PHYSICAL REVIEW B VOLUME 25, NUMBER 3 1 FEBRUARY 1982

Systematics of the dielectric constant of vortex phases in superconducting films

A. T. Fiory and A. F. Hebard
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 29 October 1981)

The magnetic field B, angular frequency ω, and temperature dependences of the complex vortex dielectric constant ϵ_v of a thin-film superconductor are deduced from measurements of the ac impedance. A temperature close to the predicted dislocation-mediated vortex-lattice melting temperature is found which delineates low-temperature behavior, $\epsilon_v \propto B$, from high-temperature behavior, $\epsilon_v \propto B/\omega$, A simple two-dimensional Coulomb gas analog successfully explains the fluid phase at high temperature.

The seminal ideas of the Kosterlitz-Thouless theory[1] of phase transitions in two-dimensional systems characterized by a complex two-component order parameter has been noted[2,3] to be relevant to an understanding of the properties of "dirty" thin-film superconductors. In zero external magnetic field the theory predicts that the motion of thermally excited bound vortex pairs drives the fluctuations in the phase of the order parameter. At the two-dimensional transition temperature T_c, less than the mean-field temperature T_{c0}, there is a discontinuous drop in the superfluid pair density n_s, and the vortex pairs of large separation unbind. The resulting dissipative motion of free vortices is thought to be responsible for the broad resistive transitions observed, for example, in thin granular films.[2] The observed decrease in the superfluid pair density n_s at finite frequencies at $T = T_c$ has been previously reported.[4]

In the presence of an external magnetic field B applied perpendicular to the superconducting film, the physics near T_c is complicated by the coexistence of externally applied and thermally excited free vortices. Recent theoretical treatments[5,6] predict a superfluid phase diagram in which there is a melting curve $T_M(B)$ which separates a high-temperature fluidlike region with negligible positional correlations between vortices from a low-temperature solidlike region with significant positional correlations between vortices. The detailed mechanism of melting is thought to be the unbinding of dislocation pairs.[7]

The discontinuity in the shear modulus predicted for the vortex lattice at T_M is analogous to the vortex-antivortex unbinding and associated discontinuity in n_s at T_c. In this Communication we present measurements of the complex vortex dielectric constant ϵ_v which convincingly show that there is a well-defined crossover temperature, presumably T_M, which is a function of B and angular frequency ω. This crossover occurs because there is pinning in the film which becomes effective only when positional correlations at $T < T_M$ are strong enough to ensure that the pinning of just a few vortices will result in the pinning of the entire vortex lattice. In contrast, at $T > T_M$ the shear modulus for long-wavelength distortions is close to zero and most vortices are free to flow around any pinned vortices. As our experimental technique[8] does not excite transverse modes in the vortex medium,[9] coupling to the shear modulus can be achieved only in the presence of pinning, and we are therefore unable to ascertain the precise mechanism of melting. However, simple phenomenological modeling of ϵ_v in the fluidlike, $T >> T_M$, and solidlike, $T < T_M$, regimes show good qualitative agreement of the melting-transition hypothesis with the data.

In order to gain an intuitive understanding of vortex dynamics in thin-film superconductors we invoke an analogy with a two-dimensional (2D) gas of uniformly charged infinitely long parallel rods.[10] The logarithmic intearaction common to both systems, but limited for the case of vortices to separations greater than $\xi_{\rm GL}$, the Ginzburg-Landau coherence length, and less than Λ, the thin-film penetration length, leads to a natural definition of vortex charge $q_v = \phi_0/2\pi\Lambda^{1/2}$. The quantum of flux is ϕ_0. In a previous treatment of thin-film superconductors it has been shown that the electric field $\vec{E}$ due to a current $\vec{K}_v$ of these charges can be written in the form[8]

$$\vec{E} = L_K \frac{\partial \vec{K}_s}{\partial t} + \frac{\phi_0}{q_v c} \hat{z} \times \vec{K}_v \quad . \tag{1}$$

The first term, involving the product of the kinetic inductance $L_K = 2\pi\Lambda/c^2$ with the time derivative of the sheet supercurrent density $\vec{K}_s$, represents the motion of the superfluid background. The second term, where $\hat{z}$ is the normal unit vector, represents the electric field produced by moving vortices. A natural consequence of this treatment is that a vortex current conservation equation, $\vec{\nabla} \cdot \vec{K}_v + \partial\rho_v/\partial t = 0$, can be written in which both $\vec{K}_v = \vec{K}_v^{\rm free} + \partial\vec{P}_v/\partial t$ and the total vortex charge density $\rho_v = \rho_v^{\rm free} - \vec{\nabla} \cdot \vec{P}_v$ are decomposed into free and bound contributions. The quantity $\vec{P}_v$ is a polarization vector associated with vortex dipoles.

Continuing with the 2D Coulomb gas analog we write *ad hoc* Gauss's law, $\vec{\nabla}\cdot\vec{E}_v=2\pi\rho_v$, and find that $q_v\vec{E}_v$, the vortex force field, and ρ_v can be written as

$$q_v\vec{E}_v=-(\phi/c)\hat{z}\times\vec{K}_s \ , \tag{2}$$

and

$$\rho_v=(q_v c L_K/\phi_0)(\vec{\nabla}\times\vec{K}_s)_z \ . \tag{3}$$

It is immediately recognized from Eq. (3) that irrotational supercurrents define the distribution of vortex charge and from Eq. (2) that the vortex force field is the well-known "Lorentz" force.[11] Taking $e^{i\omega t}$ time dependence throughout and with the two constitutive relations $\vec{K}_v^{\text{free}}=\sigma_v\vec{E}_v$, where σ_v is the vortex conductivity, and $\vec{P}_v=\chi_v\vec{E}_v$, where $\chi_v=(\epsilon_{vb}-1)/2\pi$ is the vortex susceptibility written in terms of the bound part ϵ_{vb} of the vortex dielectric constant, we derive the complex impedance

$$Z=i\omega L_K(\epsilon_{vb}+2\pi\sigma_v/i\omega)=i\omega L_K(\epsilon_v'-i\epsilon_v'') \ . \tag{4}$$

This result, which is identical to that derived by Halperin and Nelson[3] using microscopic arguments, conveniently expresses the complex impedance in terms of the complex vortex dielectric constant $\epsilon_v=\epsilon_v'-i\epsilon_v''$ equal to the sum of a contribution from bound vortices ϵ_{vb} and from free vortices $2\pi\sigma_v/i\omega$.

In the data reported here the quantity Z is directly measured[8,12] at temperatures assumed to be sufficiently below T_c so that the zero-field inductance is not renormalized by thermally excited vortex pairs and therefore is a direct measure of L_K. The inferred real $\epsilon_v'-1$ and imaginary ϵ_v'' components are displayed in Fig. 1 as a function of temperature at the indicated frequencies for a 100-Å-thick granular aluminum film in a field $B=12$ G. Additional measured film parameters are $R_N(4.2\ K)=43\ \Omega/\square$, $T_c=1.894$ K, and $T_{c0}=1.900$ K. A noteworthy feature of these data is the break in slope of $\epsilon_v''(T)$ near 1.7 K which is rather pronounced at low frequencies and which broadens and moves out to higher temperatures at higher frequencies. If we use the Bardeen-Stephen theory for free vortices[13] to calculate the dissipation at, say 1.6 K, we find a value for ϵ_v'' approximtely 10^6 greater than observed. For $T<1.7$ K we attribute this sharply reduced dissipation to the thermally activated motion of the pinned vortex lattice. The activation energy for curves A through F of Fig. 1 is found to be 12 K in this temperature region. At higher temperatures, $T>1.7$ K, the activation energies increase with decreasing frequency. Based on theory of the dynamics of 2D melting,[14] we expect the relaxation of thermally excited dislocation pairs to give a logarithmic frequency dependence in the activation energy of ϵ_v'', in qualitative agreement with the data. As the temperature approaches T_{c0} dissipation is dominated by the motion of free-vortex charges whose mobility approaches that of the Bardeen-Stephen model.

In Fig. 2 we have selected the temperature $T=1.65$ K, just below the break temperature of 1.7 K in Fig. 1, and plotted the components of ϵ_v as a function of B at the indicated frequencies. Both components are only weakly dependent on frequency and scale with B. These observations are also true as the temperature is decreased and ϵ_v'' becomes pro-

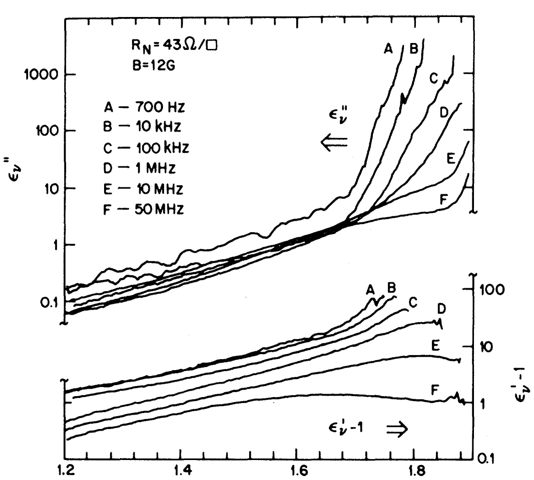

FIG. 1. Imaginary ϵ_v'' and real ϵ_v' parts of the vortex dielectric constant vs temperature at six measurement frequencies.

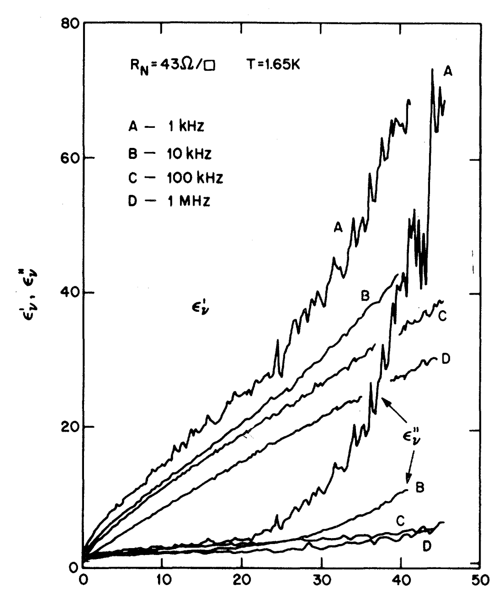

FIG. 2. Vortex dielectric constant vs external magnetic field at four measurement frequencies. The temperature is near the theoretical lattice melting point.

gressively smaller than $\epsilon_v' - 1$. We also note that Fig. 2 shows the sudden increase in ϵ_v'', and to a lesser extent in ϵ_v', for the 1-kHz data at $B \approx 20$ G. At higher frequencies this feature is broadened and moves out to higher fields. This frequency dependence bears striking similarity to that of Fig. 1 as well as that of the zero-field dielectric constant near T_c.[15]

At temperatures $T > 1.7$ K there is a marked change in the behavior of $\epsilon_v(\omega, B, T)$. Most dramatic is the sudden increase in dissipation noted in the previous dicussion of Fig. 1. This manifests itself in the sudden reversal of the inequality $\epsilon_v'' < (\epsilon_v' - 1)$ observed for $T \lesssim 1.7$ K. As the temperature is further increased and free vortices dominate the response, ϵ_v'' continues to increase relative to $\epsilon_v' - 1$ and there is a more gradual transition in the behavior of ϵ_v from a B to a B/ω dependence. This B/ω dependence might be expected on the basis of Eq. (4) where for free vortices σ_v is proportional to B.

The data, however, present an additional complication which is the approximate scaling of the nondissipative component $\epsilon_v' - 1$ with B/ω. We illustrate this complex dielectric response of the vortex fluid with the data shown in Fig. 3 taken on a film with R_N(4.2 K) $= 4130\ \Omega/\square$. The temperature $T = 1.2$ K was chosen to be sufficiently above the calculated melting temperature $T_M \approx 0.25$ K,[5,8] so that the response would manifest a predominantly fluidlike state, and

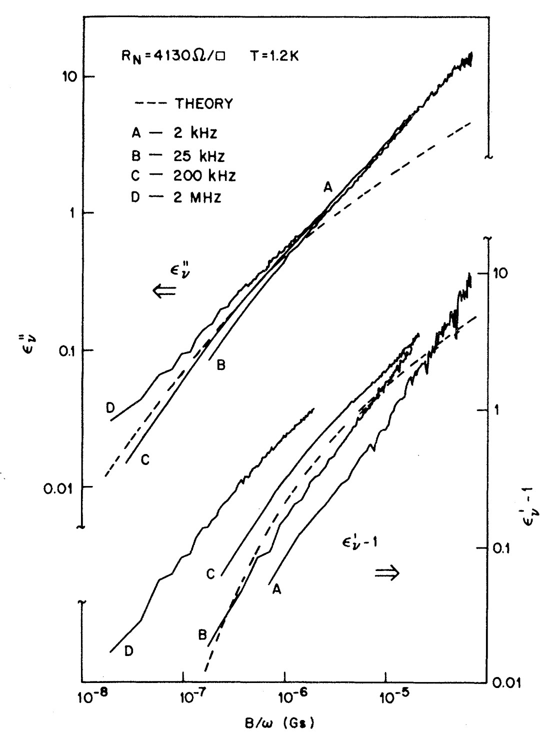

FIG. 3. Vortex dielectric constant vs ratio of external magnetic field to angular frequency. Theory curves are vortex-fluid phase response.

yet sufficiently below $T_c = 1.34$ K,[4,8] so that the contribution of thermally excited free vortices could be ignored. The data of Fig. 3 were taken at fixed frequencies as a function of B and then plotted against B/ω. We note from Fig. 3 the excellent scaling of ϵ_v'' with B/ω for a factor of 10^3 range in frequency. The scaling in $\epsilon_v' - 1$ is not as good, which we believe is caused by a residual amount of pinning.

The vortex Coulomb gas analog can be used to model the dielectric response in the fluid regime. We begin with the assumption that $\vec{E} = -i\omega\vec{A}/c$, where $\vec{A}$ is the vector potential, which when combined with Eq. (4) implies the local relation $\vec{K}_s = -\vec{A}/cL_K\epsilon_v$. This assumption of locality, together with the expressions $\rho_v^{\text{free}} = \rho_v$ and $\sigma_v = \rho_v^{\text{free}}\mu q_v$, which relate the vortex conductivity to the mobility μ, is used with Eq. (3) to derive the following expression:

$$\rho_v^{\text{free}} = -q_v \frac{B}{\phi_0} f(\omega\tau) \quad , \tag{5}$$

where $f = \frac{1}{2} i\omega\tau[(1 - 4i/\omega\tau)^{1/2} - 1]$ and $\tau = c^2 L_K/\mu B\phi_0$. Accordingly, the free-vortex density becomes real and equal to $-q_v B/\phi_0$ when $\omega\tau >> 1$. The negative sign arises because of the diamagnetic screening contribution to the vorticity in Eq. (3). Evaluating the dielectric constant, given by

$$\epsilon_v = 1 + f(\omega\tau)/i\omega\tau \quad , \tag{6}$$

as a function of B/ω, we find a rather acceptable fit of the theory (dashed lines) to the data of Fig. 3. The only adjustable parameter used for this fit is μ, which is 15% of the Bardeen-Stephen value.[13] The model successfully predicts both the inequality $\epsilon_v'' > (\epsilon_v' - 1)$ and the negative curvature for the data displayed in Fig. 3 and for other temperatures in the range $T_M < T < T_c$. A similar contribution to ϵ_v' has been calculated by Halperin and Nelson[3] for a thin-film superconductor in zero field and at a temperature $T > T_c$ high enough so that thermally excited free vortices dominate the response and by Minnhagen,[16] who used a diagrammatic analysis of the charged Coulomb-gas analog.

The observed deviations in the data of Fig. 3 of $\epsilon_v' - 1$ from the B/ω dependence predicted by our model are presumably due to a residual amount of pinning. These deviations increase as the temperature is lowered and the range of positional correlations increases and pinning becomes more effective. The B/ω dependence in both components of ϵ_v thus gradually crosses over to a B dependence at $T \approx T_M$. Long-range positional correlations then become important and there is a sudden transition from high dissipation, $\epsilon_v'' > (\epsilon_v' - 1)$, to low dissipation, $\epsilon_v'' < (\epsilon_v' - 1)$. Further studies of the role of pinning and film inhomogeneities are necessary in order to ascertain whether this observed melting transition is driven by the unbinding of dislocation pairs.[5–7]

[1]J. M. Kosterlitz and D. J. Thouless, J. Phys. C 6, 1181 (1973); J. M. Kosterlitz, *ibid.* 7, 1046 (1974); V. L. Berezinskii, Zh. Eksp. Teor. Fiz. 59, 907 (1970) [Sov. Phys. JETP 32, 493 (1970)].
[2]M. R. Beasley, J. E. Mooij, and T. P. Orlando, Phys. Rev. Lett. 42, 1165 (1979).
[3]B. I. Halperin and D. R. Nelson, J. Low Temp. Phys. 36, 599 (1979).
[4]A. F. Hebard and A. T. Fiory, Phys. Rev. Lett. 44, 291 (1980).
[5]D. S. Fisher, Phys. Rev. B 22, 1190 (1980).
[6]B. A. Huberman and S. Doniach, Phys. Rev. Lett. 43, 950 (1979).
[7]D. R. Nelson and B. I. Halperin, Phys. Rev. B 19, 2457 (1979).
[8]A. F. Hebard and A. T. Fiory, in *Ordering in Two Dimensions,* edited by S. K. Sinha (North-Holland, New York, 1980), p. 181.
[9]J. Clem (private communication).
[10]J. Pearl, in *Low Temperature Physics — LT9,* edited by J. G. Daunt, D. O. Edwards, F. J. Milford, and M. Yaqub (Plenum, New York, 1965), p. 506.
[11]See, for example, M. Tinkham, *Introduction To Superconductivity* (McGraw-Hill, New York, 1975).
[12]A. T. Fiory and A. F. Hebard, in *Inhomogeneous Superconductors—1979,* edited by T. L. Francavilla, D. V. Gubser, J. R. Leibowitz, and S. A. Wolf, AIP Conf. Proc. No. 58 (AIP, New York, 1979), p. 296.
[13]J. Bardeen and M. J. Stephen, Phys. Rev. 140, A1197 (1965).
[14]A. Zippelius, B. I. Halperin, and D. R. Nelson, Phys. Rev. B 22, 2514 (1980).
[15]A. F. Heberd and A. T. Fiory, in Proceedings of the 16th Conference on Low Temperature Physics, Los Angeles, 1981 (in press).
[16]P. Minnhagen (unpublished).