Exclusion of temperature fluctuations as the source of $1/f$ noise in metal films

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The measured coherence between the $1/f$ noise of two superimposed, thermally coupled but electrically insulated continuous gold films was found to be orders of magnitude smaller than it would be if this excess low-frequency noise were due to temperature fluctuations. Thus temperature fluctuations of either extrinsic or thermodynamic origin cannot generally be responsible for the $1/f$ noise observed in substrate-mounted metal films.

Temperature fluctuations endure as an attractive putative mechanism for generating the ubiquitous excess low-frequency resistance fluctuations in conductors known as $1/f$ noise.[1–7] We have devised an experimental test for temperature fluctuation mechanisms for generation of $1/f$ noise. We report here experiments showing that typical $1/f$ noise, in gold films on an insulating substrate, is *not* generated by either intrinsic or extrinsic temperature fluctuations.

Fluctuations of conductor temperature with power spectral density $S_T(f)$ generate voltage fluctuations, at constant current, that have a power spectrum

$$S_V(f) = \beta^2 \bar{V}^2 S_T(f) \quad , \tag{1}$$

where $\beta = (1/R)(dR/dT)$ is the temperature coefficient of resistance, and $\bar{V}$ is the average potential difference across the conductor. Since local temperature fluctuations $\delta T(\vec{x},t)$, whatever their origin, propagate in solids by diffusion, thus $\partial[\delta T(\vec{x},t)]/\partial t = D\nabla^2[\delta T(\vec{x},t)]$, where D is the appropriate diffusivity, the conductance noise developed by temperature fluctuations can be calculated.

Thermodynamic fluctuations of the temperature of freely suspended tin films near their superconducting transition temperatures (where β is very large) do account for the magnitudes and spectra of their excess conductance noise.[1] In contrast, experiments on substrate supported metal films at various temperatures generally reveal extended $1/f$ spectra[2,3,5,6,8,9] with complex temperature dependence, not attributable to equilibrium temperature fluctuations.[8] However, Voss and Clarke have proposed an alternative model with temperature fluctuations of unidentified origin that generate a $1/f$ spectrum[3] and Van Vliet *et al.* find a $1/f$ noise spectrum in a substrate-mounted metal film due to radiation fluctuations, treated as a spatially coherent, white-spectrum, surface flux.[4] Extrinsic fluctuating heat sources might be invoked to account for various excess noise spectra.

Reported measurements of spatial correlation in the $1/f$ noise along metal films[2,3,5,6] support temperature fluctuations as a cause of $1/f$ noise, since solutions of the diffusion equation are characterized by a frequency-dependent correlation length, $\lambda(f) = (D/\pi f)^{1/2}$. We have exploited this time-displaced spatial correlation to identify excess conductance noise due to temperature fluctuations. Our calculations have shown that the temperature fluctuations of two superimposed thin metal films, separated by a thin, electrically insulating layer are strongly correlated at the relevant frequencies. Therefore, any excess conductance noise due to temperature fluctuations must also be strongly correlated. We have formed appropriate superimposed films on an insulating substrate, calculated and experimentally confirmed their thermal coupling, and have sought correlations between their $1/f$ noise signals with sufficient sensitivity to detect coherence (defined below) less than one-hundredth the magnitude anticipated were the $1/f$ noise due to temperature fluctuations.

Superimposed, electrically insulated film bridges were prepared by (1) evaporating a 600-Å layer of gold onto a 0.6-mm-thick single-crystal sapphire substrate, (2) photoetching the gold to a 1-mm-long by 80-μm-wide bridge of 10 to 20 Ω resistance, (3) evaporating a layer of SiO of thickness $s = 6000$ Å on the bridge and surrounding substrate, (4) similarly fabricating a second gold bridge on top of the SiO, and (5) evaporating a thicker gold four-probe electrical contact superstructure for noise-free contacts. Film resistivity was typically 6 $\mu\Omega$ cm (twice bulk) and β was 0.003 K^{-1} (like bulk). Data were taken from two superimposed film pairs with similar results. Specimen geometry is illustrated in the inset of Fig. 2.

Conductance and Johnson noise were measured with a constant current from lead-acid batteries in series with a 1-kΩ resistor. Despite current densities approaching 10^6 A cm^{-2}, measured film temperatures rose less than 1 K because of the high thermal conductivity K (50 $\mathrm{J\,K^{-1}\,s^{-1}\,m^{-1}}$) of the substrate, mounted on a copper heat sink.

Voltage fluctuations were amplified with Ithaco 1201 or PAR 113 low-noise preamplifiers, impedance-matched to the films with PAR 190 low-noise transformers, and analyzed with an HP 5420A spectrum analyzer. The spectrum, $S_V(f)$, of the excess noise was isolated from that of the measured voltage fluctuations, $S_E(f)$, by subtracting out the background (mostly Nyquist) noise, $S_B(f)$, of an equivalent wire-wound resistor. Within the measurement range, $1 < f < 100$ Hz, both films showed typical excess noise roughly consistent with Hooge's empirical formula, $S_V(f)/\overline{V}^2 = a/(N_c f^b)$, with $1.0 < b < 1.1$ and $0.005 < a < 0.014$, where N_c is the number of carriers.[9]

In order to determine the thermal coupling between the two films we have measured the frequency dependence of the modulation amplitude of the temperature of one film generated by electrical power dissipation $\Delta P_1 \cos(2\pi f t)$ in the superimposed film. The temperature-modulation amplitude $\Delta T_2(f)$ of the second film was determined from the amplitude $\Delta v_2(f)$ of the $\cos(2\pi f t)$ component of the voltage with a steady current I_2, as $\Delta T_2 = \Delta v_2/(\beta I_2 R_2)$. Measured values of $\Delta T_2(f)/\Delta P_1$ are plotted in Fig. 1, for frequencies $0.2\ \mathrm{Hz} < f < 24$ kHz. Δv_2 was proportional to $\Delta P_1 I_2$ as expected for thermal coupling. Reversal of the roles of the two films gave identical results.

The corresponding quantities can be calculated by solving the homogeneous diffusion equation in the substrate, represented by an infinite slab normal to

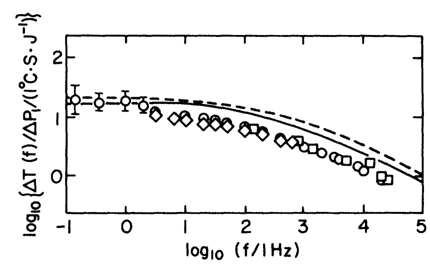

FIG. 1. Thermal response $\Delta T_2(f)/\Delta P_1$. Data points are representative of several different runs with superimposed points omitted; ○ and ◇ observed with power into top film, □ with power into bottom film. Error bars represent low-frequency uncertainties in transfer-function calibration. Solid line is the calculated thermal response $\Delta T_2(f)/\Delta P_1$. The dashed curve is the thermal auto-response $\Delta T_1(f)/\Delta P_1$. It is identical to the spectrum of the equilibrium thermodynamic temperature fluctuations of one film divided by 2×10^{-18} J K.

the z axis, having thickness d, diffusivity D, thermal conductivity K, and the bottom face ($z=0$) at constant temperature. The heater (upper) film is represented by prescribing the surface-heat flux at $z=d$ to be $(\Delta P_1/4lw)\cos(2\pi f t)$ inside the rectangular region $\{\vec{x}: |x| < l,\ |y| < w,\ z=d\}$, and zero outside. The heat capacities of the thin films are negligible. We solve for the steady-state oscillating component of the temperature averaged over the rectangle $\{\vec{x}: l_1 - l < x < l_1 + l,\ w_1 - w < y < w_1 + w,\ z = d - s\}$. The offsets l_1 and w_1 allow for imperfect alignment of the films. The average temperature modulation of the lower film is found to be

$$\Delta T_2(f) = \left| \frac{\Delta P_1}{\pi K l^2 w^2} \int_0^\infty \int_0^\infty dk_1 dk_2 \frac{\cos k_1 l_1 \cos k_2 w_1}{q} \frac{\sin^2 k_1 l}{k_1^2} \frac{\sin^2 k_2 w}{k_2^2} \frac{\sinh q(d-s)}{\cosh qd} \right| , \tag{2}$$

where $q^2 = k_1^2 + k_2^2 + i2\pi f/D$. Numerical integration yields $\Delta T_2(f)/\Delta P_1$ plotted as the solid curve in Fig. 1. Theory and experiment agree at all frequencies within the factor of 2 uncertainty due to β, K, and calibrations. The excellent agreement without adjustable parameters indicates that the model incorporates the important features of the system, and, in particular, excludes thermal boundary resistances at the interfaces.

Equation (2) yields the modulation amplitude of the temperature of the heater film $\Delta T_1(f)$ on setting $s = l_1 = w_1 = 0$. The temperatures of those portions of the two films that are directly superimposed are virtually identical for frequencies $f \ll D_{\mathrm{SiO}}/\pi s^2 \sim 1$ MHz, where s is the thickness, and D_{SiO} the diffusivity of the SiO layer. The corresponding calculation for an instantaneous point heat source in one film shows that even spatially localized temperature fluctuations are fully coupled between the two films in the relevant frequency range.

An implicit result of these calculations is the spectrum of the equilibrium thermodynamic temperature fluctuations, $S_{T,\mathrm{eq}}(f)$, for a metal film on an insulating substrate. This spectrum is proportional to the cosine transform of the impulse response[3,10] which is proportional to the amplitude of the sinusoidally driven response, that is, $\Delta T_1(f)/\Delta P_1$, plotted as the dashed curve in Fig. 1. The proportionality constant for our gold films at 300 K (2×10^{-18} J K) is obtained by normalizing the variance, $\langle(\delta T)^2\rangle = \int_0^\infty S_{T,\mathrm{eq}}(f)\,df = k_B T^2/C_V$ where C_V is the heat capacity of the film. At 10 Hz, the measured $1/f$ noise, $S_V(f)$, is more than a thousand times $\beta^2 \overline{V}^2 S_{T,\mathrm{eq}}(f)$.

Our analysis of Eq. (2) verifies the form of Voss and Clarke's Eq. (3.19) but requires that the diffusivity D of the substrate replace that of the metal film.[3] The thermal relaxation is determined almost entirely by three-dimensional diffusion into the substrate, not along the film as suggested by Voss and Clarke[3]; the one-dimensional model considered by Van Vliet *et al.* applies for frequencies $f \gg D/\pi w^2 \simeq 700$ Hz.[4]

The correlation between fluctuations of the average temperatures of the two films can be expressed by the coherence function,

$$\gamma_T^2(f) = |S_{T12}(f)|^2/[S_{T1}(f)S_{T2}(f)] \quad , \tag{3}$$

where S_{T12} is the cross-power spectral density between the average temperatures of the two films, and S_{T1} and S_{T2} are their individual power spectra.[11] (Numerical subscripts will index the bridges.) The thermal-coupling calculation indicates that with two perfectly superimposed films $\gamma_T^2(f) = 1$, $f \ll 1$ MHz. Correction for misalignment reduces γ_T^2 slightly; for our film geometry we find $0.9 < \gamma_T^2(f) < 1.0$.

Consequently, if temperature fluctuations were the cause of the $1/f$ noise, essentially the same temperature fluctuations would appear in both films and the $1/f$ noise would be correlated. From Eqs. (1) and (3) it follows that $\gamma_V^2 = \gamma_T^2$, where γ_V^2 is the coherence between the $1/f$ noise of the two films. Since the measured voltage fluctuations include both the $1/f$ noise and (mostly incoherent) background noise, the coherence γ_E^2 of the measured voltage fluctuations of the two films is reduced from γ_V^2 by

$$\gamma_E^2(f) = \frac{\gamma_V^2(f)}{[1+S_{B1}(f)/S_{V1}(f)][1+S_{B2}(f)/S_{V2}(f)]} \quad . \tag{4}$$

Using measured values of S_{V1}, S_{B1}, S_{V2}, and S_{B2}, and assuming $\gamma_V^2 = \gamma_T^2$, the right-hand side of Eq. (4) is plotted as curve (a) in Fig. 2. This curve represents the observable coherence $\gamma_E^2(f)$ that would be expected if temperature fluctuations were entirely responsible for the observed $1/f$ noise.

Experimentally γ_E^2 was found to be smaller than the measurement sensitivity; that is, $\gamma_E^2(f) \leq 5 \times 10^{-4}$, indicated by curve (b) in Fig. 2. At low frequencies the measured coherence is a factor of 10^3 smaller than expected if the $1/f$ noise were due to temperature fluctuations. At higher frequencies the Johnson noise reduces the expected $\gamma_E^2(f)$, but the disparity is still large. Since $\lambda(f) \propto f^{-1/2}$ and $\lambda(1\ \text{Hz}) \gg s$, the disparity should extend to lower frequencies. Therefore, we must conclude that the $1/f$ noise in these gold films is not due to temperature fluctuations. Extrinsic temperature fluctuations driven by hypothetical external sources are ruled out in addition to equilibrium temperature fluctuations.

Our results suggest that temperature fluctuations

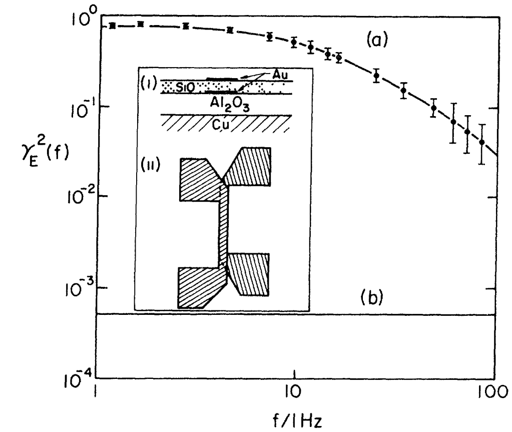

FIG. 2. (a) Computed coherence $\gamma_E^2(f)$ assuming that the measured excess noise in each film is due to temperature fluctuations. (b) Upper limit of measured coherence $\gamma_E^2(f)$. Inset: (i) Schematic cross section of a superimposed film specimen with vertical scale distorted for visibility. (ii) Plan view of a superimposed film specimen showing contact pads and imperfect alignment.

were not responsible for previously observed spatial correlation in the $1/f$ noise along metal films,[2,3,5,6] but do not rule out spatial correlations due to electrical coupling within the film. However, spatial correlations, whatever their cause, would eliminate the inverse dependence of $S_V(f)$ on any conductor dimension that is smaller than the correlation length. Thus the previously measured thickness dependence of the noise would appear to rule out spatial correlation distances > 100 Å in gold films.[9] (Weissman[10] has noted the corresponding objection to Voss and Clarke's "P source."[3])

We have put an upper bound on the coherence between the voltage fluctuations of two electrically insulated, superimposed, substrate-mounted thin gold films, demonstrated their intimate thermal contact, and calculated the coherence expected if temperature fluctuations were responsible for their $1/f$ noise. Because the measured coherence is several orders of magnitude lower than calculated, we conclude that their $1/f$ noise is not caused by temperature fluctuations of either thermodynamic or extrinsic thermal origin. We presume that these conclusions are usually applicable to $1/f$ noise in metal films.

The following paper by Black and co-workers eliminates temperature fluctuations as sources of $1/f$ noise in additional cases and thus support the generality of the result.

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