Thermoelectric power in a disordered two-dimensional electron system

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The thermoelectric power Q is studied for a two-dimensional disordered system by using two different models in the weak-scattering limit. The first is a localization theory of noninteracting electrons. No logarithmic divergence is found for Q. A second model including the effects of electron-electron interaction gives rise to a logarithmic temperature-dependent term at low temperature. In the present work the effect due to both the long-range and short-range interactions are considered.

In recent years our understanding of electronic conduction in two-dimensional disordered systems where Anderson localization of noninteracting electrons[1–6] may be important and the theory of Altshuler *et al.*,[7,8] which considers electron-electron interaction may also be important, has advanced significantly.[1–15] When these ideas were carried out in the perturbation theory, valid in the weak scattering limit, both theories predict the existence of a non-Ohmic logarithmic ω- (frequency-) dependent correction to the conductivity. The coefficient in front of the logarithmic term in localization theory is identical to that of the interaction theory if only the long-range interaction between electrons is considered. The Hall constant has also been calculated.[8,11] No logarithmic divergent term is found in the localization theory[11] while in the interaction theory the logarithmic correction is still present.[8]

The purpose of this manuscript is to point out that, only in the interaction theory,[8] there is an analogous logarithmic correction to the thermoelectric power of a two-dimensional disordered system. In the presence of both an electric field E and a temperature gradient ∇T (in the same direction), the current density is given by[16]

$$j = L_{11}E + L_{12}(-\nabla T) \quad . \tag{1}$$

Here $L_{11} = \sigma$ is the electrical conductivity and the thermoelectric power

$$Q = \frac{L_{12}}{L_{11}} \quad , \tag{2}$$

where $L_{12} = L_{21}/T$. The response functions L_{11} and L_{21} are given by linear response theory in terms of the current-current and heat current-current correlation functions, respectively. L_{11} has been obtained previously. In the following discussion the perturbation theory in the weak scattering limit shall be used to calculate L_{21}. First let us consider the localization theory, in this approach the conductivity has the form[6]

$$L_{11} = \sigma_0\left[1 - \frac{1}{\pi E_F \tau}\ln\left(\frac{L}{l}\right)\right] \quad , \tag{3}$$

where $\sigma_0 = Ne^2\tau/m$. N is the electron concentration, m and e are the electron mass and charge. E_F is the Fermi energy, τ is the scattering time, L is the sample length, and l is the mean-free path. L_{21} can be determined from the Feynman graphs shown in Fig. 1, where the solid line represents the electron, Green function $G\pm(p,\omega) = [\omega - \epsilon(p) \pm i/2\tau]^{-1}$ and the wavy line is the diffusion propagator $D(q,\omega) = u^2\tau^{-1}/(-i\omega + Dq^2)$, and $D = V_F^2\tau/2$ is the diffusion constant in two dimensions. u measures the impurity potential and V_F is the Fermi velocity. The open circle and the solid dot are, respectively, the heat current vertex $\omega\vec{v}$; and the electric current vertex $e\,\vec{v}$; here ω can be regarded as the energy variable associated with the electron Green functions in Fig. 1. $\vec{v}$ is the velocity of an electron. The evaluation of these graphs is straightforward. If L_{21} is written as $L_{21} = L_{21}^0 + \Delta L_{21}$, we have

$$L_{21} = L_{21}^0\left[1 - \frac{1}{\pi E_F \tau}\ln\left(\frac{L}{l}\right)\right] \quad ; \tag{4}$$

here L_{21}^0 corresponds to Fig. 1(a) and is given by

$$L_{21}^0 = \frac{\pi^2 e N \tau T^2}{3mE_F} \quad . \tag{5}$$

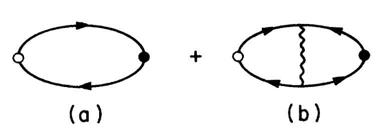

FIG. 1. Diagrams for heat current-current correlation function L_{21} in the localization theory of noninteracting electrons.

ΔL_{21} is the second term on the right-hand side of Eq. (4). Using the formulas listed from Eqs. (2) to (5), it is easy to show the $\ln(L/l)$ term in Eq. (3) cancels that of Eq. (4); therefore the thermoelectric power in localization theory does not contain a logarithmic T-dependent term

$$Q = Q_0 = \frac{\pi^2 T}{3eE_F} \ ; \tag{6}$$

here $Q_0 = L_{12}^0/\sigma_0 (L_{12}^0 = L_{21}^0/T)$ is the thermoelectric power without the contribution from the diffusion propagator.

Next we wish to consider the thermoelectric power in interaction theory. In this model the electric conductivity at finite temperature has been determined[8]

$$L_{11} = \sigma_0 \left[1 - \frac{1}{2\pi E_F \tau} \ln\left(\frac{1}{\tau T}\right)\right] \ . \tag{7}$$

The diagrams used to calculate the electric conductivity[8] also contribute to the heat current-current correlation function. They are shown in Fig. 2. These diagrams are generalized in a conserving approximation from exchange self-energy diagrams.[7] The wavy line here represents the dynamically screened Coulomb interaction dressed with repeated particle-hole scatterings through impurity potentials, and it has the form[8]

$$f(q,\omega) = \frac{v_s(q,\omega)}{(-i\omega + Dq^2)^2 \tau^2} \ , \tag{8}$$

$$v_s(q,\omega) = \frac{-i\omega + Dq^2}{2N_1(Dq^2)} \ . \tag{9}$$

$N_1 = m/2\pi$ is the single spin density of states. Using the standard finite-temperature Green-function method, these diagrams can be evaluated without difficulty. The contribution from Fig. 2(a) and its symmetry graph is

$$(\Delta L_{21})^{2a} = i\left(\frac{e}{m^2}\right)\frac{N_1 m \tau^3}{\pi \Omega} A \left(\int_\Omega^{1/\tau} d\omega f(\omega) \frac{3\Omega - 2\omega}{2T} + \int_0^\Omega d\omega f(\omega) \frac{\omega}{2T}\right) ,$$

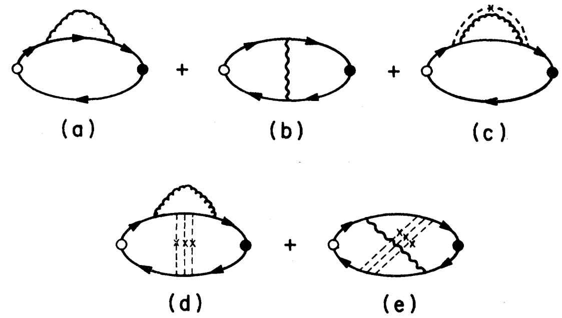

FIG. 2. Diagrams for heat current-current correlation function L_{21} in the theory of interacting electrons in the presence of weak impurity scattering.

where Ω is the external frequency which will be taken equal to zero at the end of the calculation. $f(\omega) = \sum_q f(q,\omega)$ and A is given by

$$A = \int_{-\infty}^{\infty} z^2 \operatorname{sech}^2 \frac{z}{2T} dz = \frac{4\pi^3 T^3}{3} \ . \tag{10}$$

For the expressions of Figs. 2(b) and 2(c), we, respectively, have

$$(\Delta L_{21})^{2b} = i\left(\frac{e}{m^2}\right)\frac{N_1 m \tau^3}{\pi \Omega} A \left(2\int_\Omega^{1/\tau} d\omega f(\omega) \frac{\omega - \Omega}{2T}\right) , \tag{11}$$

$$(\Delta L_{21})^{2c} = -i\left(\frac{e}{m^2}\right)\frac{N_1 m \tau^3}{\pi \Omega} A \left(\int_\Omega^{1/\tau} d\omega f(\omega) \frac{\Omega}{2T} + \int_0^\Omega d\omega f(\omega) \frac{\omega}{2T}\right) . \tag{12}$$

It can be seen immediately that the sum of Figs. 2(a)–2(c) is exactly zero so that the correction to the heat current-current correlation function comes directly from Figs. 2(d) and 2(e). The contribution from Fig. 2(d) and its symmetry graph has been evaluated. We have

$$(\Delta L_{21})^{2d} = i\left(\frac{e}{m^2}\right)\frac{(4\pi E_F N_1 \tau^3)^2}{\pi^2 E_F \Omega} A \sum_q q_x^2 \left(\int_0^{1/\tau} d\omega F_q(\omega,\Omega)\frac{\omega}{2T} + \int_\Omega^{1/\tau} d\omega F_q(\omega,-\Omega)\frac{\omega-\Omega}{2T}\right) , \tag{13}$$

$$F_q(\omega,\Omega) = f(q,\omega) D(q,\omega+\Omega) \ . \tag{14}$$

For Fig. 2(e) and its symmetry graph we obtain

$$(\Delta L_{21})^{2e} = -i\left(\frac{e}{m^2}\right)\frac{(4\pi E_F N_1 \tau^3)^2}{\pi^2 E_F \Omega} A \sum_q q_x^2 \left(\int_\Omega^{1/\tau} d\omega \frac{\omega - \Omega}{2T} [F_q(\omega,\Omega) + F_q(\omega,-\Omega)]\right) . \tag{15}$$

In the limit $\Omega \to 0$, the sum of $(\Delta L_{21})^{2d}$ and $(\Delta L_{21})^{2e}$ can be written

$$\Delta L_{21} = i\left(\frac{e}{m^2}\right)\frac{(2mE_F\tau^3)^2}{2\pi^2 E_F T} A \sum_q q_x^2 \int_\Omega^{1/\tau} d\omega F_q(\omega,\Omega) \quad . \tag{16}$$

At finite temperature and in the limit of $\Omega \to 0$, Ω should be replaced by T. After the integration in the above equation is carried out, ΔL_{21} becomes

$$\Delta L_{21} = -\frac{eT^2}{3E_F}\ln\left(\frac{1}{\tau T}\right) \quad . \tag{17}$$

From Eq. (5) and using the relation $L_{12} = (L_{21}^0 + \Delta L_{21})/T$, we obtain

$$L_{12} = L_{12}^0\left[1 - \frac{1}{\pi E_F \tau}\ln\left(\frac{1}{\tau T}\right)\right] \quad . \tag{18}$$

The thermoelectric power can be determined from Eqs. (7) and (18); the result is

$$Q = Q_0\left[1 - \frac{1}{2\pi E_F \tau}\ln\left(\frac{1}{\tau T}\right)\right] \quad . \tag{19}$$

The above result is the consequence of including only the exchange diagrams in the self-energy [see Figs. 3(a) and 3(b)]. As pointed out in Ref. 8, the Hartree diagrams in Figs. 3(c) and 3(d) may also be important if the interaction is short ranged. In the Hartree diagrams the momentum transfer in the interaction line (double wavy line)

$$V(q) = \frac{2\pi e^2}{|\vec{q}| + k}, \quad k = 4\pi e^2 N_1 \tag{20}$$

is not small and must be integrated over. It is straightforward to show that associated with Hartree diagrams of particle-hole channel in Figs. 3(a) and 3(b), an effective interaction V can be defined[8]

$$V = -u^2 \sum_{\vec{p}'\vec{p}''} G_+(\vec{p}')G_-(\vec{p}')V(\vec{p}' - \vec{p}'') \times G_+(\vec{p}'')G_-(\vec{p}'') \quad . \tag{21}$$

After the summations over $\vec{p}'$ and $\vec{p}''$ are carried out, we obtain

$$V = -\frac{F}{2N_1}, \qquad F = \int \frac{d\theta}{2\pi}\frac{1}{1 + (2k_F/k)\sin(\theta/2)} \quad . \tag{22}$$

In order to consider the effect of the Hartree diagrams [Figs. 3(c) and 3(d)] on the transport coefficient, we need only to replace $V_s(q,w)$ in Eq. (8) by V. The evaluation can be worked out easily and if the spin degeneracy of the closed loop is taken into

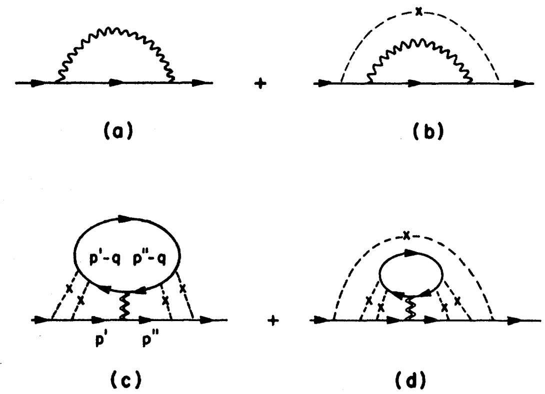

FIG. 3. Self-energy correction from exchange diagrams [(a) and (b)] and from Hartree diagrams [(c) and (d)] in particle-hole channel.

account, the result becomes

$$L_{11} = L_{11}^0\left[1 - \frac{g}{2\pi E_F \tau}\ln\left(\frac{1}{\tau T}\right)\right] \quad , \tag{23}$$

$$L_{12} = L_{12}^0\left[1 - \frac{g}{\pi E_F \tau}\ln\left(\frac{1}{\tau T}\right)\right] \quad , \tag{24}$$

where $g = 1 - F$. Thus, the inclusion of the Hartree diagrams [Figs. 3(c) and 3(d)] introduce a factor $-F$ in g. In the absence of a magnetic field the self-energy diagrams in particle-particle channel [Figs. 4(a) to 4(d)] are equally important. Their contributions to the transport coefficients can be obtained in a similar way. It is straightforward to show that the Hartree diagrams [Figs. 4(a) and 4(b)] and the exchange diagrams [Figs. 4(c) and 4(d)] give rise, respectively, to additional factors $-F$ and $\frac{1}{2}F$ in g.

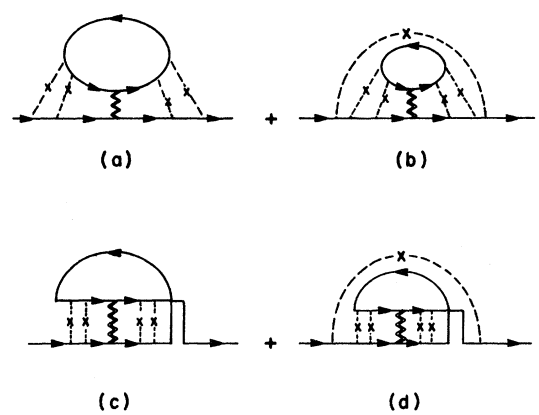

FIG. 4. Self-energy correction from Hartree diagrams [(a) and (b)] and from exchange diagrams [(c) and (d)] in particle-particle channel.

Thus we have

$$g=1-F-F+\tfrac{1}{2}F=1-\tfrac{3}{2}F \ . \tag{25}$$

In Figs. 4(c) and 4(d), the interaction lines between electrons should be ω dependent and $\omega \lesssim 1/\tau$. Since the momentum of the interaction line is quite large $\sim k_F$, we have neglected its frequency dependence and use Eq. (20) to represent this interaction. The contribution of these diagrams [Figs. 4(a) to 4(d)] to the conductivity L_{11} has been studied previously by Fukuyama.[11] The present result for L_{11} reduces to that of Altshuler *et al.*[8] if the diagrams in particle-particle channel are neglected. From Eqs. (23), (24), and (25), the thermoelectric power in the absence of a magnetic field can be written

$$Q=Q_0\left[1-\frac{1}{2\pi E_F\tau}(1-\tfrac{3}{2}F)\ln\left(\frac{1}{\tau T}\right)\right] \ . \tag{26}$$

According to Eq. (22), it is clear that F approaches unity if $2k_F/k \rightarrow 0$ and zero for large $2k_F/k$.

Finally, we also insert a note on the effects of orbital degeneracy in the Hartree diagrams for silicon inversion layers. In the absence of intervalley scattering, the Hartree term has an additional valley degeneracy factor n_v. The thermoelectric power Q for such a system can be shown to have the expression

$$Q=Q_0\left[1-\frac{1}{2\pi E_F\tau n_v}[1-(2n_v-\tfrac{1}{2})F]\ln\left(\frac{1}{\tau T}\right)\right] \ .$$

In this paper we have studied the thermoelectric power for a two-dimensional disordered system in two different models. The first model deals with the localization of noninteraction electrons. We find that there is no logarithmic T-dependent term in Q. The second model deals with interacting electrons in the presence of weak impurity scattering. The thermoelectric power is predicted to have a logarithmic correction at low temperature. It is hoped this work, which is in the same spirit of the Hall constant calculation,[8] will help to decide which of the two models is more clearly related to experiments. Moreover, it seems that the thermoelectric power measurement constitutes so far the only proposed experiment which is able to separate the logarithmic term of the second model from that of the first model in the absence of a magnetic field.

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