\phi_n(x+q_y\alpha)=\int dx\,\phi_{n'}^*(x)e^{iq_x x}e^{iq_y\alpha\frac{1}{i}\frac{\partial}{\partial x}}\phi_n(x)\ . \tag{B7}$$

Using the identity

$$e^A e^B=e^{A+B+[A,B]/2}\ , \tag{B8}$$

we have

$$F_{nn'}(\vec{q})=e^{-(i/2)q_x q_y\alpha}\int dx\,\phi_n(x)e^{iq_x x+iq_y x^2(1/i)(\partial/\partial x)}\phi_n(x)\ . \tag{B9}$$

As we are concerned with $|F_{nn'}(\vec{q})|^2$ we will ignore the prefactor from now on. Decomposing x and $(1/i)(\partial/\partial x)$ in terms of the harmonic oscillator annihilation a and creation $a^\dagger$ operators we have up to the prefactor

$$F_{nn'}(\vec{q})=\int dx\phi_{n'}^*(x)e^{i(q_x-iq_y)\sqrt{\alpha}a/\sqrt{2}+i(q_x+iq_y)\sqrt{\alpha}a^\dagger/\sqrt{2}}\phi_n(x)\ . \tag{B10}$$

Writing

$$q_x+iq_y=qe^{i\phi}\ , \tag{B11}$$
$$q=(q_x^2+q_y^2)^{1/2}\ , \tag{B12}$$

then

$$F_{nn'}(\vec{q})=\int dx\,\phi_{n'}^*(x)e^{i\sqrt{\alpha}qe^{-i\phi}a/\sqrt{2}+i\sqrt{\alpha}qe^{i\phi}a^\dagger/\sqrt{2}}\phi_n(x)\ . \tag{B13}$$

Again, as we are only concerned with $|F_{nn'}(\vec{q})|^2$ the factor $e^{i\phi}$ is irrelevant and in effect

$$F_{nn'}(\vec{q})=\int dx\phi_{n'}^*(x)e^{iqx}\phi_n(x)\ , \tag{B14}$$

proving Eq. (17).

APPENDIX C: CANCELLING OF CONTRIBUTIONS TO $\langle j_x, j_x\rangle$ AND $\langle j_y, j_x\rangle$

In this appendix we show that the contributions to $\langle j_x, j_x\rangle$ and $\langle j_y, j_x\rangle$ from diagrams 4(a), 4(b), and 4(c) cancel among themselves. We represent the contribution from the diffusion pole in (a), (b), and (c) by

$$f(\omega)=\int \frac{d^2q}{(2\pi)^2}\frac{V_s(\vec{q},\omega)}{(-i\omega+D_Hq^2)^2}\ . \tag{C1}$$

We can break up the contributions according to the signs of ϵ and $\epsilon+\Omega$. We denote by $a_{++}(x,x)$ the contribution of (a) to $\langle j_x, j_x\rangle$ when $\epsilon>0$ and $\epsilon+\Omega>0$. $a'_{++}(x,x)$ is the corresponding diagram, with the self-energy insertion in the hole line, and so on.

First we consider $a_{++}(y,x)$, Figs. 4(a) and 8. Details of the calculation for the b's and c's are similar.

$$\begin{aligned}a_{++}(y,x)=\int_\Omega^\infty d\epsilon\int_\epsilon^{1/2} d\omega\, f(\omega)\sum_{\substack{nn'\\ n''n'''}} & G^+(n)G^+(n')G^-(n'')G^+(n''')\\ &\times\int\frac{dk}{2\pi}\frac{-e}{m}\phi^*_{n'}(x_1+k\alpha)\left[k+\frac{x_1}{\alpha}\right]\phi_n(x_1+k\alpha)\\ &\times\int dx_2\phi^*_{n''}(x_2+k\alpha+q_y\alpha)e^{-iq_xx_2}\phi_{n'}(x_2+k\alpha)\\ &\times\int dx_3\phi^*_{n'''}(x_3+k\alpha)e^{-iq_xx_3}\phi_{n''}(x_3+k\alpha+q_y\alpha)\\ &\times\int dx_4\frac{ie}{m}\phi^*_n(x_4+k\alpha)\frac{\partial}{\partial x_4}\phi_{n'''}(x_4+k\alpha)\ .\end{aligned} \tag{C2}$$

Expanding the matrix elements in powers of q, we need only the leading q independent term. This sets $n'''=n''=n'$. The matrix elements are easily evaluated and we find

$$\begin{aligned}a_{++}(y,x)=\int_\Omega^\infty d\epsilon\int_\epsilon^{1/2} d\omega\, f(\omega)(-i)\frac{e^2\omega_c}{m}\sum_n\frac{n+1}{2}\{ & G^+(n)[G^+(n+1)]^2G^-(n+1)\\ & -G^+(n+1)[G^+(n)]^2G^-(n)\}\ .\end{aligned} \tag{C3}$$

Where we set $\epsilon_F+\Omega+\omega\simeq\epsilon_F+\Omega\simeq\epsilon_F$ as the energy argument of the G's which is justified since we are interested in the limit Ω, $\omega<<1/\tau<<\epsilon_F$. We obtain in a similar way the contributions from diagrams (a), (b), and (c) to $\langle j_y, j_x\rangle$ and $\langle j_x, j_x\rangle$:

$$a_{++}\begin{Bmatrix}y,x\\x,y\end{Bmatrix}=\int d\epsilon\int d\omega f(\omega)\frac{e^2\omega_c}{m}\times\begin{Bmatrix}-i\\-1\end{Bmatrix}\times\sum_n\frac{n+1}{2}\left[G^+(n)[G^+(n+1)]^2G^-(n+1)\begin{Bmatrix}-\\+\end{Bmatrix}G^+(n+1)[G^+(n)]^2G^-(n)\right],$$

$$a'_{--}\begin{Bmatrix}y,x\\x,y\end{Bmatrix}=\int d\epsilon\int d\omega f(\omega)\frac{e^2\omega_c}{m}\times\begin{Bmatrix}-i\\-1\end{Bmatrix}\times\sum_n\frac{n+1}{2}\left[G^+(n)[G^-(n)]^2G^-(n+1)\begin{Bmatrix}-\\+\end{Bmatrix}G^+(n+1)[G^-(n+1)]^2G^-(n)\right],$$

$$a_{--}\begin{Bmatrix}y,x\\y,x\end{Bmatrix}=\int d\epsilon\int d\omega f(\omega)\frac{e^2\omega_c}{m}\times\begin{Bmatrix}-i\\-1\end{Bmatrix}\times\sum_n\frac{n+1}{2}\left[G^-(n)[G^-(n+1)]^2G^+(n+1)\begin{Bmatrix}-\\+\end{Bmatrix}G^-(n+1)[G^-(n)]^2G^+(n)\right],$$

$$a'_{++}\begin{Bmatrix}y,x\\y,x\end{Bmatrix}=\int d\epsilon\int d\omega f(\omega)\frac{e^2\omega_c}{m}\times\begin{Bmatrix}-i\\-1\end{Bmatrix}\times\sum_n\frac{n+1}{2}\left[G^+(n+1)[G^+(n)]^2G^-(n)\begin{Bmatrix}-\\+\end{Bmatrix}G^+(n)[G^+(n+1)]^2G^-(n+1)\right],$$

$$a_{-+}\begin{Bmatrix} y,x \\ xx \end{Bmatrix} = \int d\epsilon \int d\omega f(\omega) \frac{e^2\omega_c}{m} \times \begin{Bmatrix} -i \\ -1 \end{Bmatrix} \times \sum_n \frac{n+1}{2} \Big[G^-(n)[G^+(n+1)]^2 G^-(n+1) \begin{Bmatrix} - \\ + \end{Bmatrix} G^-(n+1)[G^+(n)]^2 G^-(n) \Big] ,$$

$$a'_{-+}\begin{Bmatrix} y,x \\ x,x \end{Bmatrix} = \int d\epsilon \int d\omega f(\omega) \frac{e^2\omega_c}{m} \times \begin{Bmatrix} -i \\ -1 \end{Bmatrix} \times \sum_n \frac{n+1}{2} \Big[G^+(n)[G^-(n)]^2 G^+(n+1) \begin{Bmatrix} - \\ + \end{Bmatrix} G^+(n+1)[G^-(n+1)]^2 G^+(n) \Big] , \tag{C4}$$

$$c_{-+}\begin{Bmatrix} y,x \\ x,x \end{Bmatrix} = \int d\epsilon \int d\omega f(\omega) \frac{e^2\omega_c}{m} \times \begin{Bmatrix} -i \\ -1 \end{Bmatrix} \times \sum_n \frac{n+1}{2} \Big[[G^+(n+1)]^2 G^-(n) \begin{Bmatrix} - \\ + \end{Bmatrix} [G^+(n)]^2 G^-(n+1) \Big] \times u^2 \sum_n [G^+(m)]^2 G(m) ,$$

$$c'_{-+}\begin{Bmatrix} y,x \\ x,x \end{Bmatrix} = \int d\epsilon \int d\omega f(\omega) \frac{e^2\omega_c}{m} \times \begin{Bmatrix} -i \\ -1 \end{Bmatrix} \times \sum_n \frac{n+1}{2} \Big[[G^-(n)]^2 G^+(n+1) \begin{Bmatrix} - \\ + \end{Bmatrix} [G^-(n+1)]^2 G^+(n) \Big] \times u^2 \sum_m [G^-(m)]^2 G^+(m) ,$$

$$b_{++}\begin{Bmatrix} y,x \\ x,x \end{Bmatrix} = \int d\epsilon \int d\omega f(\omega) \frac{e^2\omega_c}{m} \times \begin{Bmatrix} -i \\ -1 \end{Bmatrix} \times \sum_n \frac{n+1}{2} \Big[G^-(n)G^-(n+1)G^+(n)G^+(n+1) \begin{Bmatrix} - \\ + \end{Bmatrix} G^-(n+1)G^-(n)G^+(n+1)G^+(n) \Big] ,$$

$$b_{--}\begin{Bmatrix} y,x \\ x,x \end{Bmatrix} = \int d\epsilon \int d\omega f(\omega) \frac{e^2\omega_c}{m} \times \begin{Bmatrix} -i \\ -1 \end{Bmatrix} \times \sum_n \frac{n+1}{2} \Big[G^-(n)G^-(n+1)G^+(n)G^+(n+1) \begin{Bmatrix} - \\ + \end{Bmatrix} G^-(n+1)G^-(n)G^+(n+1)G^+(n) \Big] .$$

The sums over n are evaluated easily in the limit $\omega_c << \epsilon_F$. For instance, in a_{++} we have

$$\begin{aligned}
&\sum_n \frac{n+1}{2} G^+(n)[G^+(n+1)]^2 G^-(n+1) \\
&\quad = \sum_n \frac{n+1}{2} \frac{1}{\epsilon_F-\epsilon_n+i/2\tau} \frac{1}{(\epsilon_F-\epsilon_{n+1}+i/2\tau)^2} \frac{1}{\epsilon_F-\epsilon_{n+1}-i/2\tau)} \\
&\quad = \frac{\partial}{\partial x} \sum_n \frac{n+1}{2} \frac{1}{x-\epsilon_F} \left[\frac{1}{\epsilon_n-(\epsilon_F-\omega_c)+i/2\tau} \frac{1}{\epsilon_n-x-i/2\tau} - \frac{1}{\epsilon_n-(\epsilon_F-\omega_c)+i/2\tau} \frac{1}{\epsilon_n-\epsilon_F-i/2\tau} \right]_{x=\epsilon_F-\omega_c} .
\end{aligned} \tag{C5}$$

Using the results of Sec. II, this reduces to

$$\sum_n \frac{n+1}{2} G_+(n) G_+^2(n+1) G_-(n+1) = -\frac{\epsilon_F}{2\omega_c} \frac{2\pi i N(\epsilon_F)}{p} \frac{\tau^2}{\omega_c + i/\tau} \left[1 + O\left[\frac{\omega_c}{\epsilon_F} \right] \right] , \tag{C6}$$

while in c_{-+} we have

$$\begin{aligned} u^2 \sum_m G_+^2(m) G_-(m) &= -u^2 \sum_m \frac{1}{(\epsilon_m - \epsilon_F - i/2\tau)^2} \frac{1}{\epsilon_m - \epsilon_F + i/2\tau} \\ &= -i\tau \left[1 - u^2 \left[\frac{\partial}{\partial x} \sum_m \frac{1}{\epsilon_m - x - i/2\tau} \right]_{x=\epsilon_F} \right] = -i\tau \left[1 + O\left[\frac{1}{\epsilon_F \tau} \right] \right] . \end{aligned} \tag{C7}$$

Evaluating the sums in this way we can rewrite (C4) as

$$\begin{aligned} a_{++} \begin{Bmatrix} y,x \\ x,x \end{Bmatrix} &= a'_{--} \begin{Bmatrix} y,x \\ x,x \end{Bmatrix} = \int_\Omega^\infty d\epsilon \int_\epsilon^{1/2} d\omega\, f(\omega) e^2 \frac{\epsilon_F}{m} \frac{2\pi N(\epsilon_F)\tau^4}{1+\omega_c^2\tau^2} \times \begin{Bmatrix} -\omega_c \\ 1/\tau \end{Bmatrix} , \\ a'_{++} \begin{Bmatrix} y,x \\ x,x \end{Bmatrix} &= a_{--} \begin{Bmatrix} y,x \\ x,x \end{Bmatrix} = \int_0^\infty d\epsilon \int_\epsilon^{1/2} d\omega\, f(\omega) e^2 \frac{\epsilon_F}{m} \frac{2\pi N(\epsilon_F)\tau^4}{1+\omega_c^2\tau^2} \times \begin{Bmatrix} \omega_c \\ 1/\tau \end{Bmatrix} , \\ b_{++} \begin{Bmatrix} y,x \\ x,x \end{Bmatrix} &= b_{--} \begin{Bmatrix} y,x \\ x,x \end{Bmatrix} = \int_\Omega^\infty d\epsilon \int_\epsilon^{1/2} d\omega\, f(\omega) e^2 \frac{\epsilon_F}{m} \frac{2\pi N(\epsilon_F)\tau^4}{1+\omega_c^2\tau^2} \times \begin{Bmatrix} 0 \\ -2/\tau \end{Bmatrix} , \\ a_{-+} \begin{Bmatrix} y,x \\ x,x \end{Bmatrix} &= a'_{-+} \begin{Bmatrix} y,x \\ x,x \end{Bmatrix} = \int_0^\Omega d\epsilon \int_\epsilon^\infty d\omega\, f(\omega) e^2 \frac{\epsilon_F}{m} \frac{2\pi N(\epsilon_F)\tau^4}{(1+\omega_c^2\tau^2)^2} \times \begin{Bmatrix} -\omega_c(3+\omega_c^2\tau^2) \\ -2/\tau \end{Bmatrix} , \\ c_{-+} \begin{Bmatrix} y,x \\ x,x \end{Bmatrix} &= c'_{-+} \begin{Bmatrix} y,x \\ x,x \end{Bmatrix} = \int_0^\Omega d\epsilon \int_\epsilon^\infty d\omega\, f(\omega) e^2 \frac{\epsilon_F}{m} \frac{2\pi N(\epsilon_F)\tau^4}{(1+\omega_c^2\tau^2)^2} \times \begin{Bmatrix} 2\omega_c \\ (1-\omega_c^2\tau^2)/\tau \end{Bmatrix} , \end{aligned} \tag{C8}$$

which sum to zero.

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