Interference and inertia: A superfluid-helium interferometer using an internally porous powder and its inertial interactions

R. Y. Chiao
Department of Physics, University of California, Berkeley, California 94720
(Received 31 July 1981)

A torus filled with superfluid helium and a powder of a porous solid which contains internal pores having diameters comparable to the coherence length should exhibit Sagnac-Josephson interference when rotated. Analysis is made of the effect of this interference upon the resonance frequency of a torsional oscillator. An inertial mass arising from quantum-mechanical interference is derived. Connection of this interference to general relativistic effects is explored, including the Lense-Thirring effect and gravitational radiation.

I. INTRODUCTION

There have been many attempts[1,2] to observe the Josephson effect in superfluid helium in the past. The original experiment of Richards and Anderson[1] involved a single small orifice connecting two baths of helium at different heights. Goodkind and Gregory[3] attempted to see the ac effect in a torus without any orifices. Clow and Reppy[4] did an experiment on powder without internal pores packed into a torus. Guernsey[5] attempted to see the effect with a single orifice which interrupts a torus. In most of these experiments the effective orifice diameter was much larger than the coherence length.

Here, a new approach to observe a Sagnac-Josephson interference in superfluid helium is proposed. A powder of a porous solid, which contains *internal* pores having diameters comparable to the coherence length, is packed into a torus and filled with superfluid. When the system is set into rotational motion, the superfluid within the pores, which forms a sizable fraction of the total amount of superfluid in the torus, cannot ignore the motion of the walls because of the finiteness of the coherence length. Rather, there should exist a coupling between the powder and the superfluid arising from the continuous exchange of angular momentum within the pores. Owing to the coherent quantum mechanical nature of the superfluid, as the angular speed of the torus is gradually increased, this coupling should exhibit a smoothly periodic dependence upon the circulation of the apparatus with a period h/m, where h is Planck's constant and m is the inertial mass of the helium atom. This periodic behavior can also be viewed as arising from coherent Josephson currents that pass through all the pores of the powder due to Sagnac interference. The recoil from these currents affects the moment of inertia of the system. When the torus is attached to a torsion rod and thus becomes a torsional oscillator, the change in moment of inertia of the system can be sensitively measured through the resulting shift in its resonance frequency. An important difference from many of the past experiments is that at no point during the torsional oscillation is the critical velocity for vortex formation ever exceeded.[6]

II. THE SAGNAC-JOSEPHSON INTERFERENCE

In order to understand the basic physics of the interference, let us consider a rotating torus filled

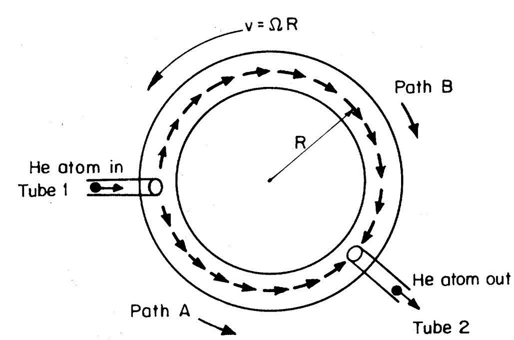

FIG. 1. Schematic of superfluid helium interferometer consisting of a superfluid-filled torus with two extremely fine tubes inserted for admitting and extracting the superfluid. The diameter of the tubes is comparable to the coherence length. Interference occurs between paths A and B upon rotation. The velocity field of the apparatus is $\vec{v}=\vec{\Omega}\times\vec{r}$, where $\vec{\Omega}$ is the angular velocity of rotation with respect to the local inertial frame, which is pointed out of the page, and $\vec{r}$ is the position vector relative to the center of the torus. The cross-sectional width of the torus is much less than its radius R.

 ©1982 The American Physical Society

only with superfluid, into which we stick two extremely fine tubes (see Fig. 1). The diameters of these tubes are chosen to be comparable to the coherence length, which at sufficiently low temperatures is comparable to the size of individual helium atoms. Let us inject superfluid through tube 1 and extract it through tube 2. An injected atom of this superfluid has zero momentum with respect to the orifice, since it is brought to rest with respect to the pore when the pore diameter is comparable to the coherence length. Upon entering the torus at the orifice of tube 1 this atom has a choice of either going along path A or path B. Upon arrival at the orifice of tube 2, the atom will self-interfere with the Sagnac phase shift[7]

$$\Delta\phi = \Delta\phi_{\text{path A}} - \Delta\phi_{\text{path B}} = \frac{m}{\hbar}\oint_\Gamma \vec{v}\cdot d\vec{l} = \frac{2\pi\Omega R^2 m}{\hbar}, \tag{1}$$

where $\vec{v}$ is the velocity field of the apparatus, Γ is the circle of radius R, where R is the radius of the torus, and Ω is the angular speed of rotation of the torus, all with respect to the local inertial frame. Since the ^{4}He atom is neutral and spinless, the interference is free from magnetic screening of the phase shift[8(a)] and from all other electromagnetic interactions. The only remaining interaction for the ^{4}He atom is the gravitational one, as evidenced by the fact that $\Delta\phi$ is proportional to the inertial mass m of the interfering atom.

Note that $\Delta\phi$ is independent of the position of tube 2, and that therefore it is identically the same for the extraction of the atom through any other tubes inserted at arbitrary points into the torus. Now let us set the torus into rotation at a speed such that $\Delta\phi=\pi$. Then universal destructive interference occurs at all orifices, so that no helium atoms can enter into or emerge from any inserted tubes. Now let us pack powder with coherence-length-sized internal pores inside the torus. The powder is tightly packed so that it rotates as a rigid body along with the torus. When $\Delta\phi=\pi$, all the pores are effectively closed and the superflow through all pores vanishes. Note that the presence of superfluid in the space between particles does not alter this argument.[8(b)] When $\Delta\phi=0$ (i.e., no rotation) all the pores are open due to universal constructive interference, but no superflow occurs since the entire system is at rest. Now the interference must be periodic when the phase shift is incremented by $2\pi n$, where n is an integer. Hence we expect nulls in the superflow when $\Delta\phi=0,\pi,2\pi,3\pi$, etc. At intermediate rotation speeds the superflow alternately rises to maxima and drops to minima in a smooth fashion. As the rotation speed is gradually changed, the system remains adiabatically in its lowest energy state. A generalization of the above argument shows that the simplest form the superflow through the internal pores of powder can have is

$$I = I_c \sin\Delta\phi, \tag{2}$$

where I_c is a constant having units of mass per unit time. This relationship can also be derived from a general argument given by Bloch[9] for a Josephson junction in a superconducting ring, and applied to superfluid helium in a torus without an internally porous powder by Schick and Zilsel.[10] However, without a porous powder, the magnitude of I_c will be very small.

III. THE CRITICAL CURRENT

The magnitude of the Josephson critical current density j_c has been calculated for a porous solid by Mamaladze and Cheishvili[11] using the Ginzburg-Pitaevskii equations[12] applied to cylindrical pores:

$$j_c = \frac{7.7\hbar^3 b}{m\beta D^3}, \tag{3}$$

where $\beta = 4\times10^{-40}$ erg cm^3 is the coefficient of the cubic term in the Ginzburg-Pitaevskii equation, and where b is the fraction by volume of pores in the solid, and D is the thickness of the solid. For Vycor with $D=1000$ Å and $b=0.28$, one obtains $j_c = 9\times10^{-4}$ g cm^{-2} s^{-1}. To calculate the critical current I_c for powder in a torus of cross-sectional area a one forms the product $I_c = g j_c a$, where g is a geometrical factor associated with the distribution of shape, size, packing, and pore orientation of the particles. Assuming a model for the powder consisting of a simple cubic close packing of spheres of uniform diameter D, one gets $g=\pi/3$, assuming random orientation of the pores. For $a=1$ mm^2 and the above numbers, one deduces $I_c = 1\times10^{-6}$ g/s.

The Josephson effect given by Eq. (3) sets in immediately below the temperature at which there is the onset of superfluidity in the pores. This occurs when the pore size is comparable to the temperature-dependent coherence length, which according to the Ginzburg-Pitaevskii theory is $\xi = 4$ Å$/(T_\Lambda - T)^{1/2}$. Thus for $\xi = d = 40$ Å, one estimates the onset of the Josephson effect to be around 10 mK below the Λ point. However, measurements show that for a 70 Å pore diameter the onset of superfluidity within the pores actually occurs around 100 mK below the Λ point.[13] Possible candidates for internally porous powders are Vycor

(40 − 70 Å pore diameter), silica gel (40 or 60 or 100 Å), activated carbons (10 − 100 Å), and zeolite (10 Å for 13X).

Inhomogeneities in pore diameter and particle size influence the magnitude I_c, but not the phase shift $\Delta\phi$, which is universally the same for all particles, as long as they all co-rotate with the walls of the torus. Inhomogeneities in pore diameter will spread out the temperature at which the onset of the Josephson effect occurs.

IV. THE TORSIONAL OSCILLATOR

Let us analyze the influence of the Sagnac-Josephson effect on the resonance frequency of a torsional oscillator. Assume that any change in the moment of inertia arising from the Josephson currents to be small, so that the motion is simple harmonic.

The total instantaneous angular speed of the torus is

$$\Omega = \Omega_e + \Omega_0 \sin\omega_0 t \ ,$$

where Ω_e is the angular velocity of the Earth's rotation projected along the axis of torsion, Ω_0 is the maximum of the oscillating angular speed of the torus, and ω_0 is the resonance frequency of the torsional oscillator. From Eqs. (1) and (2), one gets

$$I = I_c \sin(\delta + \alpha \sin\omega_0 t) \ , \tag{4}$$

where

$$\delta = \frac{2\pi m \Omega_e R^2}{\hbar} \ , \tag{5}$$

$$\alpha = \frac{2\pi m \Omega_0 R^2}{\hbar} \ . \tag{6}$$

Expanding (4) one obtains:

$$I = I_c \sin\delta \left[J_0(\alpha) + 2 \sum_{k=1}^{\infty} J_{2k}(\alpha) \cos 2k\omega_0 t \right] + I_c \cos\delta \left[2 \sum_{k=0}^{\infty} J_{2k+1}(\alpha) \sin(2k+1)\omega_0 t \right] \ . \tag{7}$$

The component at ω_0 is

$$I_1 = 2 I_c J_1(\alpha) \cos\delta \sin\omega_0 t \ . \tag{8}$$

By angular momentum conservation, the mass current I_1 through the pores will produce a recoil of the powder and thereby the rest of the system. One can express this in another way: Eq. (8) describes a co-oscillating mass at radius R of an amount

$$M_c = I_c \frac{s^2 m}{\hbar} \left[\frac{2J_1(\alpha)}{\alpha} \right] \cos\delta \ , \tag{9}$$

where $s = 2\pi R$. For small α, this becomes,

$$M_c = I_c \frac{s^2 m}{\hbar} \cos\delta \ , \tag{10}$$

independent of α. The physical significance of this quantity is that it is the *inertial mass of the superfluid-powder system arising from quantum mechanical interference.* It is analogous to the Josephson inductance of the superconducting quantum-interference device (SQUID). Numerically, $M_c = 2$ g, assuming $\cos\delta = 1$, $R = 1$ cm, a cross-sectional area of the torus $a = 1$ mm^2, and $I_c = 1 \times 10^{-6}$ g/s. This value of M_c is much larger than the classical mass of the helium in the torus, which is $M_{\text{class}} = \rho s a = 4$ mg for the above assumptions, using $\rho = 0.0675$ g/cm^3 near the Λ point. Note that whereas the classical mass M_{class} increases linearly with s, the quantum mechanical mass M_c increases quadratically, and therefore dominates over the classical mass for large s.

The resonance frequency of the torsion pendulum is affected by M_c according to the equation

$$\omega_0 = \kappa / [I_0 + M_c R^2]^{1/2} \ , \tag{11}$$

where κ is the torque constant of the torsion rod and I_0 is the background moment of inertia of the system, including that of the walls of the torus, the particles with empty pores, the normal component of the helium, and the Kelvin inertia arising from the superfluid flow in the space between particles. Assuming that $I_0 = 10$ g cm^2, one sees that the fractional change in the resonance frequency is, for the above numbers, nearly 10%. Since the experimentally determined Q for torsion oscillators is of the order of 10^5 ,[14] this means that I_c can be a factor of 10^4 smaller, and the effect can still be seen. The Josephson effect can be observed through the dependence $J_1(\alpha)/\alpha$ of the resonance frequency upon the torsional amplitude α.[15] The Sagnac effect from the Earth's rotation can be observed by changing the angle between the interferometer axis and the polar axis and observing the change in resonance frequency due to the $\cos\delta$ dependence in Eq. (10).[16] For $R = 1$ cm, one obtains from Eq. (5) that $\delta = 166°$ when the interferometer and polar axes are parallel, so that the $\Delta\phi = \pi$ null can be seen with a small interferometer.

V. AN INERTIAL ANALOG OF LENZ'S LAW

The sign of the mass M_c can be determined by a physical argument independent of arbitrary conventions about signs and directions in Fig. 1 and Eqs. (1) and (2). Let us consider a torus which is initially at rest and focus our attention on the behavior of a single representative porous particle. When one applies a torque to the torus, the particle begins to move with a certain velocity $\vec{v}$. This produces a jet of superfluid through the pores of the particle due to the Sagnac-Josephson effect either in the same direction or in the opposite direction to $\vec{v}$. Suppose that the jet is in the opposite direction to $\vec{v}$. Then there will be a recoil of the particle from this jet which will tend to further increase $\vec{v}$. But a further increase of $\vec{v}$ will cause the jet to become yet stronger, etc. Hence, the system *at rest* is no longer stable, which means that it is not the state of lowest energy. This is physically impossible.

Therefore the jet must be in the same direction as $\vec{v}$. This implies that there exists a recoil of the particle which *opposes* the further increase of $\vec{v}$. Hence it becomes harder to push on the particle. *This means that the particle's inertial mass has increased.* Therefore the sign of M_c for $\delta=\alpha=0$ (i.e., for the system at rest) in Eq. (9) is always *positive.* The inertial analog of Lenz's law can be stated thus: *The induced mass current due to Sagnac-Josephson interference will appear in such a direction that it opposes the change that produced it.*

VI. THE SPEED OF RESPONSE OF THE INTERFEROMETER

How quickly does this interferometer respond to sudden changes in $\Delta\phi$ (due, for example, to sudden changes in Ω)? By symmetry, only the azimuthally symmetric $m=0$ sound wave mode in the superfluid and in the torus walls can be excited by such a change. But this mode has zero frequency and cannot be excited. Hence, no low-frequency sound modes can be excited at all. Therefore the transit time for sound to propagate around the torus either in the superfluid or in the walls of the torus is irrelevant. Rather, the response time must be given by $\tau=\hbar/\Delta\approx 10^{-12}$ s, where Δ is the roton energy gap, since there exist no low-lying excitations of the superfluid-powder system of the proper symmetry with energy less than Δ above the ground state.[17,25] This also implies that the ac Josephson currents given by Eq. (7) should exist up to a cutoff frequency given by $\Delta/\hbar$.

VII. THE GENERAL RELATIVISTIC CONNECTION

This interferometer, if experimentally demonstrated, will form an important laboratory connection between quantum mechanics and general relativity, since the phase shift is a direct measure of the rotational components of the metric tensor $\vec{h}_0=(h_{01}, h_{02}, h_{03})$, where $h_{\mu\nu}$ are the deviations of the metric from flat spacetime, through[18–20]

$$\Delta\phi=\frac{mc}{\hbar}\oint_\Gamma \vec{h}_0\cdot d\vec{l} \ , \tag{12}$$

where Γ is a closed curve. In the limit of weak gravitational fields, one obtains the *linear* form of Eq. (2):

$$I=I_c\Delta\phi \ . \tag{13}$$

This implies that there exists a linear coupling between the field $\vec{h}_0$ and the mass current I. It also follows from Eq. (13) that the superfluid-powder system acquires an inertial mass given by[21]

$$M_c=I_c s^2 m/\hbar \ , \tag{14}$$

where s is the length of the closed curve Γ.

The Lense-Thirring effect produces a detectable phase shift through Eq. (12), which should afford us an important test for general relativity.[18–20,22]

As was shown above, the response time of the interferometer to sudden changes in $\Delta\phi$ is $\tau=\hbar/\Delta$. Now let us consider a slowly varying gravitational field, e.g., gravitational radiation, whose period is long compared with τ. Then the system changes adiabatically with time. One sees from Eqs. (12) and (13) that a time-varying $\vec{h}_0(\vec{r},t)$ will generate a time-varying $I(t)$. Specifically, consider a figure-8 tube filled uniformly along its interior with porous powder. When a low-frequency gravitational plane wave impinges on the figure-8 tube in the orientation shown in Fig. 2, it should cause mechanical

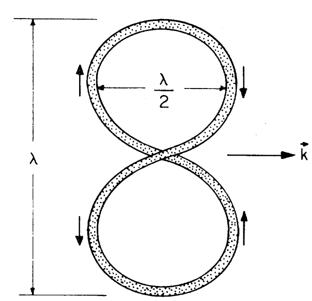

FIG. 2. Schematic of a gravitational wave antenna consisting of a figure-8 tube filled with superfluid and a powder with coherence-length-sized internal pores. The dots represent the powder. A plane-polarized gravitational plane wave with one of its planes of polarization lying in the plane of the antenna is propagating to the right. λ is the wavelength of gravitational wave.

vibrations due the recoil from $I(t)$ uniformly along its perimeter. Conversely, vibrating the figure-8 tube uniformly along its perimeter should generate gravitational radiation by reciprocity.

ACKNOWLEDGMENTS

This work was supported in part by NSF Grant No. ECS-8009834. I thank the Physics Laboratory I of the H. C. Oersted Institute of Copenhagen, Denmark, for a guest professorship during which time this work was done, and would like especially to thank M. T. Levinsen, F. B. Rasmussen, and M. Olsen for help in an initial experiment. I also thank A. A. Moulthrop for helping in the ongoing version of this experiment at Berkeley.

APPENDIX A: DERIVATION OF M_c FROM THE COUPLING ENERGY

There are actually *two* equations

$$I = I_c \sin\Delta\phi = \frac{dM}{dt} , \tag{A1}$$

$$\mu = \hbar \frac{d(\Delta\phi)}{dt} , \tag{A2}$$

which govern the Sagnac-Josephson interference. The second of these, Eq. (A2), is known as Beliaev's equation.[24] Here μ is the chemical potential difference obtained by transporting a single helium atom around the closed curve Γ, or from Eqs. (1) and (A2),

$$\mu = \oint_\Gamma \vec{F}\cdot d\vec{l} = m \oint_\Gamma \frac{\partial \vec{v}}{\partial t}\cdot d\vec{l} , \tag{A3}$$

where $\vec{F}$ is the force field exerted by the pores of the powder on the superfluid which couples to them, per atom.

The energy for producing a phase shift $\Delta\phi$ in the superfluid-powder system, for example, by spinning it up to a rotation speed corresponding to the phase shift $\Delta\phi$, is, from Eqs. (A1) and (A2),

$$W_c = \frac{\hbar}{m} I_c (1-\cos\Delta\phi) . \tag{A4}$$

This is analogous to the Josephson coupling energy. For small $\Delta\phi$, e.g., low rotation speeds, one can rewrite the coupling energy in the form

$$W_c = \tfrac{1}{2} M_c v^2 ,$$

where $\vec{v}$ is the linear velocity of the coupled superfluid-powder system. From Eqs. (1) and (A4), one obtains

$$M_c = I_c m s^2/\hbar . \tag{A5}$$

The sign of M_c is always positive by the analog of Lenz's law (see Sec. V).

There is an important corollary that follows from Eqs. (A2) and (12):

$$\mu = \oint_\Gamma \vec{F}\cdot d\vec{l} = mc\frac{d}{dt}\oint_\Gamma \vec{h}_0\cdot d\vec{l} . \tag{A6}$$

This is analogous to Faraday's law. From Eq. (A6) it follows that

$$\vec{F} = mc\frac{\partial \vec{h}_0}{\partial t} . \tag{A7}$$

The physical significance of $\vec{F}$ is discussed in Appendix B.

APPENDIX B: FURTHER PROPERTIES OF M_c

1. Its nonactivity as a gravitational mass

While M_c is the *bona fide* inertial mass of the coupled superfluid-powder system it does not obey the equivalence principle: It is *not* an active gravitational mass which can be used as a source for a longitudinal gravitational field. For it to obey the equivalence principle, it must satisfy (1) locality and (2) the conservation of mass-energy.

Locality means that the mass is localized in a sufficiently small volume of space such that it can undergo free fall along with other localized masses in its vicinity, in accordance with the weak equivalence principle. By virtue of its origin from interference over extended distances, it is obvious that M_c is a nonlocal mass, since it cannot undergo free fall, for example, along with superfluid placed in the vicinity outside of the torus. Moreover, as one lowers the temperature of the system below the Λ point, the mass M_c appears *ex nihilo,* along with the onset of the Josephson critical current I_c, apparently violating the conservation of mass-energy. Hence this mass is similar, for example, to the effective mass of an election in a crystal, which enters into cyclotron resonance. This does not mean, however, that there are no important *dynamical* consequences of this mass. In fact, because of its rigidity (see below), it drastically alters the acoustical eigenmodes of the *coupled* superfluid-powder system.

2. Its tangential rigidity

It is a remarkable fact that the single-atom interference in Fig. 1, in the absence of the condensed superfluid state, is actually very slow, whereas in its presence, it is very fast. For a single helium atom to interfere in the absence of any superfluid, it must travel from the orifice of tube 1 to the orifice of tube 2. This requires a very long transit time for either path A or path B, due to its very slow speed at liquid-helium temperatures and energies. However, in the presence of the superfluid, there exist simultaneously *(1) a Bose condensation which creates an energy gap* Δ, *and (2) an instantaneous exchange of identical Bose particles* (*^{4}He atoms) over extended distances,* ignoring for the moment the finiteness of the speed of light (see below). Hence, the helium atom can effectively make the trip from tube 1 to tube 2 in a very short time, by exchanging positions with other helium atoms in the superfluid. This time is limited by the uncertainty principle to times greater than $\hbar/\Delta$, lest the system make a transition out of the ground state, thereby destroying phase coherence during this process.[25] Then only *adiabatic* changes in the system occur, and the interference phase shift changes smoothly with time. Since $\tau = \hbar/\Delta \approx 10^{-12}$ s is a very short time, this change, although adiabatic, can be a very fast one.

Let us now consider the case when tube 2 in Fig. 1 is moved to a position very close to tube 1, as is the case for a pore in a single powder particle in the superfluid. When $\Delta\phi$ changes adiabatically, due, for example, to the motion of this particle, the Sagnac-Josephson interference mass current changes adiabatically in accordance to Eqs. (A1)–(A3), not only within the pore of the particle, but also *uniformly* throughout the longer path B, i.e., around the *entire* torus. If the interference current does not develop throughout path B, then one of the two paths essential for interference would effectively not be present, and the interference mass current given by Eq. (A1) would not develop *locally,* i.e., inside the pore of the particle, either. Therefore Eqs. (A1)–(A3) governing the behavior of the interference current are all nonlocal for the topology of a torus. Note that this argument is true, *independent of the size of the torus.*[26] (However, see below.)

Now consider a torus filled uniformly with a superfluid-powder system which is initially at rest. Let us give a tangential push on a small sector of the torus. The powder inside the pushed sector begins to move with a certain velocity $\vec{v}$. As was shown above, the Sagnac-Josephson interference sets in very quickly and an interference mass current is set up not only inside the pores of the pushed powder, but also uniformly throughout the entire torus. Now consider what happens in an unpushed sector on the opposite side of the torus. Suppose that the powder there remains at rest. Then all the pores of this power are open to the flow of this interference current through them, due to universal constructive interference for all the pores of this sector of the powder, where $\Delta\phi = 0$. But the flow of such a current through the pores of a *stationary* powder is not consistent with Eq. (13). Therefore the powder must begin to move in a self-consistent way in response to this current, such that it co-rotates with the pushed sector on the other side of the torus. Hence the torus begins to rotate as a rigid body in a time $\tau = \hbar/\Delta$, *independent of the size of the torus.* (However, see below.) The torus is extremely rigid, having been rigidized by the Sagnac-Josephson interference currents set up uniformly throughout the entire torus. This tangential rigidity can also be understood as arising from a force field $\vec{F}$ which is transmitted extremely quickly from the pores of the powder in the pushed sector to the rest of the torus by means of the Bose exchange of helium atoms. It is this same Bose force field $\vec{F}$ which enters into the chemical potential through Eq. (A3).

However, the above argument would seem to violate causality, since one could, in principle, communicate via $\vec{F}$ faster than the speed of light whenever $2\pi R \geq c\tau$. But the exchange interaction cannot propagate faster than the speed of light. Mathematically, this is already implicit in Eqs. (A6) and (A7). The Bose force field $\vec{F}$ obeys the analog of Faraday's law, which involves $\vec{h}_0(\vec{r},t)$. But $\vec{h}_0(\vec{r},t)$ is a solution to the linearized Einstein field equations, which obey causality. Hence the speed at which mechanical signals propagate through the coupled superfluid-powder system is the speed of light. Telegraphy by means of such a system should be possible.

These arguments justify the key assumption of quasirigidity of the apparatus of Ref. 23. It is also this rigidity that makes this antenna much more efficient than the classical Weber bar. The latter is not very rigid, and is therefore limited in its length to $l \lesssim v_s T/2$ where v_s is the velocity of sound in the bar and $T/2$ is the half-period of the gravitational wave, whereas the former is limited by the speed of light c to a length $l \lesssim cT/2=\lambda/2$. Since a quadrupole antenna's efficiency is proportional to l^4 for short lengths, the figure-8 antenna shown in Fig. 2 is more efficient than the Weber bar by the factor $(c/v_s)^4 \approx 10^{19}$. More importantly, the absolute radiative efficiency approaches unity for an antenna of size comparable to wavelength. When struck tangentially at any point along its perimeter, the figure-8 tube of Fig. 2 should propagate vibrations along its perimeter at the speed of light, and therefore should radiate efficiently, since its size is comparable to a wavelength. Communication by means of gravitational radiation should therefore be possible.

---

[1]P. L. Richards and P. W. Anderson, Phys. Rev. Lett. 14, 540 (1965); P. L. Richards, in *Quantum Fluids*, edited by D. F. Brewer (North-Holland, Amsterdam, 1966), p. 180; B. M. Khorana and B. S. Chandrasekhar, Phys. Rev. Lett. 18, 230 (1967); B. M. Khorana and D. H. Douglass, Jr., in *Proceedings of the Eleventh International Conference on Low Temperature Physics, St. Andrews, Scotland, 1968*, edited by D. M. Finlayson, J. F. Allen , and D. M. McCall (St. Andrews University, St. Andrews, 1968), p. 169; P. L. Richards, Phys. Rev. A 2, 1532 (1970); B. M. Khorana, in *Proceedings of the Twelfth International Conference on Low Temperature Physics, Kyoto, Japan, 1970*, edited by Eizo Kandal (Academic, Kyoto, 1971), p. 85; J. P. Hulin, B. Perrin, C. Laroche, and A. Libchaber, *ibid.*, p. 83; J. P. Hulin, C. Laroche, A. Libchaber, and B. Perrin, Phys. Rev. A 5, 1830 (1972); P. W. Anderson and P. L. Richards, Phys. Rev. B 11, 2702 (1975).

[2]G. L. Schofield, Jr., Ph.D. thesis, University of Michigan, 1971 with T. M. Sanders, Jr. (unpublished); D. L. Musinski and D. H. Douglass, Phys. Rev. Lett. 29, 1541 (1972); D. L. Musinski, J. Low Temp. Phys. 13, 287 (1973); P. Leiderer and F. Pobell, Phys. Rev. A 7, 1130 (1973); I. Rudnick, *ibid.* 8, 1969 (1973).

[3]D. Gregory, Ph.D. thesis, University of California at San Diego, with J. M. Goodkind (1972) (unpublished).

[4]J. R. Clow and J. D. Reppy, Phys. Rev. Lett. 19, 291 (1967).

[5]R. W. Guernsey, Jr., in *Proceedings of the Twelfth International Conference on Low Temperature Physics, Kyoto, Japan, 1970*, p. 79.

[6]The typical linear velocity arising from the torsional motion in this experiment is $v=\hbar/mR$, corresponding to $\alpha=2\pi$. For $R=1$ cm, $v=1.6\times10^{-4}$ cm/s. This is many orders of magnitude smaller than the critical velocity for superfluid flow through the cracks between the particles of the powder, either near the lambda point, where $v_{s,c}=v_c(1-T/T_\lambda)^{2/3}$ where $v_c=3.8\times10^2$ cm/s [Clow and Reppy (Ref. 4); J. S. Langer and M. E. Fisher, Phys. Rev. Lett. 19, 560 (1967); $v_{s,c}=48$ cm/s for $T_\lambda-T=100$ mK], or well below the lambda point, where $v_c=(\hbar/2mr)\times[\ln(8r/a)-\frac{1}{4}]$ where r is the typical radius of the hole between powder particles and $a=1$ Å is the core radius of a vortex [R. J. Donnelly, *Experimental Superfluidity* (University of Chicago Press, Chicago, 1967), p. 62; for simple cubic close-packed spheres of 1000 Å diameter, $v_c=275$ cm/s]. However, this is only 2 orders of magnitude smaller than the Josephson critical velocity $v_c=4\times10^{-2}$ cm/s assuming $\rho_s=2.4\rho(1-T/T_\lambda)^{2/3}=2.08\times10^{-2}$ g/cm^3 for $T_\lambda-T=100$ mK, and $j_c=9\times10^{-4}$ g cm^{-2} s^{-1} appropriate for 1000 Å diameter Vycor particles. The fact that $v_c>>v$ explains why $M_c>>M_{\rm class}$.

[7]M. G. Sagnac, C. R. Acad. Sci. 157, 708 (1913); 157, 1410 (1913); J. E. Zimmerman and J. E. Mercereau, Phys. Rev. Lett. 14, 887 (1965); G. Papini, Phys. Lett. 24A, 32 (1967); J. Anandan, Phys. Rev. D 15, 1448 (1977); S. A. Werner, J. L. Staudenmann, R. Colella, Phys. Rev. Lett. 42, 1103 (1979); also Refs. 19 and 20.

[8](a) Owing to magnetic screening, the interference phase of a rotating SQUID can be much reduced below that given by Eq. (1). In order for the screening to be small, the SQUID must satisfy the inequality $L \lesssim \hbar/2eI_c$ where L is its geometrical inductance and I_c is the Josephson critical current. In practice, this limits the area of the SQUID to be quite small, which can therefore only detect rapid rotations as was done by Zimmerman and Mercereau, Ref. 7. No such size limitation exists in practice for the *neutral* helium interferometer, since the gravitational analog of the geometrical inductance is extremely small: Helium mass currents in the torus do not induce classically a rotation field, except through the extremely small Lense-Thirring effect. (b) One may object that the superfluid in the space between the particles will "short circuit" the phase shift given by Eq. (1), such as would happen in the case of superconductors. But the reason why this "short circuit" affects the phase shift of the SQUID is that the phase shift is proportional to the *total* flux $\Phi_{\rm tot}=\Phi_{\rm ext}-LI_{\rm scr}$ where $\Phi_{\rm ext}$ is the externally applied flux, L is the geometrical inductance of the ring, including a short circuit around the junction, and $I_{\rm scr}$ is the screening current which flows mainly through the short circuit in response to $\Phi_{\rm ext}$. Since L is usually large enough so that the screening is a very large effect for superconductors, flux quantization in units of $h/2e$ is almost complete and the phase shift is effectively not a continous variable. In

the case of the *neutral* superfluid interferometer, however, the geometrical inductance L is exceedingly small [see footnote (a)], and the screening of the phase shift due to the "short circuit" is essentially absent. Therefore the circulation of the apparatus is *not* quantized in units of h/m as is obvious from the fact that there is no reason why the angular speed of the torus containing the coupled superfluid-powder system should be quantized. Note that there exist *three* paths for a helium atom to get from one orifice at one end to the other at the other end of a single pore passing through a powder particle: path A which is the short path just outside the particle, path B which is the long path going around the torus, and a path *inside* the pore, which we shall call path C. The *difference* in phase between paths A and B is the same as that between C and B, and is the phase difference which enters into the Josephson equation (2). This phase difference is given by Eq. (1) and must be gauge invariant. If a junction completely blocks the torus, so that the "short circuit" path A is missing, then the phase difference is that between paths C and B, which remains unaltered. This is true independent of the length of path C. Whether the short circuit path is present or not, the energy of the system is always periodic in the circulation with a period h/m.

[9]F. Bloch, Phys. Rev. Lett. 21, 1241 (1968); Phys. Rev. 166, 415 (1968); Phys. Rev. A 7, 2187 (1973).

[10]M. Schick and P. R. Zilsel, J. Low Temp. Phys. 1, 385 (1969).

[11]Yu. G. Mamaladze and O. D. Cheishvili, Zh. Eksp. Teor. Fiz. 50, 169 (1966) [Sov. Phys.—JETP 23, 112 (1966)].

[12]V. L. Ginzburg and L. P. Pitaevskii, Zh. Eksp Teor. Fiz. 34, 1240 (1958) [Sov. Phys.—JETP 7, 858 (1958)].

[13]D. F. Brewer, D. C. Champeney, and K. Mendelssohn, Cryogenics 1, 108 (1960); M. Kriss, Ph.D. thesis, University of California, Los Angeles, 1969, with I. Rudnick (unpublished).

[14]J. E. Berthold, R. W. Giannetta, E. N. Smith, and J. D. Reppy, Phys. Rev. Lett. 37, 1138 (1976): D. J. Bishop, Ph.D. thesis Cornell University, 1978, with J. D. Reppy (unpublished); J. M. Parpia, Ph.D. thesis, Cornell University, 1979, with J. D. Reppy (unpublished).

[15]Note that inhomogeneities in pore diameter and particle size do not smear out the nulls of $J_1(\alpha)/\alpha$. However, due to the finite width of the torus, there will be a washing out of the fringes for large α.

[16]This can be done by tilting the torsion axis with respect to the vertical, and stepping through the azimuthal angle. Also, note that the sidereal day, not the solar day, determines Ω_e in Eq. (5) for the Earth's Sagnac effect. This reflects the fact that this interferometer measures rotations with respect to the fixed stars (ignoring the Lense-Thirring effect). Note also that with an accurately known geometry of the torus, one should be able to measure the quantum of circulation h/m accurately.

[17]See Appendix B.

[18]G. Papini, Phys. Lett. 24A, 32 (1967). Note the analogy with the electromagnetic SQUID; one can make this analogy precise if one makes the simultaneous correspondences $m \leftrightarrow e$ and $\vec{h}_0 \leftrightarrow \vec{A}$. The physical basis for this is that the kinetic momentum in the Hamiltonian for the interfering particle is *directly* influenced by the metric so that $\vec{p} \to \vec{p} - mc\vec{h}_0$ [B. S. DeWitt, Phys. Rev. Lett. 16, 1092 (1966)], just as in the electromagnetic case $\vec{p} \to \vec{p} - (e/c)\vec{A}$. This implies that there exist not only time-independent effects, such as a Lense-Thirring phase shift, which is analogous to the Bohm-Aharanov phase shift, but also time-dependent effects, such as a gravitational-radiation-induced phase shift, which is analogous to the radio-wave induced phase shift in the SQUID. The superfluid helium interferometer should therefore be a novel gravitational antenna. This was pointed out by R. Y. Chiao, in The Symposium on Spectroscopy, Quantum Electronics, and Astrophysics to Celebrate the 65th Birthday of C. H. Townes, University of California at San Diego, 1980. See also Refs. 20 and 23.

[19]A. Widom, G. Megaloudis, J. E. Sacco, and T. D. Clark, J. Phys. A 14, 841 (1981).

[20]J. Anandan, Phys. Rev. D 24, 338 (1981); Phys. Rev. Lett. 47, 463 (1981).

[21]This is the same mass given by Eq. (10), except $\delta = 0$. A derivation is given in Appendix A. See also Appendix B.

[22]J. Anandan and R. Y. Chiao (unpublished).

[23]J. Anandan and R. Y. Chiao, J. Gen. Rel. Grav. (in press). See also Appendix B.

[24]S. T. Beliaev, Zh. Eksp. Teor. Fiz. 34, 417 (1958) [Sov. Phys.—JETP 7, 289 (1958)].

[25]The lowest-energy elementary excitation capable of being excited by the motion of the pores are rotons, and not phonons, by Landau's argument [L. D. Landau, Zh. Eksp. Teor. Fiz. 11, 592 (1941); J. Phys. Moscow 5, 71 (1941)]. No vortices can be excited, since the critical velocity for their generation is never exceeded (see Ref. 6). Fourth sound, even if somehow excited, cannot influence the Sagnac phase shift, since its velocity field contains zero circulation.

[26]The same argument applies to the case of superconductors: The Bohm-Aharanov interference for a single electron is very slow, but the SQUID is very fast.