

## Cyclotron resonance of Si(001) inversion layers

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(Received 22 January, 1981; revised manuscript received 4 May 1981)

Based on a memory-function formalism, the cyclotron data of Wagner, Kennedy, McCombe, and Tsui on inversion-layer electrons in Si metal-oxide–semiconductor field-effect transistors are analyzed. The cyclotron effective mass and the relaxation time of these electrons are evaluated as functions of density. Our theoretical results are found to be in good agreement with the data, if the impurity density is assumed to be  $0.037 \times 10^{12} \text{ cm}^{-2}$ .

## I. INTRODUCTION

Since around 1974, cyclotron resonance and magnetotransport have been effectively used for the investigation of the electronic properties of the quasi-two-dimensional electrons in Si inversion layers.<sup>1</sup> It has been revealed that the effective mass and scattering time of these electrons depend on carrier density,<sup>2,3</sup> although there are other factors which affect the dependences. These include magnetic field (frequency),<sup>4</sup> temperature,<sup>5</sup> impurity ions, etc.<sup>6</sup> In particular, Fang, Fowler, and Hartstein have found that the effects of impurity ions are surprisingly strong.<sup>6</sup>

More recently, Wagner, Kennedy, McCombe, and Tsui have reported extensive cyclotron resonance data on Si(001) inversion layers.<sup>7</sup> Their results depend on carrier densities, and in both high and low densities show interaction effects. Since it seems worthwhile to analyze theoretically such cyclotron data, we report in the present paper our results, in particular, on the density dependences of the effective mass and relaxation time.

For this purpose, we shall follow the theory given by Tzoar, Platzman, and Simons.<sup>8</sup> A similar approach, which may be called, after Götze and Wölfle,<sup>8</sup> a memory function approach, has been developed by Ting, Ying, and Quinn.<sup>8</sup> By assuming Coulomb scattering, the effective mass shift and relaxation time are then given in terms of the real and imaginary parts of the dielectric function as follows.

$$\frac{\Delta m}{m} = \frac{r_s}{2^{1/2}} \frac{n_I}{n} I(t, r_s), \quad (1.1)$$

$$\frac{1}{\tau} = \frac{r_s}{2^{1/2}} \frac{n_I}{n} (4p_F^2) J(t, r_s), \quad (1.2)$$

where  $n_I$  is the impurity concentration and

$$I(t, r_s) = \frac{2}{t^2} \int s^2 ds \left[ \epsilon_r^{-1}(s, 0) - \frac{\epsilon_r(s, t)}{|\epsilon|^2} \right], \quad (1.3)$$

$$J(t, r_s) = \frac{1}{2t} \int s^2 ds \frac{\epsilon_i(s, t)}{|\epsilon|^2}, \quad (1.4)$$

where  $s = q/p_F$ ,  $t = \omega/p_F^2$  are dimensionless variables in the units in which  $\hbar = 1$  and  $2m = 1$ ,  $m$  being the electron mass.

The random-phase approximation dielectric function of a two-dimensional (2D) electron gas has been evaluated by Stern.<sup>9</sup> Later, Isihara and Toyoda<sup>10</sup> obtained the same result for the correlation energy. We have evaluated numerically the ratio  $\epsilon_r/|\epsilon|^2$  in the integrand of Eq. (1.3) and  $\epsilon_i/|\epsilon|^2$  in Eq. (1.4) because of their important roles. For a three-dimensional (3D) case, Pines and Nozières have illustrated these ratios as functions of  $r_s$ .<sup>11</sup> We have found that the 2D graphs are generally similar to the 3D cases, except that around  $r_s = 1$  the real part ratio shows a resonance while the imaginary part ratio is zero for zero frequency.

## II. RESULTS AND DISCUSSIONS

Figure 1 illustrates the function  $I(t, r_s)$  of Eq. (1.3) which determines the cyclotron effective mass shift as a function of the reduced frequency variable  $t$ . For a given  $r_s$ ,  $I(t, r_s)$  shows a characteristic discontinuity at  $t = 1$ , i.e.,  $\omega = p_F^2$ . Below this frequency, the function drops rather fast from a limiting value for each  $r_s$  indicated by a cross. Note that the value corresponds to  $t = 0.01$ .

Although Fig. 1 shows a minimum only for the case  $r_s = 1.0$ , we have found that the curves for

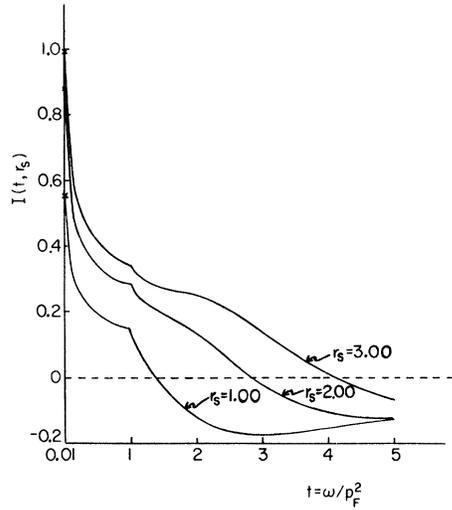


FIG. 1. Effective-mass function  $I(t, r_s)$  plotted against a dimensionless frequency variable  $t$ .

$r_s = 2$  and  $3$  also reach minima at around  $t = 5.4$  and  $7.5$ , respectively. Therefore, all three cases are similar to each other concerning frequency dependence. Before reaching the minimum, the larger the  $r_s$ , the higher  $I(t, r_s)$  and larger the effective-mass shift.

Figure 2 represents the function  $J(t, r_s)$  which determines the relaxation time. The three curves have the common feature of having a discontinuity in slope at  $t = 1$ , followed by a maximum which depends on  $r_s$ . The dotted curves represent the plasmon contribution which was not considered before. The maximum of  $J(t, r_s)$  appears in correspondence to the minimum in Fig. 1.

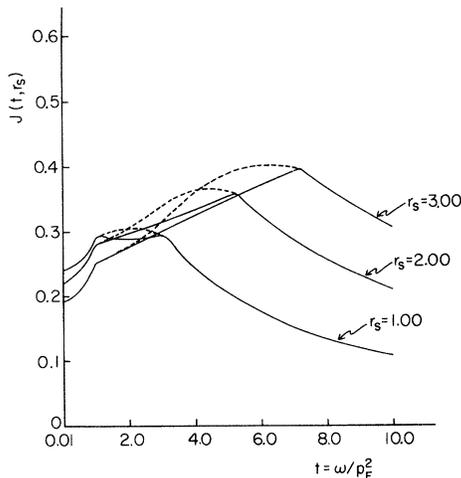


FIG. 2. Relaxation-time function  $J(t, r_s)$  as a function of  $t$  for three  $r_s$  values.

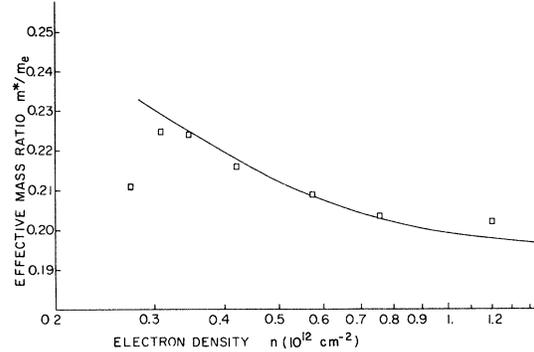


FIG. 3. Effective mass  $m^*$  in the unit of the electron mass as a function of electron density. The data which are due to Wagner *et al.* (Ref. 7) correspond to frequency  $25.4 \text{ cm}^{-1}$  and sample 3 ( $m_e = m$ ).

Figure 3 compares our theoretical result (full curve) on the density dependence of the effective mass with the data (sample 3) of Wagner *et al.*<sup>7</sup> for  $\nu = 25.4 \text{ cm}^{-1}$ , where  $\nu = \omega/2\pi c$ . The experimental transverse effective mass  $m_0$  of 0.191 has been used for our theoretical calculation. The cyclotron effective mass  $m^*$  is given by (in Fig. 3, we have used their notation  $m_e$  which is  $m$  in the present paper)

$$m^* = m_0 + \Delta m. \quad (2.1)$$

The mass shift  $\Delta m$  has been evaluated from Eq. (1.1). As we see, this formula gives the mass shift in terms of the impurity concentration  $n_I$ . We have found that  $n_I = 0.037 \times 10^{12} \text{ cm}^{-2}$  gives a reasonable fit, as illustrated in Fig. 3. This value is approximately ten times smaller than what Tzoar *et al.* suggested. However, they only estimated their impurity concentration for  $r_s$  of order 1.5. This  $r_s$  corresponds approximately to an electron density of  $6.6 \times 10^{12} \text{ cm}^{-2}$  if an average dielectric constant 7.8 and valley degeneracy 2 are used. For such a high density, the effective-mass shifts are very small. In other words, the electron density  $1 \times 10^{12} \text{ cm}^{-2}$  which they used for their estimation is approximately a factor of 10 smaller than the above value. This explains the difference in the impurity concentration, and we believe that theoretically we are consistent with Tzoar *et al.* However, we might add that we have evaluated the function  $I(t, r_s)$  for various different  $r_s$  values and also obtained  $m^*$  as a function of electron density. We believe that these new results are useful for future analyses of experimental results.

In any case, it is very interesting to observe in Fig. 3 that the theoretical curve is close to the ex-

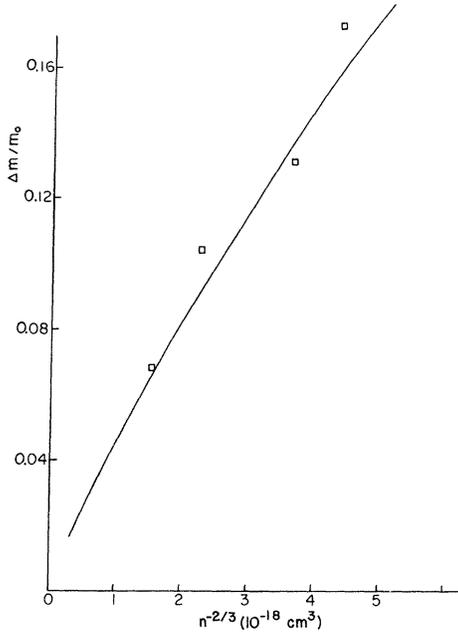


FIG. 4. Relative effective-mass shift  $\Delta m/m_0$  against  $n^{-3/2}$  where  $m_0 = 0.191 m$  and  $\Delta m = m^* - m_0$ ,  $m^*$  being the effective mass. The data (squares) are obtained from Fig. 17 of Wagner *et al.* (Ref. 7).

perimental data on the high-density side. On the other hand, the data show a maximum followed by a drop at low densities of order  $0.25 \times 10^{12} \text{ cm}^{-2}$ . Although impurity ion and some other effects<sup>6</sup> are conceivable, a certain deviation may be expected theoretically also.

Figure 4 illustrates  $\Delta m/m_0$  as a function of  $n^{-3/2}$  in view of the form of Eq. (1.1). Note that our theoretical result (full curve) is slightly convex, showing the effect of the function  $I(t, r_s)$ . The data points (triangles) are what we have obtained from the work of Wagner *et al.* for  $25.4 \text{ cm}^{-1}$ . We have used  $m_0 = 0.191 m$  as before. They plotted  $\Delta m/m_0 I$  in expectation of a straight line. Our theoretical impurity concentration is again  $0.037 \times 10^{12} \text{ cm}^{-2}$ , as in Fig. 3.

Figure 5 gives the theoretical effective mass in the unit of the electron mass as a function of  $\omega\tau$ . As we can find from Fig. 6, our theoretical relaxation time is close to the data around density of  $0.5 \times 10^{12} \text{ cm}^{-2}$ . Hence, we have obtained the data points (squares) in Fig. 5 from the data of Wagner *et al.* corresponding to  $\nu = 25.4 \text{ cm}^{-1}$  and for the same electron density. They showed in their Fig. 16(b) a least-square fit represented by a straight line. Therefore, the effective mass becomes very small for high frequencies. In our case, the solid curve approaches the bulk value of

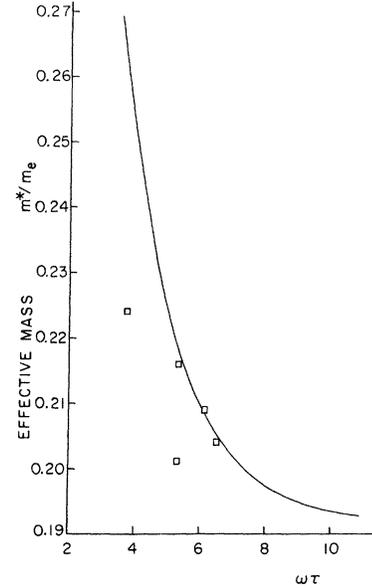


FIG. 5. Effective mass ratio  $m^*/m$  as a function of  $\omega\tau$ . The five squares near the maximum of Fig. 6 are used. Their  $m^*/m$  are obtained from Fig. 8 of Wagner *et al.* (Ref. 7).

0.19  $m$ .

Finally, we have plotted in Fig. 6 the scattering time as a function of electron density. The solid line represents our theory while the squares illustrate the data of Wagner *et al.* for  $\nu = 25.4 \text{ cm}^{-1}$ . There are systematic deviations at both low and high densities due mainly to the maximum which the present theory has failed to reproduce. We have investigated the pole contribution which must be added theoretically to the scattering time. This contribution is given by

$$\Delta J(t, r_s) = -\frac{\pi}{2t} \int_0^{s_1} ds s^2 \delta(\epsilon_r(s, t)), \quad (2.2)$$

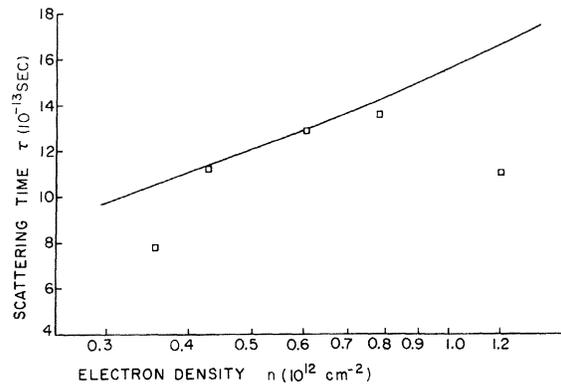


FIG. 6. Scattering time  $\tau$  against electron density  $n$ . The data are due to Wagner *et al.* (Ref. 7) for  $25.4 \text{ cm}^{-1}$ .

where  $s_1 = (1 + t)^{1/2} - 1$ . We find

$$\Delta J(t, r_s) = -\pi s_p^6 F_- F_+ / 2^{1/2} r_s t \times (-2s_p^2 F_- F_+ + G_+ F_+ - G_- F_-), \quad (2.3)$$

where the subscript  $p$  stands for pole and

$$F_{\pm} = [(s_p^2 \pm t)^2 - 4s_p^2]^{1/2}, \quad (2.4)$$

$$G_{\pm} = s_p^4 \pm 2s_p^2 t - 3t^2 + 4s_p^2.$$

We have then examined this pole contribution

numerically. For the experimental situation in which  $\nu = 25.4 \text{ cm}^{-1}$  and  $n = 0.5 \times 10^{12} \text{ cm}^{-2}$ , the contribution is found to be too small to explain the deviation. As can be seen from Fig. 2, it is effective around  $t = 2$  for  $r_s = 1$  and  $t = 4$  for  $r_s = 2$ .

#### ACKNOWLEDGMENT

This work was supported by the ONR under Contract No. N-00014-79-C-0451.

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