

## Influence of electron-phonon coupling on superconducting contacts and the properties of superconductor-semimetal (or semiconductor) systems

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Consideration of the proximity effect, electron-phonon coupling, and size-quantization results in peculiar behavior of the critical temperature  $T_c$  of a contact containing superconducting and semimetal (or semiconductor) thin films. The method of thermodynamic Green's functions is applied. The dependence of  $T_c$  on the thickness of the barrier is discussed. Size quantization leads to additional oscillations of  $T_c$  and to the possibility of observing peculiar charge-density waves. The experimental data are discussed.

### I. INTRODUCTION

It is well known that systems containing a contact of superconducting and normal films possess very interesting physical properties. In this paper we consider the situation when the normal film is a semimetal (SM) or semiconductor (SC). SM and SC films are characterized by a number of peculiarities which allow one to observe experimentally some interesting phenomena.

Systems containing thin superconducting and nonmetallic films have been studied by several experimental groups.<sup>1-4</sup> The dependence of the critical temperature  $T_c$  on the thickness of the nonmetallic film was observed. Usually, this dependence is nonmonotonic. The properties of superconductor-semimetal (S-SM) and superconductor-semiconductor (S-SC) systems depend on a number of different factors. That is why the values of different parameters and, especially,  $T_c$  can vary noticeably. We consider here the influence of several factors.

(1) The proximity effect. The Cooper pairs can move into the nonmetallic film in the presence of a good electric contact, and this results in the appearance of an induced superconducting state of the SM or SC film. Back flow of electrons also exists.

(2) Size quantization (SQ) of the transverse motion. The phenomenon is characterized by oscillations of the density of states. The best conditions for this effect are realized in thin semimetal and semiconductor films (see below).

(3) The change of the phonon spectrum caused by nonmetallic covering. This change also affects the critical temperature. Note that the peculiarities of the dispersion relation lead to the appearance of charge-density waves.

We take into account all these factors. We focus on the problem of calculation of  $T_c$ . The effect of the proximity and the size quantization on other properties of the system will be examined in the framework of the phonon model elsewhere.

The plan of the present paper is as follows. Section II addresses the problem of obtaining the main equation. We use the method of thermodynamic Green's functions and take into account the electron-phonon interaction directly. We consider the effect of covering on  $T_c$  in Sec. III. The proximity effect, size quantization, and specific charge-density waves are discussed in Secs. IV and V. In Sec. VI we examine the case when the effective constants of both films are not equal to zero. Note that the results of Secs. II, IV, and VI can also be applied to the situation when both films are metallic.

### II. MAIN EQUATIONS

Consider a system containing superconducting ( $\alpha$ ) and normal ( $\beta$ ) thin films. We shall consider in more detail (see below, Sec. V) the case when a  $\beta$  film is a semimetal (or a semiconductor). Denote by  $T_c^\alpha$  the critical temperature of the isolated superconducting film and suppose that the

thicknesses  $L_\alpha$  and  $L_\beta$  satisfy the conditions  $L_\alpha \ll \xi$ ,  $L_\beta \ll \xi$ , where  $\xi$  is the coherence length. Moreover, suppose that the metallic film is "dirty" in the Anderson sense.<sup>5</sup> Size quantization has been observed experimentally by investigation of thin films in the region  $10 \text{ \AA} \lesssim L_\beta \lesssim 10^3 \text{ \AA}$  (Refs. 6–9) (see, e.g., the excellent review in Ref. 10). Hence, the mentioned conditions are perfectly realistic.

Under these conditions we can use the McMillan model of the proximity effect.<sup>11</sup> The electron-phonon interaction was included in the McMillan model in Ref. 12 for the purpose of calculating the energy gap function.

To calculate the critical temperature of the S-SM or S-SC system it is very convenient to use the method of thermodynamic Green's functions (see, e.g., Refs. 13 and 14). Let us introduce the self-energy parts  $\Sigma_2^\alpha$  and  $\Sigma_2^\beta$  describing the pairing in the " $\alpha$ " and " $\beta$ " films. The equations for the self-energy parts are seen in Fig. 1, or, in analytical form:

$$\Sigma_2^\alpha = \Sigma_{2,\text{ph}}^\alpha + \tilde{T}^2 \int d\vec{p}' F^\beta(\vec{p}', \omega_n), \quad (1)$$

$$\Sigma_2^\beta = \tilde{T}^2 \int d\vec{p}' F^\alpha(\vec{p}', \omega_n), \quad (2)$$

$$\Sigma_{2,\text{ph}} = T \sum_{\omega_n} \int d\vec{p}' g_\alpha^2(\vec{p}, \vec{p}') \times D[\omega_n - \omega_n', \omega(q)] F^\alpha(\vec{p}', \omega_n'). \quad (3)$$

$D = \omega_{(q)}^2 / [\omega_{(q)}^2 + (\omega_n - \omega_n')^2]$  is the phonon Green's function,  $\omega_n = (2n + 1)\pi T$ ,  $g_\alpha^2(\vec{p}, \vec{p}')$  describes the electron-phonon interaction, and  $F^\alpha$  and  $F^\beta$  are anomalous Green's functions<sup>15–17</sup>

$$F^\alpha(\omega_n, \vec{p}) = \frac{-\Sigma_2^\alpha(\omega_n, \vec{p})}{[\omega_n^2 Z_\alpha^2 + \xi_\alpha^2(\vec{p}) + \Sigma_2^{(\alpha)^2}(\omega_n, \vec{p})]} \quad (4)$$

$$F^\beta(\omega_n, \vec{p}) = \frac{-\Sigma_2^\beta(\omega_n, \vec{p})}{[\omega_n^2 Z_\beta^2 + \xi_\beta^2(\vec{p}) + \Sigma_2^{(\beta)^2}(\omega_n, \vec{p})]}.$$

Here  $\xi_{\alpha;\beta}$  is the energy of an ordinary electron referred to the Fermi level,  $Z_{\alpha;\beta}$  is the renormalization function, and  $\tilde{T}$  is the tunneling matrix element.<sup>18</sup> The Coulomb pseudopotential  $\mu^*$  can be included in the usual way.

Equations (1)–(3) are written for the case when  $g_\beta = 0$ . Hence, the existence of a pair condensate in the  $\beta$  film is due to the proximity effect only. The case  $g_\beta \neq 0$  is considered below (see Sec. VI).

The electron-phonon interaction is taken into account directly. We restrict ourselves to the consideration of the phonon model with weak coupling (see, e.g., Refs. 19 and 20). The consideration of the strong coupling is straightforward and will be given elsewhere.

The renormalization functions are connected (in the weak electron-phonon-coupling approximation) with the proximity effect only, and they are equal to (see Ref. 11)

$$Z_\alpha(\omega_n) = 1 + \Gamma^{\alpha\beta} / |\omega_n| \quad (4')$$

$$Z_\beta(\omega_n) = 1 + \Gamma^{\beta\alpha} / |\omega_n|. \quad (5)$$

Here

$$\Gamma^{\alpha\beta} = \pi \tilde{T}^2 v_\beta V_\beta, \quad (6)$$

$$\Gamma^{\beta\alpha} = \pi \tilde{T}^2 v_\alpha V_\alpha,$$

where  $v_\alpha$  and  $v_\beta$  are the densities of states (per unit of volume). The quantity  $\Gamma^{\beta\alpha}$  can be written in the form (see Ref. 11)

$$\Gamma^{\beta\alpha} = v_{F_1} \sigma / 2BL_\beta \quad (6')$$

where  $v_{F_1}$  is the Fermi velocity,  $\sigma$  is the barrier penetration probability, and  $B$  is a function of the ratio of the mean free path to the film thickness.

Our goal is to calculate  $T_c$ .  $T_c$  can be evaluated from Eqs. (1)–(4). If  $T = T_c$ , we should put  $\Sigma^\alpha = 0$  and  $\Sigma^\beta = 0$  in the denominators of (4). We introduce the function

$$C(\omega_n)|_{T_c} = [\Sigma_2(\omega_n)|_{T_c}] / (1 - T/T_c)^{1/2}.$$

According to Eqs. (1)–(4) we obtain

$$C^\alpha(\omega_n) = C_{2,\text{ph}}^\alpha(\omega_n) + \Gamma_{\alpha\beta} \frac{C^\beta(\omega_n)}{|\omega_n Z^\beta(\omega_n)|}, \quad (7)$$

$$C^\beta = \Gamma_{\beta\alpha} \frac{C^\alpha(\omega_n)}{|\omega_n Z^\alpha(\omega_n)|}. \quad (8)$$

The term  $C_{\text{ph}}^\alpha(\omega_n)$  is described by Eq. (3) and can be written in the form

$$C_{\text{ph}}^\alpha(\omega_n) = \pi T \sum_{\omega_n'} \int d\omega g_\alpha(\omega) \frac{\omega^2}{\omega^2 + (\omega_n - \omega_n')^2} \times \frac{C^\alpha(\omega_n')}{|\omega_n' Z^\alpha(\omega_n')|_{T_c}}. \quad (9)$$

We transformed to integration over frequencies and introduced the function (see, e.g., Refs. 19 and 20)

FIG. 1. The temperature self-energy parts.

$$g(\omega) = \frac{\xi}{p_F^2} \sum_j \frac{q dq}{d\omega} \frac{s_j^2 q^2}{\omega_j^2} \gamma_j, \quad (10)$$

where  $\xi$  is the Fröhlich parameter,  $q$  is the phonon momentum,  $s$  is the velocity of sound,  $p_F$  is the Fermi momentum,  $\gamma_j(q) \sim 1$ , and  $j$  is the order number of the phonon branch.

The function  $g(\omega)$  can be written in the well-known form<sup>21</sup>  $g(\omega) = \alpha^2(\omega)F(\omega)$ , where  $\alpha^2(\omega)$  describes the electron-phonon interaction. This function can be found from tunnel measurements.<sup>22</sup> Based on Eqs. (7) and (8), one can express  $C^\alpha$  in terms of  $C_{\text{ph}}^\alpha$ :

$$C^\alpha(\omega_n) = C_{\text{ph}}^\alpha \frac{(1 + \Gamma_{\alpha\beta}/\omega_n)(1 + \Gamma_{\beta\alpha}/\omega_n)}{1 + (\Gamma_{\alpha\beta} + \Gamma_{\beta\alpha})/\omega_n}. \quad (11)$$

Using this expression and Eq. (9) we arrive at the following equation:

$$1 = 2\pi T \sum_{\omega_n' > 0} \int d\omega g_\alpha(\omega) \frac{\omega^2}{\omega^2 + \omega_n'} \frac{1}{\omega_n'} \times \frac{1 + \Gamma_{\beta\alpha}/\omega_n'}{1 + (\Gamma_{\alpha\beta} + \Gamma_{\beta\alpha})/\omega_n'} |_{T_c}. \quad (12)$$

We took advantage of the weak-coupling approximation. If one calculates  $T_c$  in this approximation, one can neglect the term  $\omega_n$  in the denominator of the integrand in Eqs. (3) and (9). As was shown by the author in Ref. 19, this approximation is valid to within small corrections in the coefficient before the exponent in the expression describing  $T_c$ . This approximation allows us to neglect the dependence of  $C^\alpha$  on  $\omega_n$  and we arrive at Eq. (12).

Equation (12) allows us to calculate the critical temperature. It can be rewritten in the form

$$1 = \int d\omega g_\alpha(\omega) 2\pi T \sum_{\omega_n > 0} \frac{\omega^2}{(\omega^2 + \omega_n^2)\omega_n} - \Gamma_{\alpha\beta} \int d\omega g_\alpha(\omega) 2\pi T \sum_{\omega_n > 0} \frac{\omega^2}{(\omega^2 + \omega_n^2)\omega_n} \frac{1}{\omega_n + \Gamma}. \quad (13)$$

The sums on the right-hand side of Eq. (13) can be evaluated and we obtain

$$1 = I_1 + I_2, \quad (14)$$

$$I_1 = \int d\omega g_\alpha(\omega) \ln \frac{2\omega\gamma}{\pi T_c}, \quad (15)$$

$$I_2 = -\frac{\Gamma_{\alpha\beta}}{\Gamma} \int d\omega g_\alpha(\omega) \left\{ \left[ \psi \left( \frac{1}{2} + \frac{\Gamma}{2\pi T_c} \right) - \psi \left( \frac{1}{2} \right) \right] \frac{\omega^2}{\omega^2 + \Gamma^2} + \frac{\Gamma^2}{\omega^2 + \Gamma^2} \ln \frac{2\omega\gamma}{\pi T_c} \right\}, \quad (16)$$

where  $\Gamma = \Gamma_{\alpha\beta} + \Gamma_{\beta\alpha}$ . We neglect the small term  $\{-\pi\omega\Gamma/[2(\omega^2 + \Gamma^2)]\}$  in the brackets in Eq. (16);  $\gamma \simeq 1.78$ .

We see that the value of  $T_c$  depends on the function  $g_\alpha(\omega)$  and on the term  $I_2$  describing the proximity effect. According to Eqs. (6) and (16)

the term  $I_2$  depends on the density of states  $\nu^\beta$ . We shall consider (see below, Sec. VI) the case when  $\beta$  film is a semimetal or semiconductor. The interesting features of S-SM and S-SC systems are connected with the peculiarities of the density of states in these films. It is worth noting that the

function  $g_\alpha(\omega)$  differs from the function  $g_\alpha^0(\omega)$  describing the phonon spectrum and the electron-phonon interaction in an isolated metallic film. We consider thin metallic films and they are very sensitive to the covering due to the change of  $g_\alpha^0(\omega)$ .

The function  $g_\alpha(\omega)$  can be written in the form

$$g_\alpha(\omega) = g_\alpha^0(\omega) + g_\alpha^{(1)}(\omega), \quad (17)$$

$$\ln \frac{T_c}{T_c^\alpha} = \lambda_\alpha^{-1} \int d\omega g_\alpha^{(1)}(\omega) \ln \frac{2\omega\gamma}{\pi T_c^\alpha} - \frac{\Gamma_{\alpha\beta}}{\Gamma} \lambda_\alpha^{-1} \int d\omega g_\alpha(\omega) \left\{ \left[ \psi \left( \frac{1}{2} + \frac{\Gamma}{2\pi T_c} \right) - \psi \left( \frac{1}{2} \right) \right] \times \frac{\omega^2}{\omega^2 + \Gamma^2} + \frac{\Gamma^2}{\omega^2 + \Gamma^2} \ln \frac{2\omega\gamma}{\pi T_c} \right\}, \quad (19)$$

where

$$\lambda_\alpha = \int g_\alpha(\omega) d\omega. \quad (20)$$

Equations (14)–(16) [or Eq. (19)] are the basic equations of the theory. The first and second terms on the right-hand side of Eq. (19) describe the influence of different factors on  $T_c$ .

### III. THE CHANGE OF THE PHONON SPECTRUM

As is well known, covering results in a distortion of the initial phonon spectrum, in the appearance of new modes, and so on. It is particularly important in the investigation of thin superconducting films. The influence of covering can be considered by analogy with the author's paper<sup>23</sup> on the change in  $T_c$  in superconductors which contain complex molecules.

The function  $g_\alpha(\omega)$  can be written in the form Eq. (17). Suppose  $g_\alpha^{(1)} > 0$ ; that is, the covering leads to an additional attraction between electrons. The critical temperature  $T_c^\alpha$  of the isolated  $\alpha$  film satisfies Eq. (18). Denote by  $\Delta T_c \equiv T_c' - T_c^\alpha$  the change of the critical temperature caused by the term  $g_\alpha^{(1)}(\omega)$ . The quantity  $T_c'$ , which is equal to the value of the critical temperature in the absence of the proximity effect, satisfies the equation [see Eqs. (14) and (15)]

$$1 = \int d\omega g_\alpha(\omega) \ln \frac{2\omega\gamma}{\pi T_c'}. \quad (21)$$

According to Eq. (18) and (21) we obtain

where  $g_\alpha^{(1)}(\omega)$  is due to the influence of the covering. We denote by  $T_c^\alpha \equiv T_c(L_\beta=0)$  the critical temperature of an isolated  $\alpha$  film.  $T_c^\alpha$  is satisfied by the equation

$$1 = \int d\omega g_\alpha^0(\omega) \ln \frac{2\omega\gamma}{\pi T_c^\alpha}. \quad (18)$$

Using (18), we can reduce Eq. (14) to the form

$$\ln \frac{T_c'}{T_c^\alpha} = \frac{\lambda_1}{\lambda_\alpha} \ln \frac{2\langle\omega\rangle^{(1)}\gamma}{\pi T_c^\alpha}, \quad (22)$$

where  $\langle\omega\rangle^{(1)}$  denotes the mean value in the sense

$$\ln \frac{\langle\omega\rangle^{(1)}}{T_c^\alpha} = \frac{1}{\lambda_1} \int d\omega g_\alpha^{(1)}(\omega) \ln \frac{\omega}{T_c^\alpha},$$

$$\lambda_1 = \int d\omega g_\alpha^{(1)}(\omega),$$

and  $\lambda_\alpha$  is defined by Eq. (20). According to Eq. (22) we obtain

$$T_c' = T_c^\alpha (2\langle\omega\rangle^{(1)}\gamma/\pi T_c^\alpha)^{\lambda_1/\lambda_\alpha}. \quad (23)$$

Usually  $\lambda_1 \ll \lambda_\alpha$ . We find then

$$\frac{\Delta T_c}{T_c^\alpha} = \frac{\lambda_1}{\lambda_\alpha} \ln \frac{2\langle\omega\rangle^{(1)}\gamma}{\pi T_c^\alpha}. \quad (24)$$

If the function  $g_\alpha^{(1)}(\omega)$  corresponds to the appearance of an additional peak at  $\omega \simeq \omega_1$ , then  $\langle\omega\rangle^{(1)} \simeq \omega_1$  and we obtain

$$\frac{\Delta T_c}{T_c^\alpha} \simeq \frac{\lambda_1}{\lambda_\alpha} \ln \frac{2\omega_1\gamma}{\pi T_c^\alpha}. \quad (25)$$

The change of  $T_c$  can be noticeable because of the presence of large logarithmic factors. If, for example,  $\lambda_1/\lambda_\alpha \approx 0.1$  and  $T_c^\alpha/\omega_1 \approx 0.1$ , then  $\Delta T_c/T_c^\alpha \approx 25\%$ . Even if  $\lambda_1/\lambda_\alpha \approx 10^{-2}$  and  $T_c^\alpha/\omega_1 \approx 10^{-2}$ , the increase of  $T_c$  can reach several percent. Equations (24) and (25) relate  $\Delta T_c'$  to measurable quantities. The quantities  $\lambda_\alpha, \lambda_1$  can be obtained from tunnel measurements. If the covering is characterized by the existence of soft modes, this leads to an increase of  $\lambda_1$  and  $T_c$ . Size

quantization results in the appearance of an additional softening mechanism (see below, Sec. V).

The increase of  $T_c$  as a function of  $L_\beta$  relative to  $T_c^\alpha$  is observed often experimentally in the region of small  $L_\beta \sim a$  (Refs. 1–4) ( $a$  is atomic distance) by the investigation of the systems Tl-Ge,<sup>1</sup> Mo-C, Te-C, V-C,<sup>2</sup> Al-SiO,<sup>3</sup> Pb-Ge, and Pb-Si.<sup>4</sup> This increase is due to the considered mechanism's role in the region of small  $L_\beta$ . Note that this increase of  $T_c$  cannot be coupled with the term  $I_2$  in

$$\ln \frac{T_c}{T_c'} = -\frac{\Gamma_{\alpha\beta}}{\Gamma} \frac{1}{\lambda_\alpha} \int d\omega g(\omega) \left\{ \left[ \psi \left( \frac{1}{2} + \frac{\Gamma}{2\pi T_c} \right) - \psi \left( \frac{1}{2} \right) \right] \frac{\omega^2}{\omega^2 + \Gamma^2} + \frac{\Gamma^2}{\omega^2 + \Gamma^2} \ln \frac{2\omega\gamma}{\pi T_c} \right\}. \quad (26)$$

Here  $T_c'$  is the solution of Eq. (21) and corresponds to the value of  $T_c$  in the absence of the proximity effect. It is worth noting that, generally speaking,  $T_c' \neq T_c^\alpha$  (see Sec. III). The equality  $T_c' = T_c^\alpha$  is valid if it is possible to neglect the change of the phonon spectrum.

Consider the case  $\Gamma \gg T_c$ . Using Eq. (26) and the asymptotic expression of the digamma function [ $\psi(z) \simeq \ln z$ ], we obtain

$$\psi \left( \frac{1}{2} + \frac{\Gamma}{2\pi T_c} \right) - \psi \left( \frac{1}{2} \right) \simeq \ln \frac{2\Gamma\gamma}{\pi T_c}. \quad (27)$$

Hence, we arrive at the following equation:

$$\ln \frac{T_c}{T_c'} = -\frac{\Gamma_{\alpha\beta}}{\Gamma} \ln \frac{2\langle u(\omega) \rangle \gamma}{\pi T_c}, \quad (28)$$

where the mean value is to be understood in the sense

$$\ln \frac{\langle u(\omega) \rangle}{T_c} = \frac{1}{\lambda_\alpha} \int d\omega g_\alpha(\omega) \ln \frac{u(\omega)}{T_c} \quad (29)$$

and

$$u(\omega) = \Gamma^{1-\delta} \omega^\delta, \quad \delta = \frac{\Gamma^2}{\omega^2 + \Gamma^2}. \quad (30)$$

After simple calculations we finally obtain

$$T_c = T_c' \left[ \frac{\pi T_c'}{2\langle u \rangle \gamma} \right]^\rho, \quad (31)$$

where

$$\rho = \Gamma_{\alpha\beta} / \Gamma_{\beta\alpha}, \quad \Gamma = \Gamma_{\alpha\beta} + \Gamma_{\beta\alpha} = \Gamma_{\beta\alpha} (1 + \rho), \quad (32)$$

and  $T_c'$  and  $\langle u \rangle$  are defined by Eqs. (21) and (29).

Eq. (14) because the proximity effect tends to diminish the value of  $T_c$  (see below).

#### IV. PROXIMITY EFFECT

##### A. Critical temperature

Let us consider again the basic Eqs. (14)–(16). Equation (14) can be rewritten in the form

Equation (31) is valid for an arbitrary ratio of  $\langle \omega \rangle$  and  $\Gamma$ .

According to Eq. (6) we obtain

$$\rho = (\nu_\beta / \nu_\alpha) (L_\beta / L_\alpha). \quad (32')$$

Note that if films “ $\alpha$ ” and “ $\beta$ ” are ordinary metallic films, then (in the effective-mass approximation)

$$\nu_\alpha = m_\alpha p_F^\alpha / \pi^2, \quad \nu_\beta = m_\beta p_F^\beta / \pi^2, \quad (33)$$

and hence

$$\rho = (m_\beta p_F^\beta / m_\alpha p_F^\alpha) (L_\beta / L_\alpha). \quad (34)$$

Here  $m_\alpha$ ,  $m_\beta$  are the effective masses and  $p_F^\alpha$ ,  $p_F^\beta$  are the Fermi momenta. If film “ $\beta$ ” is a semimetal or degenerate semiconductor and is not size quantizing, Eqs. (31)–(34) are also valid. In this connection  $p_F^\beta \sim n^{1/3}$ , where  $n$  is the electron concentration. Then  $\rho$  and  $T_c$  can vary as functions of  $n$ . The special case of size quantization will be considered below (Sec. V).

It is very convenient to present the dependence Eq. (30) in the form

$$u = \Gamma \left[ \frac{\omega}{\Gamma} \right]^\delta \quad (35)$$

or in the following dimensionless form:

$$y = t^{(1/1+t^2)}, \quad (36)$$

where  $y = u/\Gamma$ ;  $t = \omega/\Gamma$ . The function  $y(t)$  is shown in Fig. 2. We see that if  $t$  is small ( $\omega \ll \Gamma$ ),  $y \simeq t$  and  $u \simeq \omega$ . In the opposite case of large  $t$  ( $\omega \gg \Gamma$ ), the quantity  $y \rightarrow 1$  and  $u \simeq \Gamma$ .

Consider Eq. (31) and examine the limiting cases. Equation (29) can be written in the form

$$\ln \frac{\langle u \rangle}{T_c} = K_1 \ln \frac{\Gamma}{T_c} + K_2 \ln \frac{\langle \omega \rangle}{T_c}, \quad (37)$$

where

$$K_1 = \frac{1}{\lambda_\alpha} \int d\omega g(\omega) \omega^2 / (\omega^2 + \Gamma^2), \quad (37')$$

$$K_2 = \frac{1}{\lambda_\alpha} \int d\omega g(\omega) \frac{\Gamma^2}{\omega^2 + \Gamma^2} \ln \left[ \frac{\omega}{T_c} \right] \bigg/ \ln \frac{\langle \omega \rangle}{T_c}.$$

Suppose that  $\Gamma \ll \langle \omega \rangle$ , or, more exactly,  $K_2 \ll 1$ . Then  $K_1 \simeq 1$  and, according to Eq. (37),  $\langle u \rangle \simeq \Gamma$ . Then we arrive at the following equation:

$$T_c = T_c' \left[ \frac{\pi T_c'}{2\Gamma\gamma} \right]^\rho, \quad (38)$$

where  $\rho$  is defined by Eq. (32). If the change of the phonon spectrum is small, then  $T_c' = T_c^\alpha$  and we obtain

$$T_c = T_c^\alpha \left[ \frac{\pi T_c^\alpha}{2\Gamma\gamma} \right]^\rho. \quad (38')$$

This expression was obtained by McMillan.<sup>11</sup> If  $\Gamma$  is large ( $\Gamma \gg \langle \omega \rangle$ ), or, more precisely,  $K_1 \ll 1$ , then  $K_2 \simeq 1$  and  $\langle u \rangle \simeq \langle \omega \rangle$ . Naturally, these estimates are consistent with the estimates of the function  $y(t)$  (see above). Then we arrive at the expression (if  $T_c' = T_c^\alpha$ ) which corresponds to Cooper's case<sup>11,24</sup>:

$$T_c = T_c^\alpha \left[ \frac{\pi T_c^\alpha}{2\langle \omega \rangle \gamma} \right]^\rho. \quad (39)$$

In the general case one should use Eqs. (28)–(31) which are valid for an arbitrary relation between  $\Gamma$  and  $\langle \omega \rangle$ .

If  $\Gamma \leq T_c$ ,  $\langle \omega \rangle \gg T_c$ , Eq. (31) is not applicable. According to Eq. (26), we obtain in this case

$$\ln(T_c/T_c') = -F(\rho, \Gamma/T_c), \quad (40)$$

where

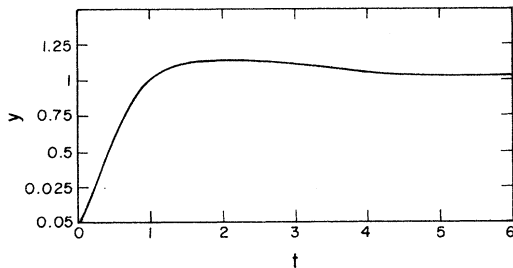


FIG. 2. Universal function  $y(t)$ ;  $y = u/\Gamma$ ,  $t = \omega/\Gamma$ .

$$F(\rho, \Gamma/T_c) = (\Gamma_{\alpha\beta}/\Gamma) \left[ \psi \left( \frac{1}{2} + \frac{\Gamma}{2\pi T_c} \right) - \psi \left( \frac{1}{2} \right) \right]$$

$$\rho = \Gamma_{\alpha\beta}/\Gamma_{\beta\alpha}.$$

The function  $F$  does not depend on the phonon spectrum directly. Equation (40) was considered in Ref. 11 (in the case when  $T_c' = T_c^\alpha$ ). Note that if  $\rho \ll 1$ , we obtain

$$T_c = T_c' \exp[-F(\rho, \Gamma/T_c')]. \quad (41)$$

According to Eq. (31),  $T_c$  strongly depends on the quantity  $\langle u \rangle$ . The value of  $\langle u \rangle$  [see Eqs. (29) and (30)] is related to  $\Gamma$  and to the function  $g(\omega)$ , or, roughly speaking, the value of  $\langle u \rangle$  depends on the relation between  $\Gamma$  and  $\langle \omega \rangle$ .  $\Gamma$  and  $\langle \omega \rangle$  can be varied noticeably and this results in a corresponding change of  $T_c$ .

## B. Dependence $T_c(d)$

The peculiar behavior of  $\langle u \rangle$  (see Fig. 2) allows us to propose the following experiment. The parameter  $\Gamma = \Gamma_{\alpha\beta} + \Gamma_{\beta\alpha}$  contains [see Eq. (6)] the tunneling matrix element and its value depends on the thickness of the barrier, e.g., on the thickness of the oxide,  $d$ . One should choose a superconductor with a small value of  $\langle \omega \rangle$ , that is, with a low phonon mode. A realistic situation is that of  $\Gamma \gg \langle \omega \rangle$  in the region of small  $d$ . The critical temperature is described by Eq. (31) with  $\langle u \rangle \simeq \langle \omega \rangle$ . Let the thickness “ $d$ ” increase gradually and consider the dependence  $T_c(d)$ . Gradual increase of  $d$  means the gradual decrease of the coupling. An increase in “ $d$ ” results in a decrease in  $\Gamma$ . The critical temperature does not change noticeably until the inequality  $\Gamma \gg \langle \omega \rangle$  holds. Then we come to the region  $d > d_c$ , the quantity  $\langle u \rangle \simeq \Gamma$  (see Fig. 2), and the subsequent increase of  $d$  and the corresponding decrease of  $\Gamma$  result, according to Eq. (31), in the dependence  $T_c(d)|_{d > d_c}$ . According to Eq. (31),  $T_c$  increases with an increase of  $d$  in the region  $d > d_c$ . The increase of  $T_c(d)$  described by Eq. (31) continues until  $\Gamma \gg T_c$ . When  $\Gamma \leq T_c$ , Eq. (31) is not applicable, but then we can use Eqs. (40) and (41). It is easy to see that if  $\Gamma \ll T_c$ , the critical temperature tends to  $T_c'$  (if it is possible to neglect the term  $g_\alpha^{(1)}$  which is due to the oxide, then  $T_c' = T_c^\alpha$ ). Hence, an increase in  $d$  gradually leads to the value  $T_c = T_c'$ , and this equality is natural for large values of  $d$ .

Hence, the change of  $T_c$  is described by the fol-

lowing. If  $d < d_c$ , the critical temperature remains almost constant. An increase in  $d$  in the region  $d > d_c$  is accompanied by a noticeable increase of  $T_c$  up to  $T'_c$ . We have seen that  $\Gamma(d_c) \simeq \langle \omega \rangle$ . If  $g(\omega)$  and the quantities  $\nu^\alpha$  and  $\nu^\beta$  are known, it is possible to calculate  $\langle \omega \rangle$  and  $\Gamma(d_c)$ . Then the dependence  $T_c(L_\beta)$  (if  $d = d_c$ ) can be verified experimentally.

### C. Maximum $T_c$

We see from Eqs. (26) and (31) that the proximity effect results in a decrease of  $T_c$ . The dependence  $T_c(L_\beta)$ , which is caused by the proximity effect, can be nonmonotonic (see below), but the inequality  $T_c < T'_c$  always holds. Hence, the contributions of the terms  $I_1$  (if  $g_\alpha^{(1)} > 0$ ) and  $I_2$  to a change of  $T_c$  [see Eq. (14)] have opposite signs. If  $L_\beta$  is small ( $L_\beta \sim a$ ,  $a$  is atomic distance), the contribution of  $I_2$  is small, because  $(\Gamma_{\alpha\beta}/\Gamma) \sim L_\beta/L_\alpha$ . The term  $I_1$  also contains the small parameter  $\lambda_1/\lambda^\alpha$  [see Eq. (24)], but a realistic situation is when the contribution of  $I_1$  dominates. Then the covering results in an increase of  $T_c$  relative to  $T_c^\alpha$  (in the region of  $L_\beta \sim a$ ). If  $L_\beta \gg a$ , the term  $I_2$  becomes more important. Indeed, the range of the effect of the phonons of the  $\beta$  film on the electrons of the  $\alpha$  film is limited by the atomic distance. On the other hand, the term  $I_2$  increases as  $L_\beta$  increases. Hence the quantity  $\partial T_c / \partial L_\beta |_{L_\beta \gg a}$

depends mainly on  $I_2$ , and  $T_c$  decreases with the increase of  $L_\beta$ . Therefore we come to the conclusion that  $T_c$  should have a maximum  $T_{cm} \sim T'_c$  in the region  $L_\beta \sim a$ .

A maximum of  $T_c$  in the region of small  $L_\beta$  was observed experimentally in Refs. 1–4 (see Fig. 3). Mikheeva *et al.*<sup>2</sup> have developed a very precise method of determination of the thickness of films. They observe  $T_{c_{max}}$  in the Mo-C, Te-C, and V-C systems when  $L_c \simeq 5 \text{ \AA}$ . The system Al-SiO was considered by Sixl.<sup>3</sup> The dependence of  $T_c$  on the

thickness of the SiO coating is characterized by  $T_{c_{max}}$ , when  $L_{\text{SiO}} \simeq 3 \text{ \AA}$ . The same result was obtained by Orlov *et al.* by the investigation of Pb-Ge and Pb-Si systems.<sup>4</sup> The first observation of  $T_{c_{max}}$  was made by Naugle<sup>11</sup> for the system Tl-Ge in the region of small  $L_{\text{Ge}} \simeq 10 \text{ \AA}$ . Note that the investigations<sup>2–4</sup> are characterized by a more precise determination of the thickness of the films. It is worth noting that the existence of this maximum  $T_c$  cannot be explained by the proximity effect because the proximity effect in the absence of a change of the phonon spectrum results in the inequality  $T_c < T_c^\alpha$ , which contradicts the experimental data.<sup>1–4</sup>

The appearance of  $T_{c_{max}}$  can be explained by the combination of the change of the phonon spectrum (term  $I_1$ ) and the proximity effect (term  $I_2$ ). Note that the absence of  $T_{c_{max}}$  in the Sn-Ge (Ref. 1) and Pb-C (Ref. 4) systems is connected with the small value of the function  $g_\beta^{(1)}$ .

According to our explanation, an increase of  $L_\alpha$  does not affect noticeably the position of the maximum, but it does affect the value of  $T_{c_{max}}$ . This conclusion is consistent with the experimental data.<sup>2–4</sup> If  $\beta$  film is not size-quantizing, the critical temperature is described by Eq. (31) or Eq. (38) (if  $\Gamma \ll \langle \omega \rangle$ ;  $\Gamma \gg T_c$ ). In this case  $\rho$  is described by Eq. (34) and depends significantly on  $L_\beta$ . The quantity  $\Gamma$  depends on  $L_\beta$  also [see Eqs. (6') and (32)], but the dependence  $\rho(L_\beta)$  is more essential. The dependence of  $\rho$  on  $L_\beta$  leads to a monotonic decrease of  $T_c$  with an increasing  $L_\beta$ . If  $\beta$  is a size-quantizing film, the situation changes drastically (see Sec. V).

## V. SIZE QUANTIZATION

### A. Density of states

According to Eqs. (31) and (32),  $T_c$  depends on the function  $\Gamma_{\alpha\beta}$ , which [see Eq. (6)] is related

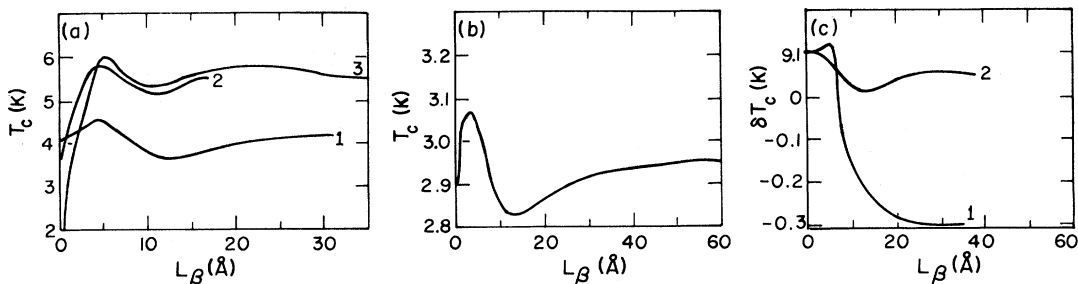


FIG. 3. Functional dependence  $T_c(L_\beta)$  for (a) 1—Al-SiO (Ref. 3),  $L_\alpha = 21 \text{ \AA}$ ; 2—Mo-C (Ref. 2),  $L_\alpha = 60 \text{ \AA}$ ; 3—Te-diamond (Ref. 2),  $L_\alpha = 50 \text{ \AA}$ , (b) Al-Si (Ref. 3),  $L_\alpha = 70 \text{ \AA}$ , (c) 1—Pb-Ge (Ref. 4), 2—Pb-Si (Ref. 4),  $\delta T_c = T_c - T_c^\alpha$ .

directly to the density of the states of the nonmetallic film. Consider now a realistic situation when the semimetal (or semiconductor) film is size-quantizing. As is well-known (see, e.g., review<sup>10</sup>), size quantization (SQ) is very sensitive to the structure of the film and to the quality of its surface. The best conditions for SQ are realized in a semimetal film where a number of factors (such as low electron density and a small value of the transverse effective mass) cause the de Broglie wavelength to greatly exceed the atomic distance; it is this which makes the surface, in fact, specular.

SQ is observed experimentally by the investigation of films of Bi,<sup>6,7</sup> Sb,<sup>8</sup> and InSb.<sup>9</sup> The observation of this effect in thin metallic films is also possible (see, e.g., Refs. 25 and 26), but is considerably more complicated. Consider a thin semimetal (or semiconductor) film. SQ results in a situation where the energy  $\epsilon(\vec{\kappa}, n)$  is determined by the longitudinal two-dimensional quasimomentum  $\vec{\kappa}$  and by the transverse quantum number  $n$ . Instead of a Fermi surface we have a group of two-dimensional subbands. The density of states can be written in the form (see, e.g., Ref. 10)

$$\nu_{\beta} = \sum_i \nu_{\beta}^i,$$

where

$$\nu_{\beta}^i = \frac{1}{L_{\beta}} \frac{\tilde{m}^i}{\pi \hbar^2} r, \quad \tilde{m}^i = (m_1^i, m_2^i)^{1/2}. \quad (42)$$

The summation is taken over the filled subbands;  $r$  is the number of two-dimensional valleys. The dispersion relation has (in the effective-mass approximation) the following form:

$$\begin{aligned} \epsilon(\vec{\kappa}, n_1) &= \frac{\hbar^2}{2m_1} (\kappa_x - \kappa_{x0})^2 \\ &+ \frac{\hbar^2}{2m_2} (\kappa_y - \kappa_{y0})^2 + \epsilon_1. \end{aligned}$$

If only one subband is filled, the density of states is equal to

$$\nu_{\beta} = \frac{1}{L_{\beta}} \frac{\tilde{m}}{\pi \hbar^2} r. \quad (43)$$

Indeed, if only one subband is filled, the number of states in the region  $\kappa, \kappa + d\kappa$  is equal to  $4\pi\kappa d\kappa S / (2\pi\hbar)^2$  ( $S$  is the area of the film;  $\epsilon = \kappa^2/2m$ ). The density of states (per unit volume) is equal to  $\nu = mL^{-1}/\pi\hbar^2$  ( $L$  is the thickness of the film) and decreases with an increase of  $L$ . If the film is characterized by several filled

subbands and by several valleys, we come to Eq. (42).

The number of filled subbands  $\nu$  depends on the electron concentration "n". If  $n < L^{-3}$ , only the lowest subband is filled (more exact conditions are given below). For example, in Bi films only one subband remains filled up to  $L \sim 5 \times 10^2$  Å. The film, which remains a three-dimensional system in coordinate space ( $L \gg a$ ) becomes a two-dimensional system in momentum space, and this results in peculiar behavior.

An increase of  $L$  leads to a decrease of the distance between transverse levels. When  $L$  becomes equal to some value  $L_c$ , the next subband begins to be filled, and this is accompanied by a jump of the density of states. Hence, the density of states is an oscillating function of  $L$ .<sup>10,27,28</sup>

Oscillations of the density of states  $\nu_{\beta}$  result, according to Eqs. (6), (31), (32), and (42), in a non-monotonic dependence  $T_c(L_{\beta})$ . The values  $L_{\beta c}$  correspond to jumps of the density of states. They depend on the dispersion relation. To estimate their values we use a simple model, where the longitudinal motion is described by the relation

$$\epsilon = \kappa^2/2\tilde{m}, \quad \Delta\epsilon_1^{(1)} = 3\pi^2\hbar^2/2m_1L_{\beta}^2 \quad (44)$$

(the potential box or the quantization

$\kappa_1 = \pi\hbar n/L_{\beta}$ ). The second subband begins to be filled if  $L_{\beta}$  satisfies the condition

$$\Delta\epsilon^{(1)} = \epsilon_F,$$

where  $\epsilon_F$  is the Fermi level.

It is necessary to take into account the fact that  $\kappa_F = (2\tilde{m}\epsilon_F)^{1/2}$  also depends on  $L_{\beta}$ . The electron concentration  $n$  is equal to  $n = \kappa_F^2/2\pi\hbar^2L_{\beta}$ . Therefore

$$\kappa_F^2 = 2\pi\hbar^2nL_{\beta}.$$

Based on Eqs. (44), we obtain the following value for  $L_{\beta c}^{(1)}$ :

$$L_{\beta c}^{(1)} = a(\tilde{m}/m_1n)^{1/3}, \quad a \simeq 1.7. \quad (45)$$

If the condition  $nL_{\beta}^3 < a^3(\tilde{m}/m_1)$  [see Eq. (45)] is satisfied, only the lowest subband is filled. Let us examine the value  $L_{\beta c}^{(2)}$ , which corresponds to the next rapid change of  $T_c$ . Generally speaking, values of the masses for different subbands can be different. Solving the equation

$$\begin{aligned} \Delta E^{(2)} &= \kappa_{F2}^2/2\tilde{m}_2, \\ (\kappa_{F2}^2/2\tilde{m}_2) + \Delta E^{(1)} &= \kappa_{F1}^2/2\tilde{m}_1, \\ \kappa_{F1}^2 + \kappa_{F2}^2 &= 2\pi\hbar^2nL \end{aligned} \quad (46)$$



results in the following value for  $\Delta L = L_{\beta c}^{(2)} - L_{\beta c}^{(1)}$ :

$$\Delta L = a \left[ \frac{\tilde{m}_1}{m_1 n} \right]^{1/4} \{ b [1 + c (\tilde{m}_2 / \tilde{m}_1)]^{1/3} - 1 \}, \quad (47)$$

where  $a, b, c \sim 1$  (in the model of the box  $a \approx 1.7$ ,  $b \approx 1.4$ ,  $c \approx 0.6$ ). For example,  $\Delta L \approx 30 \text{ \AA}$  for Sb films.<sup>8,10</sup> The quantity  $\Delta L$  in the Bi films can be changed in the region  $10^2 \text{ \AA} < \Delta L < 4 \times 10^2 \text{ \AA}$ , depending on the structure of the film.<sup>6</sup> We see that  $\Delta L \neq L_1$  and the oscillations are aperiodic.

### B. Nonmonotonic behavior of $T_c$

The critical temperature is described by Eqs. (26), (31), and (40). If the " $\beta$ " film is a semimetal (or semiconductor) film with SQ, the densities of states  $\nu_\alpha$  and  $\nu_\beta$  are described by Eq. (33) and Eq. (42), respectively.

SQ causes  $\rho$  and, consequently,  $T_c$  to change step-wise. If  $L_\beta$  becomes equal to  $L_{\beta c}^{(1)}$  [see Eq. (45)],  $\nu_\beta$  increases very rapidly and this leads to a rapid decrease of  $T_c$ . The detailed picture of the dependence  $T_c(L_\beta)$  varies with the conditions of the experiment. Consider the case when the ratio  $L_\beta/L_\alpha = \text{const}$ . If  $L_\beta < L_{\beta c}^{(1)}$ , the quantity  $\nu_\beta$  decreases with the increase of  $L_\beta$  [see Eq. (42)], and one can observe an increase in  $T_c$  (of course, it always holds that  $T_c < T_c'$ ). When  $L_\beta$  becomes equal to  $L_{\beta c}^{(1)}$ ,  $\nu_\beta$  makes a stepwise increase and  $T_c$  decreases, according to Eq. (31) (if  $\Gamma \gg T_c$ ). Hence an appearance of a maximum of  $T_c$  is possible. This second maximum differs from the maximum of  $T_c$  in the region  $L_\beta \sim a$  (see Sec. III). The subsequent increase of  $L_\beta$  in the region  $L_\beta > L_{\beta c}^{(1)}$  again leads to a decrease of  $\nu_\beta$  and an increase of  $T_c$ , and this continues until  $L_\beta = L_{\beta c}^{(2)}$ , and so on (see Fig. 4). Hence SQ leads to the possibility of oscillations of  $T_c$ . If  $L_\alpha = \text{const}$  (this case corresponds to the typical experimental situation; see Refs. 1–4), then [see Eqs. (32') and (42)]

$$\rho \approx \frac{\pi \tilde{m}_\beta r \nu_0}{m_\alpha p_F^\alpha L_\alpha} \quad (48)$$

[cf. Eq. (34)], where  $\nu_0$  is the number of filled subbands.

We would like to emphasize that SQ leads to a peculiar dependence of  $\rho(L_\beta)$  on  $L_\beta$  [cf. Eq. (34)]. According to Eq. (48),  $\rho(L_\beta)$  is a step-wise function and, if  $L_\beta < L_{\beta c}^{(1)}$  or  $L_{\beta c}^{(1)} < L_\beta < L_{\beta c}^{(2)}$ , etc. [see Eqs. (45) and (47)], the quantity  $\rho$  does not depend

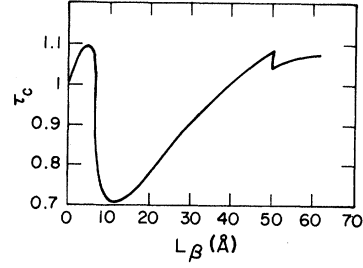


FIG. 4. The dependence  $T_c(L_\beta)$  ( $L_\alpha/L_\beta = \text{const}$ ,  $\tau_c = T_c/T_c^\alpha$ ,  $\tilde{\Gamma} = \Gamma/T_c^\alpha$ ,  $\tau_c' = T_c'/T_c^\alpha$ ) for parameters  $\rho(L_\beta = 20 \text{ \AA}) = 0.05$ ,  $\Gamma(L_\beta = 20 \text{ \AA}) = 40$ ,  $\lambda'/\lambda = 0.02$ ,  $\langle \omega \rangle / T_c^\alpha = 10^2$ .

on  $L_\beta$  at all. Then the dependence  $T_c(L_\beta)$  is caused only by the dependence  $\Gamma(L_\beta)$  [see Eqs. (6') and (38)]; it is assumed that  $\Gamma \ll \langle \omega \rangle$ . According to Eqs. (6'), (32), and (38), the increase of  $L_\beta$  results in an increase in  $T_c$ . This increase continues until  $L_\beta = L_{\beta c}^{(1)}$ . Then we get  $\nu_0 = 2$ , and  $T_c$  makes a stepwise decrease and so on. Hence the dependence  $T_c(L_\beta)$  becomes nonmonotonic.

Moreover, if  $\Gamma \gg \langle \omega \rangle$ , one should use Eq. (39). Then an increase of  $L_\beta$  does not affect  $T_c$  until the decrease of  $\Gamma_{\beta\alpha}$  [see Eq. (6')] gradually results in a situation where  $\Gamma < \langle \omega \rangle$ . Then the dependence  $T_c(L_\beta)$  is described by Eqs. (31), (38), and (40), and  $T_c(L_\beta)$  becomes an increasing function. Therefore, SQ leads to a nonmonotonic dependence  $T_c(L_\beta)$ . Note that the distance between the maximum  $T_c$  in the region  $L_\beta \sim a$  (see Sec. III) and the maximum  $T_c$ , if  $L_\beta = L_{\beta c}^{(1)}$ , is not equal to the period of oscillation because they have different origins. As is known (see, e.g., Ref. 10), the thin film can be size-quantizing if its thickness  $L_\beta > 10 \text{ \AA}$ , and, hence, the described increase of  $T_c$  can be observed if  $L_\beta > 10 \text{ \AA}$ . Therefore,  $T_c$  has a minimum in the region  $L_\beta \sim 10 \text{ \AA}$ .

The nonmonotonic dependence  $T_c(L_\beta)$  [see Sec. III, Eqs. (6'), (38), and (48)] has been observed experimentally (see Fig. 1) by the investigation of Mo, Te, and V films covered by C (Ref. 2) and the systems Al-SiO (Ref. 3) and Pb-Si.<sup>4</sup> (For a general description of the experimental situation see Ref. 29. The oscillations in the region  $L_\beta > 10 \text{ \AA}$  are observed only in those cases when the  $\beta$  films are semiconductor or semimetal.) Note that SQ is not a universal phenomenon and it is very sensitive to the structure of the films (see, e.g., Ref. 10). It is not surprising that the increase of  $T_c(L_\beta)$  has not been discovered by the investigation of some systems [e.g., Pb-Ge, Pb-C (Ref. 4), Tl-Ge, and Sn-Ge (Ref. 1)]. In the absence of SQ,  $T_c$  decreases

with increased  $L_\beta$ , in accordance with Eqs. (34) and (38). Of course, it is necessary to have information about phonon and electron spectra of the films in order to make detailed comparison with experimental data.<sup>1-4</sup> We present the theoretical dependence  $T_c(L_\beta)$  (see Fig. 5) for some values of the parameters. The quantity  $\Gamma^{\beta\alpha}$  is described<sup>11</sup> by Eq. (6') (see also Ref. 22). The S-N sandwiches with Cu and Ag normal layers have been studied in Ref. 12. According to measurements in Ref. 12, the best description of the experimental data is obtained if the values  $\Gamma_M^{\beta\alpha}$  are the following:  $\Gamma_M^{\beta\alpha} = 8$  mV ( $L_\beta = 100$  Å),  $\Gamma_M^{\beta\alpha} = 4$  mV ( $L_\beta = 200$  Å), and  $\Gamma_M^{\beta\alpha} = 2$  mV [ $L_\beta = 400$  Å; cf. Eq. (6')], where  $\Gamma_M^{\beta\alpha} \equiv \Gamma^{\beta\alpha}$  for metallic film. We discussed the method of obtaining the value  $\Gamma^{\beta\alpha}$  (see Sec. IV B), which can be applied if  $\beta$  film is SM or SC film also. It is possible to estimate the values  $\Gamma_{SQ}^{\beta\alpha}$  ( $\Gamma_{SQ}^{\beta\alpha} \equiv \Gamma^{\beta\alpha}$  for size-quantizing film) using Eq. (6') and data<sup>12</sup>:

$$\Gamma_{SQ}^{\beta\alpha} \sim p_{F_1} \sigma / m_\perp L_\beta \sim \pi \hbar \sigma / m_\perp L_\beta^2 \\ \sim \Gamma_M^{\beta\alpha} (a/L_\beta) (m_\perp^* / m_\perp).$$

Here  $m_\perp^*$  and  $m_\perp$  are the effective transverse masses of the metallic and SM (or SC) films. We consider the case where only the lowest subband is filled and  $p_{F_1} \sim \pi \hbar / L_\beta$ ; in the metal film  $p_F \sim \hbar / a$ . The smallness  $a/L_\beta$  can be compensated by the ratio  $m_\perp^* / m_\perp$ . For example, in Bi film  $m_\perp = 0.01 m_e$ . If, for instance,  $L_\beta = 75$  Å, the value  $\Gamma^{\beta\alpha}$  in Bi film is  $\Gamma^{\beta\alpha} \sim 15$  mV. We present the theoretical dependence  $T_c(L_\beta)$  (see Fig. 5) for some values of the parameters.

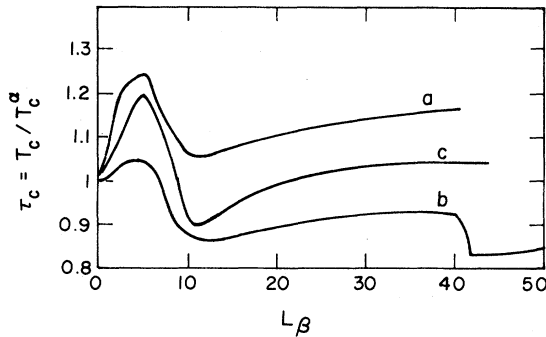


FIG. 5. Transition temperature  $T_c$  versus  $L_\beta$  ( $L_\alpha = \text{const}$ ) for (a)  $\rho = 0.05$ ,  $\bar{\Gamma}(L_\beta = 30 \text{ Å}) = 10$ ,  $(\lambda_1/\lambda) = 0.09$ ,  $\langle \omega \rangle / T_c^\alpha = 15$ ; (b)  $\rho = 0.05$ ,  $\bar{\Gamma}(L_\beta = 20 \text{ Å}) = 20$ ,  $\lambda_1/\lambda = 0.01$ ,  $\langle \omega \rangle / T_c^\alpha = 150$ ; (c)  $\rho = 0.1$ ,  $\bar{\Gamma}(L_\beta = 20 \text{ Å}) = 10$ ,  $\lambda_1/\lambda = 0.04$ ,  $\langle \omega \rangle / T_c^\alpha = 10^2$ .

### C. Charge-density waves

SQ results in appearance of peculiar charge-density waves. This problem has been considered by Kokotov and the author in Ref. 30. This kind of instability can affect the properties of S-SM and S-SC systems.

Consider the case when only the lowest subband is filled. The film is characterized by the Fermi line (see above, Sec. V A). If several subbands are filled, there is a set of Fermi lines. A perfectly realistic situation is one in which the Fermi line has linear sections. For example, the radius curvature of sections of the Fermi line in Bi films (see Ref. 6) is larger than the dimension of the Fermi line itself and even the dimension of the Brillouin zone. These sections can be regarded as straight lines with high degree of accuracy.

The electron-phonon interaction leads to instability of the lattice in the presence of the linear sections of the Fermi line.<sup>30</sup> The calculation of the polarization operator

$$\Pi(q) = 2i \int dp G(p + q/2) G(p - q/2)$$

$$[q = (\vec{q}_{||}, \omega), p = (\vec{\kappa}, \epsilon)]$$

leads to the expression<sup>30</sup>

$$\Pi \sim q_0 \ln(8\epsilon_F / i\omega) \quad (49)$$

which contains a logarithmic singularity (it is assumed that the linear section corresponds to the region  $|\kappa_x| < q_0$ ). The presence of a logarithmic singularity in  $\Pi$  leads to the appearance of an imaginary pole in the phonon Green's function  $D = D_0^{-1} - g\Pi g^\epsilon [g^\epsilon = (1 + V\Pi)^{-1}]$  (see, e.g., Ref. 31), and to lattice instability. The temperature  $T_p$  which corresponds to the appearance of static-deformation waves and the structural transition is

$$T_p \simeq \epsilon_F e^{-1/\lambda}. \quad (50)$$

The SM and SC films are characterized by a small value of  $\epsilon_F$  (e.g., for Bi films  $\epsilon_F \simeq 10^{-2}$  eV) relative to  $\epsilon_F$  of metals. That is why the value  $T_p$  is small ( $T_p \lesssim 1^\circ$ ). The minimum of the resistance, which was observed in an experimental investigation of thin Bi films in the low-temperature region,<sup>32</sup> is described by Eq. (50).

The temperature  $T = T_p$  is characterized by the appearance of a static-deformation wave. If  $T > T_p$ , the gradual decrease of  $T$  leads to a softening of the phonon mode. Because of smallness of  $T_p$  in the SM and SC size-quantizing films, one can consider the situation when  $T_c$  of the S-SM or

S-SC system is larger than  $T_p$ . Then the region  $T \sim T_c$  is characterized by the existence of a low-phonon mode, and this results in an increase of the constant describing the electron-phonon interaction in the "β" film. The value of this constant can exceed the value of the Coulomb pseudopotential and the effective constant  $\lambda_\beta$  becomes different from zero.

## VI. THE CASE $\lambda_\beta \neq 0$

Previously we considered the case  $g_\beta = 0$ . Suppose that film "β" is characterized by a nonzero value of  $g_\beta$ . This means that the value of the effective constant  $\lambda_\beta$  describing the electron-phonon interaction in film "β" exceeds the value of the Coulomb pseudopotential  $\mu^*$ . Then one should write the following equations  $T = T_c$  [cf. Eqs. (7) and (8)]:

$$C^\alpha(\omega_n) = C_{\text{ph}}^\alpha(\omega_n) + \Gamma_{\alpha\beta} C^\beta(\omega_n) / (|\omega_n| Z^\beta), \quad (51)$$

$$C^\beta(\omega_n) = C_{\text{ph}}^\beta(\omega_n) + \Gamma_{\beta\alpha} C^\alpha(\omega_n) / (|\omega_n| Z^\alpha), \quad (52)$$

where the additional term is equal to

$$C_{\text{ph}}^\beta(\omega_n) = \pi T \sum_{\omega_n' > 0} \int d\omega g_\beta(\omega) \frac{\omega^2}{\omega^2 + (\omega_n - \omega_n')^2} \times \frac{C^\beta(\omega_n')}{|\omega_n'| Z^\beta(\omega_n')}. \quad (53)$$

Equation (52) can be written in the form

$$C^\beta(\omega_n) = \Gamma_{\beta\alpha} C^\alpha / (|\omega_n| Z^\alpha) + R, \quad (54)$$

where

$$R \simeq \Gamma_{\beta\alpha} \pi T \sum_{\omega_n'} \int d\omega g_\beta(\omega) \frac{\omega^2}{\omega^2 + \omega_n'^2} \frac{C^\alpha(\omega_n')}{\omega_n'^2 Z^\alpha(\omega_n') Z^\beta(\omega_n')}. \quad (55)$$

Therefore, we see that Eq. (54) for the function

$$\ln \frac{T_c}{T_c'} = - \frac{\Gamma_{\alpha\beta}}{\lambda_\alpha \Gamma} \int d\omega g_\alpha(\omega) f(\omega, \Gamma, T_c) + \frac{\Gamma_{\alpha\beta} \Gamma_{\beta\alpha}}{\lambda_\alpha \Gamma^2} \int d\omega g_\alpha(\omega) f(\omega, \Gamma, T_c) \int d\omega g_\beta(\omega) f(\omega, \Gamma, T_c), \quad (62)$$

where [cf. Eqs. (26) and (27)]

$$f(\omega, \Gamma, T_c) = \frac{\omega^2}{\omega^2 + \Gamma^2} \ln \frac{2\Gamma\gamma}{\pi T_c} + \frac{\Gamma^2}{\omega^2 + \Gamma^2} \ln \frac{2\omega\gamma}{\pi T_c} \quad (63)$$

$C^\beta$  contains, besides the usual term [see Eq. (8)], the additional term  $R$ . Substituting Eqs. (54) and (55) into Eq. (51) and performing calculations by analogy with the derivation of Eq. (13), we arrive at the following equation:

$$1 = 2\pi T \sum_{\omega_n > 0} \int d\omega g_\alpha(\omega) \frac{\omega^2}{\omega^2 + \omega_n^2} \frac{K(\omega_n)}{\omega_n Z^\alpha(\omega_n)}, \quad (56)$$

where

$$K(\omega_n) = \frac{1}{S(\omega_n)} + \frac{\Gamma_{\alpha\beta} \Gamma_{\beta\alpha}}{\omega_n Z^\beta(\omega_n) S(\omega_n)} \times 2\pi T \sum_{\omega_n' > 0} \int d\omega g_\beta(\omega) \frac{\omega^2}{\omega^2 + \omega_n'^2} \times \frac{1}{\omega_n'^2 Z^\alpha(\omega_n') Z^\beta(\omega_n') S(\omega_n')}. \quad (57)$$

Here

$$S(\omega_n) = 1 - \Gamma_{\alpha\beta} \Gamma_{\beta\alpha} / \omega_n^2 Z^\alpha(\omega_n) Z^\beta(\omega_n). \quad (58)$$

According to Eqs. (4) and (5), we obtain

$$S^{-1}(\omega_n) = Z^\alpha(\omega_n) Z^\beta(\omega_n) / (1 + \Gamma/\omega_n). \quad (59)$$

According to Eqs. (13), (15), (57), and (59), Eq. (56) can be reduced to the form

$$1 = \int d\omega g_\alpha(\omega) \ln \frac{2\omega\gamma}{\pi T_c} - \frac{\Gamma_{\alpha\beta}}{\Gamma} L_\alpha + \frac{\Gamma_{\alpha\beta} \Gamma_{\beta\alpha}}{\Gamma^2} L_\alpha L_\beta, \quad (60)$$

where

$$L_i = 2\pi \Gamma T \sum_{\omega_n > 0} \int d\omega g_i(\omega) \frac{\omega^2}{\omega^2 + \omega_n^2} \frac{1}{\omega_n(\omega_n + \Gamma)}, \quad i \equiv \{\alpha, \beta\} \quad (61)$$

Using Eq. (19) and summing in Eq. (61) over  $\omega_n$ , we obtain

and  $T_c'$  is defined by Eq. (21); it is assumed that  $\Gamma \gg T_c$ . Equation (62) can be written in the form

$$\ln \frac{T_c}{T_c'} = - \frac{\Gamma_{\alpha\beta}}{\Gamma} \ln \frac{2\gamma \langle u \rangle_\alpha}{\pi T_c} + \phi \quad (64)$$

where

$$\phi = \frac{\Gamma_{\alpha\beta}\Gamma_{\beta\alpha}}{\Gamma^2} \lambda_{\beta} \ln \frac{2\gamma\langle u \rangle_{\alpha}}{\pi T_c} \ln \frac{2\gamma\langle u \rangle_{\beta}}{\pi T_c}. \quad (65)$$

The quantity  $u$  is defined by Eqs. (30) and (35), and  $\langle u \rangle_i$  denotes the mean value in the sense

$$\ln \frac{\langle u \rangle_i}{T_c} = \frac{1}{\lambda_i} \int d\omega g_i(\omega) \ln \frac{u}{T_c}.$$

If  $\Gamma \ll \langle \omega \rangle_{\alpha}$ , and  $\Gamma \ll \langle \omega \rangle_{\beta}$ , then  $\langle u \rangle_i = \Gamma$  (see above). Then

$$\phi \simeq \lambda_{\beta} \frac{\Gamma_{\alpha\beta}\Gamma_{\beta\alpha}}{\Gamma^2} \left[ \ln \frac{2\Gamma\gamma}{\pi T_c} \right]^2. \quad (66)$$

The critical temperature can be evaluated from Eq. (64). Note that the terms on the right-hand side of Eq. (64) have opposite signs. Therefore the inequality  $\lambda_{\beta} \neq 0$  results in an increase of  $T_c$  relative to the situation when  $\lambda_{\beta} = 0$ .

If, for example,  $\lambda_{\beta}$  is small [ $T_c^{\beta} \ll T_c^{\alpha}$ ;  $\lambda_{\beta} \simeq \ln^{-1}(\langle \omega \rangle_{\beta}/T_c^{\beta})$ ] and  $\Gamma_{\alpha\beta} \ll \Gamma$ , the increase of  $T_c$  due to the second term on the right-hand side of Eq. (64) is equal to

$$\frac{\Delta T_c}{T_c} \sim \lambda_{\beta} \frac{\Gamma_{\alpha\beta}}{\Gamma} \left[ \ln \frac{2\Gamma\gamma}{\pi T_c} \right]^2. \quad (67)$$

The smallness of the factor  $\lambda_{\beta}\Gamma_{\alpha\beta}/\Gamma$  is compensated to some extent by the large logarithmic factor  $(\ln 2\Gamma\gamma/\pi T_c)^2$ . If, for example,  $\langle \omega \rangle_{\beta} \sim 10$  mV,  $T_c^{\alpha} \sim 10^{-3}$ ,  $\Gamma_{\alpha\beta}/\Gamma \sim 0.1$ , and  $\Gamma/T_c \sim 10$ , we obtain  $(\Delta T_c/T_c) \sim 5\%$ . Hence, we see that the inequality  $\lambda_{\beta} \neq 0$  leads to an increase of  $T_c$ .

## VII. DISCUSSION

The S-SM and S-SC systems are characterized by a number of parameters. A variety of parameters allows one to change the properties and, especially,  $T_c$  in the desired direction. The critical temperature of S-SM and S-SC systems in the presence of size quantization is described by Eqs. (19), (31), (41), and (48). In the absence of SQ one should use Eq. (34) instead of Eq. (48). The relation  $T_c' > T_c^{\alpha}$  is caused by the change of the phonon spectrum (see Sec. III). On the other hand, the proximity effect tends to decrease the critical temperature. SQ leads to oscillations of the density of states and, hence, to oscillations of  $T_c$ . The combination of these factors results in complex nonmonotonic dependence  $T_c(L_{\beta})$ . Estimates (see above) show that the quantitative change of  $T_c$  can

be very noticeable. Equations (19), (29)–(32'), and (48) allow one to carry out detailed calculations. These equations express the value of  $T_c$  in terms of measured quantities. Namely, the value of  $T_c$  depends on the function  $g(\omega)$  and the properties of the dispersion relations of films in the presence of SQ.

As is well known there are several ways to determine the function  $g(\omega)$ . The most powerful method is the method of tunnel spectroscopy. A very interesting investigation was carried out by Chaikin, Arnold, and Hansma.<sup>12</sup> The authors of Ref. 12 have studied the system superconductor-normal metal. They took advantage of the proximity effect to get information about the quantity  $\lambda_n$  describing the electron-phonon interaction (EPI) and the phonon spectrum of the normal metal. A similar investigation of semimetal and semiconductor films would be very interesting. One can get the same interesting information about the EPI and the phonon spectrum in these types of solids. Moreover, the change of the thickness of the oxide (see, e.g., Ref. 33) allows one to get information about the function  $g_{\alpha}^{(1)}$  (see Sec. III).

Investigation of the dispersion relation in size-quantizing thin films is a very interesting problem. This problem was discussed by Kokotov and the author in Ref. 30. As was mentioned above, in the presence of SQ, electrons are characterized not by a Fermi surface but by a Fermi line. The Fermi line can differ from the usual section through a three-dimensional Fermi surface because of the specifics of the film state brought about by sputtering conditions and, moreover, because, strictly speaking, the transverse quasimomentum is not defined for these thin films. One can suggest (see Ref. 30) several methods (e.g., investigation of sound absorption in a magnetic field, absorption of an electromagnetic field, and so on) which allow one to reconstruct the Fermi line by using experimental data. The development in this direction is important, because it allows one to study the properties of the film state. In our case, it will be possible to calculate the density of states  $\nu_{\beta}$  and the corresponding contribution to the change in  $T_c$ .

The best conditions for observations of SQ are realized in the SM and SC films. In principle, it is possible, although more complicated, to observe SQ in thin metallic films.<sup>25,26</sup> The effect of SQ on  $T_c$  was considered by Blatt<sup>27</sup> and by Tavger and the author.<sup>28</sup>

It is supposed in this paper that the metallic film is not size-quantizing. SQ of the metallic

film with a number of filled subbands can also affect  $T_c$  of S-SC and S-SM systems. This additional mechanism was considered in Ref. 29. It was assumed that it is possible to separate variables; this corresponds to specular reflection. The change of  $T_c$  is caused by the boundary conditions.<sup>29</sup> This mechanism is not connected with oscillations of the density of states of SM or SC films and with EPI, and is very sensitive to the quality of the metallic film.

We have discussed (see above) the effect of different factors on the dependence  $T_c(L_\beta)$ . The nonmonotonic dependence  $T_c(L_\beta)$  was observed experimentally. A whole set of experimental data can be explained in terms of the present theory (see Secs. IV C, and VI). Hence, we see that the investigation of S-SM and S-SC systems in the presence of SQ leads to the possibility of peculiar changes of  $T_c$ . The development in this direction and subsequent experiments promise to be very interesting.

#### SUMMARY

In this paper we develop an approach which allows an evaluation of  $T_c$  for different systems (S-SM, S-SC, S-S, superconductor-normal metal). The electron-phonon interaction is taken into account directly. The results of the paper can be summarized as follows.

(1) We have considered S-SM and S-SC systems containing thin superconducting and semimetal (or semiconductor) films. Using the method of thermodynamic Green's functions, we considered the

influence of the proximity effect, size quantization, and the change of the phonon spectrum on  $T_c$ .

(2) The dependence of  $T_c$  on the thickness of nonmetallic films is nonmonotonic. One can observe  $T_{c_{\max}}$  in the region  $L_\beta \sim a$ .

(3) Equation (31) describing  $T_c$  in the framework of the phonon model in the presence of the proximity effect has been obtained. The peculiar dependence  $T_c(d_{\text{bar}})$  was discussed.

(4) Size quantization leads to oscillations of  $T_c(L_\beta)$ .

(5) Features of the dispersion relation in a size-quantizing film and the appearance of charge density-waves were discussed.

(6) The increase of  $T_c$  caused by the inequality  $\lambda_\beta \neq 0$  was considered.

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<sup>1</sup>D. Naugle, Phys. Lett. **25A**, 688 (1967).

<sup>2</sup>V. Golyanov, M. Mikheeva, and M. Tsetlin, Zh. Eksp. Teor. Fiz. **68**, 736 (1975) [Sov. Phys.—JETP **41**, 365 (1975)]; V. Golyanov and M. Mikheeva, *ibid.* **70**, 2236 (1976) [ **43**, 1167 (1976)].

<sup>3</sup>H. Sixl, Phys. Lett. **53A**, 333 (1975); H. Sixl, C. Bubeck, and W. Sanwald, J. Low Temp. Phys. **40**, 469 (1980).

<sup>4</sup>A. Orlov, A. Milai, and V. Dmitriev, Fiz. Tverd. Tela (Leningrad) **18**, 1470 (1976) [Sov. Phys.—Solid State **18**, 854 (1976)].

<sup>5</sup>P. Anderson, Phys. Chem. Solids **11**, 20 (1959).

<sup>6</sup>Yu. Ogrin, V. Lutskii, and M. Elinson, Pis'ma Zh. Eksp. Teor. Fiz. **3**, 114 (1966) [JETP Lett. **3**, 71 (1966)]; V. Lutskii, Phys. Status Solidi **1**, 199 (1970).

<sup>7</sup>Yu. Komnik and E. Bukhshtab, Zh. Eksp. Teor. Fiz. **54**, 63 (1968) [Sov. Phys.—JETP **27**, 34 (1968)].

<sup>8</sup>Yu. Komnik and E. Bukhshtab, Pis'ma Zh. Eksp. Teor.

Fiz. **6**, 536 (1967) [JETP Lett. **6**, 58 (1967)].

<sup>9</sup>O. Philatov and I. Karpovitch, Pis'ma Zh. Eksp. Teor. Fiz. **10**, 224 (1969) [JETP Lett. **10**, 224 (1969)].

<sup>10</sup>B. Tavger and V. Demikhovskii, Usp. Fiz. Nauk **96**, 61 (1968) [Sov. Phys.—Usp. **11**, 644 (1969)].

<sup>11</sup>W. McMillan, Phys. Rev. **175**, 537 (1968).

<sup>12</sup>P. Chaikin, G. Arnold, and P. Hansma, J. Low Temp. Phys. **26**, 229 (1977).

<sup>13</sup>A. Abrikosov, L. Gor'kov, and I. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics* (Prentice-Hall, Englewood Cliffs, New Jersey, 1963).

<sup>14</sup>E. Lifshitz and L. Pitaevski, in *Statistical Physics, Part II*, Vol. 9 of *Course of Theoretical Physics*, edited by L. Landau and E. Lifshitz (Pergamon, New York, 1980).

<sup>15</sup>L. Gor'kov, Zh. Eksp. Teor. Fiz. **34**, 735 (1958) [Sov. Phys.—JETP **7**, 505 (1958)].

<sup>16</sup>J. Schrieffer, *Theory of Superconductivity* (Benjamin, New York, 1964).

<sup>17</sup>G. Eliashberg, Zh. Eksp. Teor. Fiz. **39**, 1437 (1960)

- [Sov. Phys.—JETP 13, 1000 (1961)].
- <sup>18</sup>M. Cohen, L. Falicov, and J. Phillips, Phys. Rev. Lett. 8, 316 (1962).
- <sup>19</sup>V. Kresin, J. Low Temp. Phys. 5, 565 (1971).
- <sup>20</sup>B. Geilikman, V. Kresin, and N. Masharov, J. Low Temp. Phys. 18, 241 (1975); B. Geilikman, *ibid.* 4, 181 (1971).
- <sup>21</sup>J. Schrieffer, D. Scalapino, and J. Wilkins, Phys. Rev. Lett. 10, 336 (1963); Phys. Rev. 148, 263 (1966).
- <sup>22</sup>W. McMillan and J. Rowell, in *Superconductivity*, edited by R. Parks (Dekker, New York, 1969), Vol. 1, p. 561.
- <sup>23</sup>V. Kresin, Phys. Lett. 49A, 117 (1974).
- <sup>24</sup>L. Cooper, Phys. Rev. Lett. 6, 689 (1961).
- <sup>25</sup>N. Alekseevskii and S. Vedenev, Pis'ma Zh. Eksp. Teor. Fiz. 6, 865 (1967) [JETP Lett. 6, 302 (1967)]; Yu. Komnik and E. Bukhshtab, *ibid.* 8, 9 (1968) [*ibid.* 8, 4 (1968)].
- <sup>26</sup>R. Jacklevic and J. Lambl, Phys. Rev. B 12, 4146 (1975).
- <sup>27</sup>J. Blatt and C. Thompson, Phys. Rev. Lett. 10, 332 (1963).
- <sup>28</sup>V. Kresin and B. Tavger, Zh. Eksp. Teor. Fiz. 50, 1689 (1966) [Sov. Phys.—JETP 23, 1124 (1966)].
- <sup>29</sup>Y. Kagan and L. Dubovskii, Zh. Eksp. Teor. Fiz. 72, 646 (1977) [Sov. Phys.—JETP 45, 339 (1977)].
- <sup>30</sup>V. Kresin and B. Kokotov, Zh. Eksp. Teor. Fiz. 75, 1066 (1978) [Sov. Phys.—JETP 48, 537 (1978)].
- <sup>31</sup>B. Geilikman, J. Low. Temp. Phys. 9, 539 (1972); Usp. Fiz. Nauk 115, 403 (1975) [Sov. Phys.—Usp. 18, 190 (1975)].
- <sup>32</sup>Yu. Komnik, E. Bukhstab, Yu. Nikitin, and V. Andrievskii, Zh. Eksp. Teor. Fiz. 60, 699 (1971) [Sov. Phys.—JETP 33, 364 (1971)].
- <sup>33</sup>Y. Vrba and S. Woods, Phys. Rev. B 3, 2243 (1971).