Transverse magnetoresistance in spin-glass alloys

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The scattering of the conduction electrons by magnetic ions in the spin-glass phase in the presence of a magnetic field has been investigated. The transverse magnetoresistance of spin-glasses has been calculated using the method of the double-time Green's function and the Kubo-Greenwood formula. The higher-order Green's functions have been decoupled into the lower-order Green's functions using Nagaoka's decoupling approximation. In the first approximation we have neglected the correlation function describing the quasibound states between the conduction electron and the impurity spin. The self-energy of the Green's function has been obtained to the second order in normal and exchange interactions V_0 and J, respectively. It is found that the self-energy consists of two parts: one involving the spin-glass order parameter Q and the other spin-deviation correlation function. An expression for the transverse magnetoresistance has been obtained by evaluating the relaxation time at the Fermi surface.

I. INTRODUCTION

Much interest has been generated in recent years in the alloys like *Au*Fe, *Cu*Mn, etc. A sharp cusp[1] in the static susceptibility has been discovered with the magnetic impurity concentration in the range 0.1–10 at. %. The system is supposed to undergo a new kind of magnetic phase transition called spin-glass at a characteristic temperature T_g. There are some other experiments indicating a sharp change of physical behavior at T_g, e.g., Mössbauer effect,[2] Hall effect,[3] and muon depolarization.[4] In contrast to this a broad maximum appears in the specific heat.[5] The electrical resistivity[6] also has a broad maximum at a temperature T_m which is higher than T_g. Neutron scattering measurements[7] do not indicate any long-range magnetic order below T_g. There are some other measurements also which show a smooth behavior around T_g, e.g., thermoelectric power,[1] ultrasonic velocity,[8] and NMR.[9]

The above transition appears to be very sensitive to the external magnetic fields. The susceptibility, the Hall effect, and the muon depolarization peaks are all smoothed out even at a field of a few hundred gauss. So the study of magnetoresistance should be useful in understanding the spin-glass phase.

Several workers[10] have measured the magnetoresistance of spin-glasses. Recently Nigam and Majumdar[11] have made systematic studies of the transverse magnetoresistance (TMR) in *Au*Fe, *Ag*Mn, *Cu*Mn, and *Au*Mn systems. The general features are almost the same for all the systems studied. The magnetoresistance is negative at all temperatures and fields H. It is quadratic in H at low fields and fairly independent of temperature below the freezing temperature. The TMR may be fitted to $\Delta\rho_H/\rho_0 = -a(T)H^n$, where $a(T)$ is a temperature-dependent factor, $n=2$ for low fields and $n<2$ for higher fields.

In this paper we have attempted to explain the above results on the magnetoresistance within the framework of the Edwards and Anderson (EA) model.[12] Taking a symmetric Gaussian distribution of exchange forces, Edwards and Anderson could demonstrate within a novel form of mean-field theory that a quenched system undergoes a thermodynamic phase transition at a characteristic temperature T_g. They introduced an order parameter Q and identified the new phase with the spin-glass. This model has been further studied by several authors.[13–15] Much activity[16] is still going on to improve the model. The EA model explains the cusp in the static susceptibility of spin-glasses, but it also predicts sharp cusp in the specific heat, contrary to the experimental observation. It is therefore still necessary to test this model for other experimental observations. With this point in view we have presently undertaken work on the magnetoresistance. Our plan of the paper is as follows: in Sec. II we use the method of double-time Green's function to calculate self-energy of the conduction electron in the presence of magnetic field. In Sec. III the spin deviation correlation function is evaluated, while in Sec. IV we calculate the transverse magnetoresistance for small magnetic field, and compare it with the experimental findings of Nigam and Majumdar.[11]

II. FORMULATION

Let us consider a system of noninteracting conduction electrons and a small concentration of impurities with localized magnetic moment. The interaction between electrons and the magnetic impurities is described by the usual normal and s-d exchange components. In addition there is a magnetic field also applied in the z direction. The corresponding Hamiltonian is described by[17]

$$H=\sum_{k,s}\epsilon_{ks}a_{ks}^{\dagger}a_{ks}+\frac{V_0}{N}\sum_{k,k',s,j}e^{i(\vec{k}-\vec{k}')\cdot\vec{R}_j}a_{ks}^{\dagger}a_{k's}-\frac{J}{N}\sum_{k,k',j}e^{i(\vec{k}-\vec{k}')\cdot\vec{R}_j}[(a_{k\uparrow}^{\dagger}a_{k'\uparrow}-a_{k\downarrow}^{\dagger}a_{k'\downarrow})S_j^z+a_{k\downarrow}^{\dagger}a_{k'\uparrow}S_j^{+}+a_{k\uparrow}^{\dagger}a_{k'\downarrow}S_j^{-}]-g\mu_BH\sum_j S_j^z\ . \tag{1}$$

The first term is the Hamiltonian for the electrons in the magnetic field H such that $\epsilon_{ks}=\hbar^2k^2/2m-E_F-\mu_B sH$, and are measured from the Fermi energy E_F. The quantities $a_{ks}^{\dagger}$ and a_{ks} are, respectively, the creation and annihilation operators for the wave vector k and spin s. The second and third terms are normal and exchange interactions with the strengths V_0 and J, respectively, and are assumed to be independent of k and k'. N is the total number of electrons. The last term describes the interaction of magnetic impurity (spin operator S) with the magnetic field $\vec{H}$ acting in the z direction. The summation j runs over the impurity sites.

To investigate the effect of magnetic field on the resistivity in the spin-glass phase we follow the two-time Green's-function method as used by Fullenbaum and Falk.[18] We define the retarded double-time single-particle Green's function[19] for $s=s'=\uparrow$

$$G_{k\uparrow k'\uparrow}(t)=-i\theta(t)\langle[a_{k\uparrow}(t),a_{k'\uparrow}^{\dagger}(0)]_+\rangle\ , \tag{2}$$

where the average $\langle\cdots\rangle$ is taken over the grand canonical ensemble and $\theta(t)$ is the step function. The equation of motion for the Fourier transform

$$G_{kk'}(\omega)=\int_{-\infty}^{\infty}G_{kk'}(t)e^{i\omega t}dt \tag{3}$$

is given by

$$(\omega-\epsilon_{k\uparrow})G_{k\uparrow k'\uparrow}(\omega)=\delta_{kk'}+\frac{V_0}{N}\sum_{q,j}e^{i(\vec{k}-\vec{q})\cdot\vec{R}_j}G_{q\uparrow k'\uparrow}(\omega)-\frac{J}{N}\sum_{q,j}e^{i(\vec{k}-\vec{q})\cdot\vec{R}_j}[\Gamma^j_{q\uparrow k'\uparrow}(\omega)+M^j_{q\downarrow k'\uparrow}(\omega)]\ . \tag{4}$$

Here we have introduced the Fourier transform of the two higher-order Green's functions,

$$\Gamma^j_{q\uparrow k'\uparrow}(t)=-i\theta(t)\langle[a_{q\uparrow}(t)S_j^z,a_{k'\uparrow}^{\dagger}(0)]\rangle \tag{5a}$$

and

$$M^j_{q\downarrow k'\uparrow}(t)=-i\theta(t)\langle[a_{q\downarrow}(t)S_j^{-}(t),a_{k'\uparrow}^{\dagger}(0)]\rangle\ , \tag{5b}$$

which obey the equations of motion

$$\begin{aligned}(\omega-\epsilon_{q\uparrow})\Gamma^j_{q\uparrow k'\uparrow}(\omega)=\;&\delta_{qk'}\langle S_j^z\rangle+\frac{V_0}{N}\sum_{q',j'}\exp[(\vec{q}-\vec{q}')\cdot\vec{R}_{j'}]\Gamma^{j'}_{q'\uparrow k'\uparrow}(\omega)\\&-\frac{J}{N}\sum_{q',j'}\exp[i(\vec{q}-\vec{q}')\cdot\vec{R}_{j'}][\langle\langle a_{q'\uparrow}(t)S_j^z(t)S_{j'}^z(t)|a_{k'\uparrow}^{\dagger}(0)\rangle\rangle_\omega\\&+\langle\langle a_{q'\downarrow}(t)S_{j'}^{-}(t)S_j^z(t)|a_{k'\uparrow}^{\dagger}(0)\rangle\rangle_\omega]-\frac{J}{N}\sum_{q',q''}\exp[i(\vec{q}'-\vec{q}'')\cdot\vec{R}_j]\\&\times[\langle\langle a_{q\uparrow}(t)a_{q'\downarrow}^{\dagger}(t)a_{q''\uparrow}(t)S_j^{+}(t)|a_{k'\uparrow}^{\dagger}(0)\rangle\rangle_\omega\\&-\langle\langle a_{q\uparrow}(t)a_{q'\uparrow}^{\dagger}(t)a_{q''\downarrow}(t)S_j^{-}(t)|a_{k'\uparrow}^{\dagger}(0)\rangle\rangle_\omega]\ ,\end{aligned} \tag{6}$$

$$
\begin{aligned}
(\omega-\epsilon_{q\downarrow}+g\mu_B H)M^{j}_{q\downarrow k'\uparrow}(\omega) &= \frac{V_0}{N}\sum_{q',j'}\exp[i(\vec{q}-\vec{q}\,')\cdot\vec{R}_{j'}]M^{j'}_{q'\downarrow k'\uparrow}(\omega) \\
&\quad +\frac{J}{N}\sum_{q',j'}\exp[i(\vec{q}-\vec{q}\,')\cdot\vec{R}_{j'}][\langle\langle a_{q'\downarrow}(t)S^z_{j'}(t)S^-_j(t)\,|\,a^\dagger_{k'\uparrow}(0)\rangle\rangle_\omega \\
&\qquad -\langle\langle a_{q'\uparrow}(t)S^+_{j'}(t)S^-_j(t)\,|\,a^\dagger_{k'\uparrow}(0)\rangle\rangle_\omega] \\
&\quad -\frac{J}{N}\sum_{q',q''}\exp[i(q'-q'')\cdot\vec{R}_j] \\
&\qquad \times\{\langle\langle a_{q\downarrow}(t)[a^\dagger_{q'\uparrow}(t)a_{q''\uparrow}(t)-a^\dagger_{q'\downarrow}(t)a_{q''\downarrow}(t)]S^-_j(t)\,|\,a^\dagger_{k'\uparrow}(0)\rangle\rangle_\omega \\
&\qquad -2\langle\langle a_{q\downarrow}(t)a^\dagger_{q'\downarrow}(t)a_{q''\uparrow}(t)S^z_j(t)\,|\,a^\dagger_{k'\uparrow}(0)\rangle\rangle_\omega\}\;, \qquad (7)
\end{aligned}
$$

where we have used the notation $\langle\langle\cdots\rangle\rangle_\omega$ for the Fourier transform of the quantity $-i\theta(t)\langle[A(t),B(0)]_+\rangle$ and used the commutation relations

$$[S^z_j,S^\pm_{j'}]=\pm S^\pm_j\delta_{jj'},\quad [S^+_j,S^-_{j'}]=2S^z_j\delta_{jj'}\;. \qquad (8)$$

To solve Eqs. (5)–(7) we now decouple the higher-order Green's functions into the lower-order Green's functions. We have

$$
\begin{aligned}
&\langle\langle a_{q'\uparrow}(t)S^z_{j'}(t)S^z_j(t)\,|\,a^\dagger_{k'\uparrow}(0)\rangle\rangle=\langle S^z_{j'}S^z_j\rangle\langle\langle a_{q'\uparrow}(t)\,|\,a^\dagger_{k'\uparrow}(0)\rangle\rangle\;,\\
&\langle\langle a_{q'\downarrow}(t)S^-_{j'}(t)S^z_j(t)\,|\,a^\dagger_{k'\uparrow}(0)\rangle\rangle=\langle S^z_j\rangle\langle\langle a_{q'\downarrow}(t)S^-_{j'}(t)\,|\,a^\dagger_{k'\uparrow}(0)\rangle\rangle\;,\\
&\langle\langle a_{q\uparrow}(t)a^\dagger_{q'\uparrow}(t)a_{q''\downarrow}(t)S^-_j(t)\,|\,a^\dagger_{k'\uparrow}(0)\rangle\rangle=\langle a_{q\uparrow}a^\dagger_{q'\uparrow}\rangle\langle\langle a_{q''\downarrow}(t)S^-_j(t)\,|\,a^\dagger_{k'\uparrow}(0)\rangle\rangle\;,\\
&\langle\langle a_{q'\uparrow}(t)S^+_j(t)S^-_j(t)\,|\,a^\dagger_{k'\uparrow}(0)\rangle\rangle=\langle S^+_jS^-_j\rangle\langle\langle a_{q'\uparrow}(t)\,|\,a^\dagger_{k'\uparrow}(0)\rangle\rangle\;, \qquad (9)\\
&\langle\langle a_{q\downarrow}(t)a^\dagger_{q'\downarrow}(t)a_{q''\uparrow}(t)S^z_j(t)\,|\,a^\dagger_{k'\uparrow}(0)\rangle\rangle=\langle a_{q\downarrow}a^\dagger_{q'\downarrow}\rangle\langle\langle a_{q''\uparrow}(t)S^z_j(t)\,|\,a^\dagger_{k'\uparrow}(0)\rangle\rangle\;,\\
&\langle\langle a_{q\downarrow}(t)a^\dagger_{q'\downarrow}(t)a_{q''\downarrow}(t)S^-_j(t)\,|\,a^\dagger_{k'\uparrow}(0)\rangle\rangle=\langle a^\dagger_{q'\downarrow}a_{q''\downarrow}\rangle\langle\langle a_{q\downarrow}(t)S^-_j(t)\,|\,a^\dagger_{k'\uparrow}(0)\rangle\rangle\\
&\qquad\qquad +\langle a_{q\downarrow}a^\dagger_{q'\downarrow}\rangle\langle\langle a_{q''\downarrow}(t)S^-_j(t)\,|\,a^\dagger_{k'\uparrow}(0)\rangle\rangle\;.
\end{aligned}
$$

The thermal average $\langle BA\rangle$ appearing in the above decoupling approximation is related to the corresponding Green's function $\langle\langle A\,|\,B\rangle\rangle$ by

$$\langle BA\rangle=-\frac{1}{2\pi}\int_{-\infty}^{\infty}f(\omega)(\langle\langle A\,|\,B\rangle\rangle_{\omega+i\epsilon}-\langle\langle A\,|\,B\rangle\rangle_{\omega-i\epsilon})\,d\omega\;,\quad f(\omega)=(e^{\beta\omega}+1)^{-1},\quad \beta=(k_BT)^{-1}\;, \qquad (10)$$

while decoupling the higher-order Green's functions, we neglected correlation functions of the type $\langle a^\dagger_{q'\uparrow}a_{q''\downarrow}S^-_j\rangle$, as these describe quasibound states[20] between the conduction electron and the impurity spin. Such states characterize the Kondo effect and are responsible for the logarithmic divergence in the resistivity. The present decoupling scheme is therefore valid for temperatures greater than the Kondo temperature.

From Eqs. (6), (7), and (9) we get the following equations of motion for $\Gamma^{J}_{q\uparrow k'\uparrow}(\omega)$ and $M^{J}_{q\downarrow k'\uparrow}(\omega)$:

$$(\omega-\epsilon_{q\uparrow})\Gamma^{J}_{q\uparrow k'\uparrow}(\omega)=\delta_{qk'}\langle S^z_J\rangle+\frac{V_0}{N}\sum_{q',J'}\exp[i(\vec{q}-\vec{q}')\cdot\vec{R}_{J'}]\Gamma^{J'}_{q'\uparrow k'\uparrow}(\omega)-\frac{J}{N}\sum_{q',J'}\exp[i(\vec{q}-\vec{q}')\cdot\vec{R}_{J'}]\langle S^z_{J'}S^z_J\rangle G_{q'\uparrow k'\uparrow}(\omega)$$
$$-\frac{J}{N}\sum_{q',J'}\exp[i(\vec{q}-\vec{q}')\cdot\vec{R}_{J'}]\langle S^z_J\rangle M^{J'}_{q'\downarrow k'\uparrow}(\omega)+\frac{J}{N}\sum_{q'}\exp[i(\vec{q}-\vec{q}')\cdot\vec{R}_J]\langle a_{q\uparrow}a^{\dagger}_{q'\uparrow}\rangle M^{J}_{q'\downarrow k'\uparrow}(\omega)\ , \tag{11}$$

$$(\omega-\epsilon_{q\downarrow}+g\mu_B H)M^{J}_{q\downarrow k'\uparrow}(\omega)=\frac{V_0}{N}\sum_{q',J'}\exp[i(\vec{q}-\vec{q}')\cdot\vec{R}_{J'}]M^{J'}_{q'\downarrow k'\uparrow}(\omega)+\frac{J}{N}\sum_{q',J'}\langle S^z_{J'}\rangle\exp[i(\vec{q}-\vec{q}')\cdot\vec{R}_J]M^{J}_{q'\downarrow k'\uparrow}(\omega)$$
$$-\frac{J}{N}\sum_{q',J'}\exp[i(\vec{q}-\vec{q}')\cdot\vec{R}_{J'}]\langle S^+_{J'}S^-_J\rangle G_{q'\uparrow k'\uparrow}(\omega)$$
$$+\frac{J}{N}\sum_{q'}\exp[i(\vec{q}-\vec{q}')\cdot\vec{R}_J]\langle a_{q\downarrow}a^{\dagger}_{q'\downarrow}\rangle M^{J}_{q'\downarrow k'\uparrow}(\omega)$$
$$+\frac{2J}{N}\sum_{q'}\exp[i(\vec{q}-\vec{q}')\cdot\vec{R}_J]\langle a_{q\downarrow}a^{\dagger}_{q'\downarrow}\rangle\Gamma^{J}_{q'\uparrow k'\uparrow}(\omega)\ . \tag{12}$$

In the above two equations if we put $H=0$ and hence $\langle S^z_J\rangle=0$ and add them together, we recover Nagaoka's[20] equation (2.14) with $m_{k'}=0$ in the single-impurity approximation.

The Eqs. (4), (11), and (12) are the set of coupled equations. In principle these equations can be solved to any desired order in V_0 and J. However, to get a closed equation for the Green's function we solve Eqs. (11) and (12) to the first order in J and V_0 and substitute them in (4). Thus we obtain

$$G_{k\uparrow k'\uparrow}(\omega)=G^0_{k\uparrow}(\omega)\delta_{kk'}+\frac{V_0}{N}\sum_{q,J}\exp[i(\vec{k}-\vec{q})\cdot\vec{R}_J]G_{q\uparrow k'\uparrow}(\omega)-G^0_{k\uparrow}(\omega)\frac{J}{N}\sum_{J}\exp[i(\vec{k}-\vec{k}')\cdot\vec{R}_J]\langle S^z_J\rangle G^0_{k'\uparrow}(\omega)$$
$$-G^0_{k\uparrow}(\omega)\frac{JV_0}{N^2}\sum_{J,J',q}\exp[i(\vec{k}-\vec{q})\cdot\vec{R}_J]\exp[i(\vec{q}-\vec{k}')\cdot\vec{R}_J]\langle S^z_J\rangle G^0_{q\uparrow}(\omega)G^0_{k'\uparrow}(\omega)$$
$$+G^0_{k\uparrow}(\omega)\left(\frac{J}{N}\right)^2\sum_{J,J',q,q'}\exp[i(\vec{k}-\vec{q})\cdot\vec{R}_J]\exp[i(\vec{q}-\vec{q}')\cdot\vec{R}_{J'}]$$
$$[\langle S^z_J S^z_{J'}\rangle G^0_{q\uparrow}(\omega)+\langle S^+_{J'}S^-_J\rangle G^0_{q\downarrow}(\omega+\omega_0)-\delta_{JJ'}2(1-n_{q\downarrow})\langle S^z_J\rangle G^0_{q\downarrow}(\omega+\omega_0)]G_{q'\uparrow k'\uparrow}(\omega)\ , \tag{13}$$

where we have introduced

$$G^0_{q\uparrow}(\omega)=(\omega-\epsilon_{q\uparrow})^{-1},\quad \omega_0=g\mu_B H,\quad n_{q\downarrow}=a^{\dagger}_{q\downarrow}a_{q\downarrow}\ .$$

Equation (13) can be solved by the use of the usual multiple-scattering theory. However, in the spin-glass phase each quantity appearing in the iterated equation is to be exchange averaged. We note also that Eq. (13) contains explicitly the spin correlation function in the form of a scalar product. In the spin-glass phase the spins are randomly locked in space and no direction is preferred. For the exchange averaging we assume symmetric probability distribution $P(J_{ij})=P(-J_{ij})$ and write

$$[\langle S^z_J S^z_{J'}\rangle]_{\rm av}=\tfrac{1}{3}Q(H)\delta_{JJ'}+[\langle\delta S^z_J\delta S^z_{J'}\rangle]_{\rm av}\ ,$$
$$[\langle S^{\pm}_J S^{\mp}_{J'}\rangle]_{\rm av}=[\tfrac{2}{3}Q(H)\pm M(H)]\delta_{JJ'}$$
$$+\tfrac{1}{2}\{[\langle\delta S^+_J\delta S^-_{J'}\rangle]_{\rm av}+[\langle\delta S^-_J\delta S^+_{J'}\rangle]_{\rm av}\}\ , \tag{14}$$

where $M(H)=[\langle S_j^z\rangle]_{av}$. The subscripts "av" denotes the exchange averaging and $Q(H)$ and $M(H)$ are the spin-glass order parameter[14] and magnetization, respectively, in the presence of the magnetic field. $[\langle \delta S_j^+ \delta S_j^- \rangle]_{av}$ is the spin-deviation correlation function. In the spin-glass phase, the spin deviations are the diffusive modes[21] and are important at low temperatures only. Using (14), in the multiple-scattering approximation, we obtain

$$[G_{k\uparrow k\uparrow}(\omega)]_{av}=[\omega-\epsilon_{k\uparrow}-\Sigma_{k\uparrow\uparrow}(\omega)]^{-1} \,, \tag{15}$$

where $\Sigma_{k\uparrow\uparrow}(\omega)$ is the self-energy for the electron with spin $\uparrow$, and to the first order in concentration C we have

$$\Sigma_{k\uparrow\uparrow}(\omega)=\frac{C}{N}\sum_q\left[\frac{V_0^2-2JV_0M(H)+\frac{1}{3}Q(H)+P_{jj}^{zz}}{\omega-\epsilon_{q\uparrow}}+J^2\frac{\frac{2}{3}Q(H)-M(H)+2n_{q\downarrow}M(H)+\frac{1}{2}(P_{jj}^{+-}+P_{jj}^{-+})}{\omega-\epsilon_{q\downarrow}+\omega_0}\right] . \tag{16}$$

In a similar manner one can write down the equation of motion of $G_{k\downarrow k\downarrow}(\omega)$ for electron with spin $\downarrow$. The resulting expression for the self-energy $\Sigma_{k\downarrow\downarrow}(\omega)$ comes out to be

$$\Sigma_{k\downarrow\downarrow}(\omega)=\frac{C}{N}\sum_q\left[\frac{V_0^2-2JV_0M(H)+\frac{1}{3}Q(H)+P_{jj}^{zz}}{\omega-\epsilon_{q\downarrow}}+J^2\frac{\frac{2}{3}Q(H)+M(H)-2n_{q\uparrow}M(H)+\frac{1}{2}(P_{jj}^{+-}+P_{jj}^{-+})}{\omega-\epsilon_{q\uparrow}-\omega_0}\right] , \tag{17}$$

where

$$P_{jj}^{zz}=[\langle \delta S_j^z \delta S_j^z\rangle]_{av} \,,$$
$$P^{\pm\mp}=[\langle \delta S_j^{\pm}\delta S_j^{\mp}\rangle]_{av} \,. \tag{18}$$

From (16) and (17) it is obvious that the self-energy can be separated into two parts,

$$\Sigma(\omega)=\Sigma_{el}(\omega)+\Sigma_{inel}(\omega) \,,$$

where $\Sigma_{el}(\omega)$ is the elastic part of the self-energy which arises from the scattering of conduction electrons with the frozen-in impurity-spin moments. The inelastic part of self-energy $\Sigma_{inel}(\omega)$ is due to the scattering from the elementary spin excitations which become important at well below the spin-glass transition temperature. For an explicit evaluation of the self-energy $\Sigma(\omega)$, we require $Q(H)$, $M(H)$, and the spin-deviation correlation function for the Heisenberg spin system. For $Q(H)$ and $M(H)$ we have extended the work of Sherrington and Southern[14] by including an external magnetic field H. We get the following coupled equations for $Q(H)$ and $M(H)$[22]:

$$S(S+1)-Q(H)=\frac{k_BT}{\tilde{J}}\left[\frac{3}{Q}\right]^{1/2}\int\frac{d^3R}{(2\pi)^{3/2}}\frac{1}{R}\left[R^2-\left[\frac{3}{Q}\right]^{1/2}\vec{R}\cdot\vec{P}\right]\left[\exp\left\{-\frac{1}{2}\left[\vec{R}-\left[\frac{3}{Q}\right]^{1/2}\vec{P}\right]^2\right\}\right]SB_s(A) \,, \tag{19}$$

$$M(H)=\int\frac{d^3R}{(2\pi)^{3/2}}\frac{\vec{R}\cdot\vec{P}}{RP}\left[\exp\left\{-\frac{1}{2}\left[\vec{R}-\left[\frac{3}{Q}\right]^{1/2}\vec{P}\right]^2\right\}\right]SB_s(A) \,, \tag{20}$$

where

$$\vec{P}=(\tilde{J}_0/\tilde{J})\vec{M}+(g\mu_B/\tilde{J})\vec{H}$$

and

$$A=(\tilde{J}/k_BT)(Q/3)^{1/2}R \,.$$

It is not difficult to see that the above equations reduce to those of Sherrington and Southern[14] in the limit of P going to zero.

III. CALCULATION OF SPIN-DEVIATION CORRELATION FUNCTIONS

To evaluate the contribution of elementary spin excitations to the self-energy of the conduction electron, we define the following Green's function:

$$G_{jj}^{+-}=[\langle\langle \delta S_j^+ | \delta S_j^- \rangle\rangle]_{av} \,. \tag{21}$$

The averages $[\langle \delta S_j^+ \delta S_j^- \rangle]_{av}$ are related to the above

Green's function in the following way:

$$[\langle \delta S_j^- \delta S_j^+ \rangle]_{\rm av} = \frac{1}{2\pi} \int \frac{d\omega}{e^{\beta\omega}-1} [G_{jj}^{+-}(\omega + i\epsilon) - G_{jj}^{+-}(\omega - i\epsilon)] \ . \quad (22)$$

Rivier[23] has argued that $G_{jj}^{+-}(\omega)$ being dynamical susceptibility of the spin-glass, is the Green's function of a diffusion equation. The subscripts "av" denote the ensemble average over all possible impurity configurations. Now we define the space Fourier transform

$$G_{jj}^{+-}(\omega) = \frac{1}{N} \sum_q G^{+-}(q,\omega) \ . \quad (23)$$

An expression for $G^{+-}(q,\omega)$ has been given by Rivier[23] in the absence of the magnetic field. Following his approach, we find that, in the presence of the magnetic field,

$$G^{+-}(q,\omega) = \frac{-iS^2}{\mp i(\omega - \omega_0) + \Delta(q)} \ , \quad (24)$$

where $\omega_0 = g\mu_B H$, q labels the diffusive modes, and $\Delta(q) = \Lambda q^2$ for small q. Λ is the diffusion constant. The minus and plus signs in $i(\omega - \omega_0)$ apply to retarded and advanced Green's functions, respectively. Similarly for $G^{-+}(q,\omega)$ we get an expression

$$G^{-+}(q,\omega) = \frac{-iS^2}{\mp i(\omega + \omega_0) + \Delta(q)} \ . \quad (25)$$

From (22), (24), and (25), we get

$$\tfrac{1}{2}(P_{jj}^{+-} + P_{jj}^{-+}) = \frac{S^2}{2\pi}\frac{1}{2} \sum_q \int \frac{d\omega}{e^{\beta\omega}-1} \left[\left(\frac{1}{\omega - \omega_0 + i\Delta(q)} + \frac{1}{\omega - \omega_0 - i\Delta(q)} \right) + \left(\frac{1}{\omega + \omega_0 + i\Delta(q)} + \frac{1}{\omega + \omega_0 - i\Delta(q)} \right) \right] \ . \quad (26)$$

Let us now replace $\Delta(q)$ by Λq^2 and change the summation over q by an integration. As the integral over q is highly covergent, we extend the upper limit to infinity. The q integration leads to

$$\tfrac{1}{2}(P_{jj}^{+-} + P_{jj}^{-+}) = \frac{\Omega_{\rm at} S^2}{8\sqrt{2}\pi^2 \Lambda^{3/2}} \int_0^\infty \frac{d\omega}{e^{\beta\omega}-1} \left(\frac{\omega - \omega_0}{|\omega - \omega_0|^{1/2}} + \frac{\omega + \omega_0}{(\omega + \omega_0)^{1/2}} \right) , \quad (27)$$

where $\Omega_{\rm at}$ is the atomic volume. In the limit $\omega_0 = 0$, (27) reduces to

$$P_{jj}^{+-} = \frac{\Omega_{\rm at} S^2}{6\pi^2 \sqrt{2} \Lambda^{3/2}} (k_B T)^{3/2} J_{3/2} \ , \quad (28)$$

where

$$J_{3/2} = \int_0^\infty \frac{x^{3/2}\, dx}{(e^x - 1)(1 - e^{-x})} = \Gamma(\tfrac{5}{2})\zeta(\tfrac{3}{2})$$

with Γ a γ function $\zeta(n)$ a Riemann ζ function. Since the resistivity is proportional to the imaginary part of the self-energy, Rivier's[25] result, i.e., $\rho \propto T^{3/2}$, automatically follows from (16) and (28).

IV. MAGNETORESISTANCE

The magnetoresistance is calculated from the formula

$$\rho(H) = -\frac{6\pi^2 m}{e^2 k_F^3} \frac{1}{\langle \tau_+ \rangle + \langle \tau_- \rangle} \ , \quad (29)$$

where τ_+ and τ_- are the relaxation times for spin-up and spin-down electrons. The average $\langle \cdots \rangle$ is defined as

$$\langle O_\pm \rangle = \int O \frac{\partial f_\pm}{\partial \epsilon_\pm} d\epsilon_\pm \ , \quad (30)$$

in which $f_\pm$ is the Fermi distribution function and $\epsilon_\pm$ is the sum total of kinetic and Zeeman electron energies measured from the Fermi surface. However in the present calculation we shall replace the averages $\langle \tau_+ \rangle$ and $\langle \tau_- \rangle$ by their values at the Fermi surface. In doing so only slight error appears in the calculation of $\rho(H)$ as shown by Béal-Monod and Weiner[24] in their calculation of magnetoresistance for normal transition-metal alloys. Our calculation for $\rho(H)$, therefore, reduces to the evaluation of the relaxation times τ_+ and τ_- at the Fermi surface. The relaxation time τ is given by the self-energy $\Sigma(\omega)$ of the Green's function

$$\tau^{-1} = -\hbar^{-1} {\rm Im}\,\Sigma(\omega + i\epsilon) \ , \quad (31)$$

where ϵ is a small imaginary part. At the Fermi surface $\omega = 0$ and from Eqs. (16), (17), (27), and (31) we get the following expression for the relaxation time:

$$\tau_{\pm}(0)=\frac{\hbar}{\pi N(0)C}\left[\frac{1}{V_0^2 \mp M(H)[J^2+2V_0J-2J^2 f(\pm\omega_0)]+J^2[Q(H)+I(H)]}\right], \tag{32}$$

where

$$I(H)=\frac{3S^2\Omega_{\rm at}}{8\sqrt{2}\pi^2}\int_0^\infty \frac{d\omega}{e^{\beta\omega}-1}\left[\frac{\omega-\omega_0}{(\omega-\omega_0)^{1/2}}+\frac{\omega+\omega_0}{(\omega+\omega_0)^{1/2}}\right]. \tag{33}$$

In arriving at (32) it has been assumed for convenience and simplicity that the imaginary part of $G^0(\omega)=N^{-1}\sum_q(\omega-\epsilon_q)^{-1}$ is independent of ω and we have $\mathrm{Im}G^0(\omega)=i\pi N(0)$, where $N(0)$ is the electron density of states near the Fermi surface. For small magnetic field let us expand (32) as a power series in $[V_0^2+J^2Q(H)+J^2I(H)]^{-1}$ and obtain

$$\tau_+(0)+\tau_-(0)=2[\pi CN(0)]^{-1}\frac{1}{V_0^2+J^2Q(H)+J^2I(H)}\times\left[1+\frac{1}{2T}\frac{M(H)\alpha J^2}{V_0^2+J^2Q(H)+J^2I(H)}+\frac{4M^2(H)V_0^2J^2}{[V_0^2+J^2Q(H)+J^2I(H)]^2}+\cdots\right]. \tag{34}$$

From (29) and (34) we arrive at the following expression for the magnetoresistance $[\rho(H)=\Delta\rho_H+\rho_0]$:

$$\frac{\Delta\rho_H}{\rho_0}=\frac{J^2[Q_1(H)-Q_1(0)]}{V_0^2+J^2Q_1(H)}-\frac{[\alpha M(H)J^2/2T][V_0^2+J^2Q_1(H)]+4M^2(H)V_0^2J^2}{[V_0^2+J^2Q_1(0)][V_0^2+J^2Q_1(H)]}, \tag{35}$$

where $Q_1(H)=Q(H)+I(H)$, $Q_1(0)=Q(0)+I(0)$, $\alpha=g\mu_BH/k_B$, and $\rho_0\propto N(0)[V_0^2+J^2Q_1(0)]$ is the resistivity in the absence of the magnetic field. $Q(0)$ is the spin-glass order parameter in the absence of the magnetic field. Our calculation of the resistivity in the absence of the magnetic field agrees with that obtained by Fischer.[25]

The expression (35) can be further simplified if we use the fact that for alloys under consideration the magnitude of the exchange interaction J is very small in comparison to normal scattering strength V_0. We therefore expand the expression for $\Delta\rho_H/\rho_0$ in (35) as a power series in J/V_0 and retain terms of order $(J/V_0)^2$. We thus obtain

$$\frac{\Delta\rho_H}{\rho_0}=-\frac{J^2}{V_0^2}\left[4M^2(H)-\Delta Q(H)-\Delta I(H)+\frac{\alpha M(H)}{2T}\right] \tag{36}$$

where

$$\Delta Q(H)=Q(H)-Q(0)$$

and

$$\Delta I(H)=B\int_0^\infty\frac{dx}{e^x-1}\left[\frac{x-\beta\omega_0}{|x-\beta\omega_0|^{1/2}}+\frac{x+\beta\omega_0}{(x+\beta\omega_0)^{1/2}}-2x\right], \tag{37}$$

with $B=(3S^2/4\sqrt{2})(k_BT/\Lambda k_0^2)^{3/2}$. We have replaced $\Omega_{\rm at}$ by $6\pi^2/k_0^2$ with k_0 as the cutoff wave vector in the conduction band.

In order to see the variation of $\Delta\rho_H$ with the applied magnetic field, we have solved Eqs. (19) and (20) for $Q(H)$ and $M(H)$ numerically which have been plotted in Figs. 1(a) and 1(b) for different temperatures and magnetic fields. We have also evaluated $\Delta I(H)/B$ of Eq. (37) numerically and have plotted it in Fig. 2 for different values of the magnetic field and temperature. In Fig. 3, $\Delta\rho_H/\rho_0$ has been plotted against H^2 for $T_g=8$ and $\tilde{J}_0/\tilde{J}=0.65$. J/V_0 has been obtained by matching our calculation with

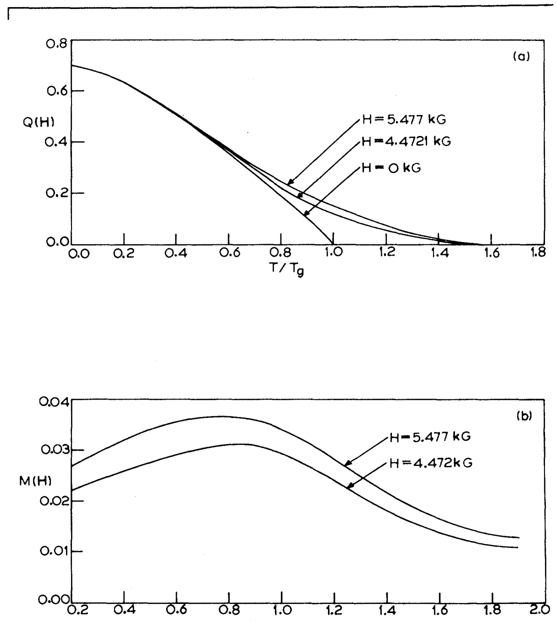

FIG. 1. Plot of (a) $Q(H)$ and (b) $M(H)$ against the reduced temperature T/T_g for Heisenberg spin ($S=\frac{1}{2}$) at $T_g=8$ K and $\tilde{J}_0/\tilde{J}=0.65$ for two sets of magnetic fields.

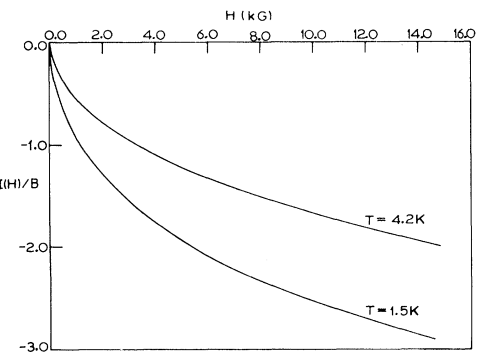

FIG. 2. Plot for $\Delta I(H)/B$ against H for two sets of temperature (i) $T=1.5$ K and (ii) $T=4.2$ K.

one point of the experimental curve at $T=6$ K. The value of J/V_0 comes out to be 0.68, which is rather high. The value of J/V_0 depends upon the parameter $\tilde{J}_0/\tilde{J}$. Higher value of $\tilde{J}_0/\tilde{J}$ will give lower value of J/V_0. Since our main interest is in the qualitative behavior of our results which is hardly affected by the change in J/V_0, we have, therefore, not carried out calculation further to get precise value of J/V_0. In the inset of Fig. 3, $a(T)$ for $n=2$ has been plotted against T. The broken lines represent the experimental curves of Nigam and Majumdar[11] which have been drawn for the sake of comparison. Qualitatively our calculation agrees with the experiment. We get negative magnetoresistance at all temperatures and fields. The order of magnitude is the same as in the experimental data. The magnetoresistance shows a quadratic dependence on H at low field. Below T_g magnetoresistance is insensitive to the variation in temperature. However, at high temperatures ($T>T_g$) we get rather small magnetoresistance as compared to the experimental data. This is also obvious from the curve in the inset of Fig. 3 where at high temperature the fall in our curve is steeper. But at low temperature our curve is flat like the experimental curve. The reason for the low value of the calculated magnetoresistance may be that we have considered the effect of internal field partially through $Q(H)$ and $M(H)$ and also some scattering processes have been neglected as mentioned earlier. However, even at high temperatures there is qualitative agreement between our calculation and the experimental data. It should be noted that our calculation of magnetoresistance does not include the spin-wave contribution. Because it would obliterate the quadratic dependence of $\Delta\rho_H/\rho_0$ on H in Fig. 3. The spin-wave contribution may be important at temperatures lower than 1.5 K.

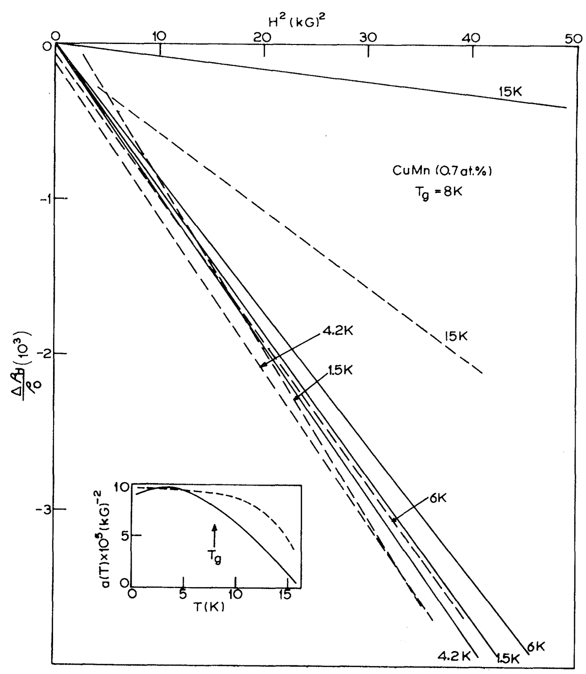

FIG. 3. Plot of $10^3\,\Delta\rho_H/\rho_0$ against H^2 for Heisenberg spin ($S=\frac{1}{2}$) and for $T_g=8$ K, $\tilde{J}_0/\tilde{J}=0.65$, and $J/V_0=0.68$. In the inset $10^5 a(T)$ [$=(1/H^2)(\Delta\rho_H/\rho_0)\times 10^5$] has been plotted against T. Continuous lines represent our calculation and the broken lines represent the experimental curves of Nigam and Majumdar (Ref. 11) for *Cu*Mn (0.7 at.%) and $T_g=8$ K.

Klein[26] has argued that in many cases Ising distribution of the internal field gives better agreement with the experiment than the Heisenberg distribution. Just to see how far Ising distribution of internal field gives result in agreement with the experiment we have carried out calculation for this case also. The expression for $\Delta\rho_H/\rho_0$ in Eq. (36) will be slightly modified. Equation (14) would read as

$$[\langle S_j^z S_{j'}^z\rangle]_{av}=Q(H)\delta_{jj'}+[\langle\delta S_j^z\delta S_{j'}^z\rangle]_{av}\ ,$$

$$[\langle S_j^{\pm}S_{j'}^{\mp}\rangle]_{av}=0\ . \tag{38}$$

Equation (36) would become

$$\frac{\Delta\rho_H}{\rho_0}=-\frac{J^2}{V_0^2}\left[4M^2(H)+\frac{\alpha M(H)}{2T}+Q_0-Q_H\right]\ . \tag{39}$$

Here we have used the following expression[15] for $Q(H)$ and $M(H)$ for Ising spins:

$$Q(H)=\int_{-\infty}^{\infty}\frac{dz}{(2\pi)^{1/2}}e^{-z^2/2}\tanh^2\left[\frac{T_0M(H)}{T}+\frac{T_g}{T}Q^{1/2}(H)z+\frac{\alpha}{T}\right], \tag{40}$$

$$M(H)=\int_{-\infty}^{\infty}\frac{dz}{(2\pi)^{1/2}}e^{-z^2/2}\tanh\left[\frac{T_0M(H)}{T}+\frac{T_g}{T}Q^{1/2}(H)z+\frac{\alpha}{T}\right]. \tag{41}$$

Our calculation for $Q(H)$ and $M(H)$ for various values of H and T are shown in Figs. 4(a) and 4(b). The magnetoresistance has been plotted against H^2 for $T_g=8$ in Fig. 5. The calculation has been performed for $\tilde{J}_0/\tilde{J}=0.375$ and $J/V_0=0.21$. $a(T)$ $(n=2)$ vs T has been plotted in the inset of Fig. 5. The dotted lines represent the experimental curves of Nigam and Majumdar.[11] Here also we find qualitatively the same agreement as in the case of Heisenberg distribution. At high temperature there is the same type of discrepancy here also. We therefore conclude that the Ising distribution gives equally good result.

V. CONCLUSION

We have studied the scattering of conduction electrons due to the magnetic impurities in the presence of finite but small magnetic field in the spin-glass phase. For this we have used the double-time Green's-function method, as used by Fullenbaum and Falk.[18] The self-energy of the Green's function has been obtained to second order in J and V_0 and from it we have determined the relaxation time at the Fermi surface. We have derived an expression for magnetoresistance in the low-field approximation. A comparison has been made between our result and the experimental data of Nigam and Majumdar[11] for the transverse magnetoresistance. A qualitative agreement is found. Below T_g the magnetoresistance is fairly independent of temperature. It is quadratic in field and negative at all temperatures and fields.

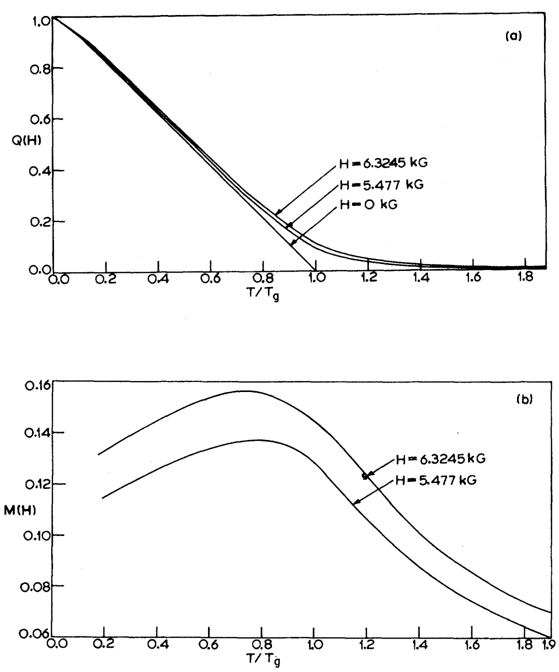

FIG. 4. Plot of (a) $Q(H)$ and (b) $M(H)$ against the reduced temperature T/T_g for Ising spin $(S=\frac{1}{2})$ at $T_g=8$ K and $\tilde{J}_0/\tilde{J}=0.375$ for two sets of magnetic field.

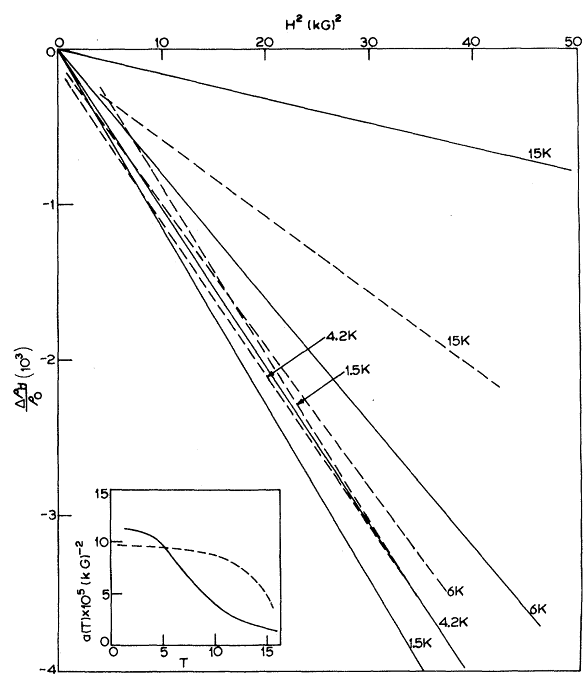

FIG. 5. Plot of $10^3\,\Delta\rho_H/\rho_0$ against H^2 for Ising spin $(S=\frac{1}{2})$ and for $T_g=8$ K, $\tilde{J}_0/\tilde{J}=0.375$, and $J/V_0=0.21$. In the inset $10^5 a(T)$ $[=(1/H^2)(\Delta\rho_H/\rho_0)\times 10^5]$ has been plotted against T. Continuous lines represent our calculation and the broken lines represent the experimental curves of Nigam and Majumdar (Ref. 11) for CuMn (0.7 at.%) and $T_g=8$ K.

At high temperatures ($T > T_g$) also our result is qualitatively in agreement with the experiment but there is somewhat difference in the magnitude. We have carried out calculations both for the Heisenberg and the Ising expression for $Q(H)$ and $M(H)$. We find that the Ising and the Heisenberg expressions give equally good results for the magnetoresistance. Our theory for the magnetoresistance may be improved by carrying out the calculation for higher order in J and V_0. The calculation may be further improved by including fluctuations in the mean-field expressions for $Q(H)$ and $M(H)$.

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