Quasiparticle lifetime in disordered two-dimensional metals

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(Received 27 July 1981)

A calculation of the electron quasiparticle lifetime in a disordered metal due to electron-electron scattering is given. The calculation takes account of the diffusive nature of electron motion which leads to enhancement of the diagonal exchange term of the electron self-energy. The lifetime, as a function of temperature, behaves as $T^{-d/2}$ where d is the dimensionality, in contrast to the T^{-2} behavior of ordinary Fermi-liquid theory. At $d = 2$, a logarithmic singularity occurs which leads to $(T \ln T)^{-1}$ behavior of the lifetime and a failure of the quasiparticle picture near the Fermi surface. The calculated lifetime agrees in temperature, Fermi energy, and elastic mean-free-path dependence with recent experiments on silicon inversion layers.

I. INTRODUCTION

In this paper we calculate perturbatively the effects of electron-electron scattering on the lifetime of the one-electron eigenstates in a disordered metal. In the pure metal, it is well known that the electron-electron interaction produces a quasiparticle decay rate $1/\tau_{ee}$ of order $v_F \kappa (k_B T/E_F)^2$ in all dimensionalities. Here κ^{-1} is the screening length of the electron gas whose Fermi velocity is v_F. Recently it has been pointed out that in the presence of static impurity scattering, the diffusive nature of the electron density propagation causes important modifications in the influence of electron-electron scattering on the thermodynamic and transport properties.[1] In particular, one expects anomalous behavior in the density of states, the electrical conductivity, the Hall effect,[2] and the magnetoresistance.[3] In addition to depending on the degree of disorder, these anomalies depend on dimensionality and many of them appear to have been observed experimentally.[4-7]

The scaling theory of localization[8] predicts that for weak disorder in two dimensions (2D), the conductivity depends logarithmically on a characteris-

tic length scale. At finite temperature this scale is the Thouless length L_T which is determined by inelastic scattering processes.[9] L_T is the distance an electron diffuses in an inelastic time τ_{in} and is given by $L_T^2 = D\tau_{in}$ where D is the elastic diffusion constant. Since $\tau_{in} \propto T^{-p}$, the extra conductivity due to the localization effect in 2D may be written as[10]

$$\Delta\sigma = (e^2/\pi^2 \hbar)(p/2) \ln T/T_0 . \tag{1.1}$$

Information on the exponent p is best obtained from magnetoresistance experiments on 2D systems.[11,12] The strength of the inelastic coupling, i.e., the scale T_0, may be estimated from experiments on thin wires[13] where the behavior is, instead of $\ln(T/T_0)$, $\Delta\sigma \propto (T/T_0)^{-p/2}$.

From the point of view of the localization theory, two difficulties arise in connection with Eq. (1.1). One is that experiments in dirty metal films and inversion layers and in thin wires appear to favor $p \simeq 1$, and so far the only process proposed[14] for τ_{in} (scattering from two-level tunneling states) which has $p = 1$ independent of dimensionality is not strong enough to account for the large T_0 which is observed in thin wires.[15] At the same

time, contributions from electron-electron scattering to the 2D resistivity[16] are not distinguishable from the purely localization effects since the former have precisely the form of Eq. (1.1) with, in some cases, $p \simeq 1$ and $k_B T_0 = \hbar/\tau$ where τ^{-1} is the elastic scattering rate from the static disorder.

We have remarked above that electron-electron scattering gives $p = 2$ for the inelastic rate in clean samples. In this paper we show that electron-electron scattering gives an additional contribution with $p = 1$ in the dirty 2D case. In fact, in thin metal films, the $p = 1$ contribution dominates the conventional T^2 ($p = 2$) contribution at low T even if the films are rather clean.

The question of the electron-electron scattering rate in the disordered case has been discussed previously, at $d = 3$, by Schmid[17] and by Atschuler and Aronov[18] who have extracted $1/\tau_{ee}$ from the Boltzmann equation for quasiparticle excitations. From such calculations, or from a simple argument we offer in Sec. II, one expects $1/\tau_{ee} \propto T^{d/2}$. However, we shall find that at $d = 2$, logarithmic corrections have a structure which makes it essential to calculate the full energy and frequency-dependent electron self-energy rather than restrict oneself to the energy shell as in Refs. 17 and 18. The result becomes $1/\tau_{ee} \propto T \ln T$ for $d = 2$.

In Sec. II we formulate the problem in terms of the eigenstates of the disordered noninteracting problem. A simple calculation of the density of states correction is presented.[19] The calculation makes it clear that the origin of the singular corrections comes from the assumed diffusive nature of density fluctuations, which in turn imply strong spatial correlations among eigenstates that are nearby in energy. In Sec. III a more rigorous treatment of the dynamically screened interaction is developed, and the decay rate due to electron-electron scattering is given. We compare our results to experiment on Si inversion layers in Sec. IV.

II. FORMULATION OF THE CALCULATION

In the disordered metal we denote the one-electron eigenenergies and eigenfunctions by E_m, $\psi_m(\vec{r})$. We add an electron in the state m and ask for its time evolution in the presence of the electron-electron interaction. For the latter, we take the dynamically screened Coulomb interaction which we treat in the lowest order of perturbation theory. The screening is effected by the average dielectric constant in the presence of impurities. In

the long wavelength, low-frequency limit, at $d = 2$, we have[16]

$$V_c(q,\Omega) = \frac{(2\pi e^2/q)(|\Omega| + Dq^2)}{(|\Omega| + D\kappa q)}, \qquad (2.1)$$

where Ω is a Matsubara frequency $\pi k T n$ (n is an even integer) and κ is the inverse screening length $2me^2$.

Equation (2.1) is appropriate for inversion layers and extremely thin films. For thicker films, we need the 3D result

$$V_c(q,\Omega) = \frac{4\pi e^2}{q^2} \frac{|\Omega| + Dq^2}{|\Omega| + D\kappa_3^2}. \qquad (2.2)$$

Here the symbols are as in Eq. (2.1) except that κ_3^2 is given by $4me^2 k_F/\pi$. Typically, in calculations only those momentum transfers satisfying $Dq^2 < |\Omega| \approx kT$ will be important. Then if the film thickness t is less than $\sqrt{D/kT}$ only the smallest transverse component $q_\perp = 0$ will enter and a 3D sum over momentum transfers becomes

$$\frac{1}{\text{vol}} \sum_{\vec{q}} \rightarrow \frac{1}{t} \int \frac{d^2q}{(2\pi)^2} \sum_{q_\perp} \delta_{q_\perp,0}. \qquad (2.3)$$

One then finds that a thin-film 3D calculation is identical to the 2D calculation except that for each Coulomb interaction there is an extra factor $\pi/k_F t$ reflecting both the phase space restrictions of Eq. (2.3) and the difference between κ_3 and κ.

In this paper we focus our attention on the one-electron properties which are described by the Green's function for a given electron configuration

$$G_{mm}(\omega) = \langle m | (\omega - H)^{-1} | m \rangle. \qquad (2.4)$$

The effect of interaction is included in the self-energy correction $\Sigma_m(\omega)$ so that

$$G_{mm}(\omega) = [\omega - E_m - \Sigma_m(\omega)]^{-1}. \qquad (2.5)$$

We are keeping only the diagonal terms in the self-energy because we shall find that it is enhanced by wave-function correlation. Writing $\Sigma_m = \Delta_m + i\Gamma_m$, we perform the standard expansion[20]

$$\Delta_m(\omega) = \Delta_m(\widetilde{E}_m) + (\omega - \widetilde{E}_m) \frac{\partial \Delta_m}{\partial \omega}, \qquad (2.6)$$

where

$$\widetilde{E}_m = E_m + \Delta_m(\widetilde{E}_m). \qquad (2.7)$$

We obtain

$$G_{mm}(\omega) = Z/[(\omega - \widetilde{E}_m) - i\gamma_m], \qquad (2.8)$$

where

$$Z^{-1} = \left[1 - \frac{\partial \Delta_m}{\partial \omega} \right]_{\omega = \widetilde{E}_m}$$ (2.9)

and

$$\gamma_m = Z \Gamma_m(\omega = \widetilde{E}_m) \ .$$ (2.10)

In Eq. (2.8) $\widetilde{E}_m$ is interpreted as the quasiparticle energy, γ_m the decay rate of the quasiparticle, and Z is the fractional weight of the quasiparticle excitation.

To discuss the average energy shift and the average decay rate we have to study the impurity average of the self-energy for a fixed E_m, i.e.,

$$\widetilde{\Sigma}_E(\omega) = \frac{1}{N_0} \left\langle \sum_m \delta(E - E_m) \Sigma_m(\omega) \right\rangle_{\text{av}} ,$$ (2.11)

where N_0 is the one-spin density of states and $\langle \ \rangle_{\text{av}}$ denotes impurity averaging. Writing $\widetilde{\Sigma}_E$

$= \widetilde{\Delta}_E + i \widetilde{\Gamma}_E$, we see that the average density of quasiparticle states is corrected by the amount

$$\delta N_Q = \frac{\partial \widetilde{\Delta}_E}{\partial E} \ ,$$ (2.12)

and the average quasiparticle decay rate is given by

$$\widetilde{\gamma}_E = \widetilde{Z} \widetilde{\Gamma} \approx \widetilde{\Gamma}_E \ ,$$ (2.13)

where $\widetilde{\Gamma}_E = (1/N_0) \Sigma_m \delta(E - E_m) \Gamma_m(\omega = \widetilde{E}_m)$. We work to lowest order in the screened Coulomb interaction so that we can evaluate ω at $\omega = E$ in Eqs. (2.12) and (2.13), and set $\widetilde{Z} = 1$ as we have done in Eq. (2.13). (As we shall see, an exception to the former is in the evaluation of $\widetilde{\Gamma}_E$ in 2D.)

As an illustration we consider a model problem of electrons interacting via a static interaction $v(\vec{r})$. Expanding in terms of the one-electron eigenstates ψ_m, we have

$$H_{\text{int}} = \int d\vec{r} \, d\vec{r}' \sum_{mnpq} \psi_m^*(\vec{r}') \psi_n^*(\vec{r}) v(\vec{r} - \vec{r}') \psi_p(\vec{r}) \psi_q(\vec{r}') a_m^+ a_n^+ a_p a_q \ .$$ (2.14)

In the Hartree-Fock approximation, the four-Fermion term is factorized. Let us focus our attention on the exchange term; the Hartree term can be shown to be small if the potential $v(r)$ has a range larger than the interparticle spacing. We shall see that the diagonal exchange term is enhanced because of wave-function correlation, so that we write $H_{\text{int}} = \sum_m \Sigma_m a_m^+ a_m$, where

$$\Sigma_m = - \sum_{n \text{ occupied}} \int d\vec{r} \, d\vec{r}' \psi_m^*(\vec{r}) \psi_n^*(\vec{r}') \psi_m(\vec{r}') \psi_n(\vec{r}) v(\vec{r} - \vec{r}) \ .$$ (2.15)

Suppose we insert a particle at energy E. Its energy will be shifted on the average by the amount

$$\widetilde{\Sigma}_E = \frac{1}{N_0} \sum_m \langle \delta(E - E_m) \Sigma_m \rangle_{\text{av}} = - \int_{-\infty}^0 dE' F(E, E'; \vec{r}, \vec{r}') v(\vec{r} - \vec{r}') \ ,$$ (2.16)

where

$$F(E, E'; \vec{r}, \vec{r}') = \sum_{m,n} \delta(E - E_m) \delta(E' - E_n) \psi_m^*(\vec{r}) \psi_n^*(\vec{r}') \psi_m(\vec{r}') \psi_n(\vec{r}) \ .$$ (2.17)

In Eq. (2.17) we need to know the average of a product of four wave functions. Since we do not have an explicit solution of the impurity problem, we seem to be faced with a hopeless task. Fortunately we notice that the combination required in Eq. (2.17) is precisely that which enters in a density-density correlation function defined as

$$A(q, \omega) = \int_{-\infty}^\infty dt \, d\vec{r} \, d\vec{r}' e^{i\omega t} e^{i \vec{q} \cdot (\vec{r} - \vec{r}')} \langle [\rho(\vec{r}, t), \rho(\vec{r}', 0)] \rangle \ .$$ (2.18)

In a metal the density fluctuation decays by diffusion. With this assumption it can be shown that[21]

$$A(q, \omega) = \frac{\partial n}{\partial \mu} \text{Im} \left[\frac{Dq^2}{-i\omega + Dq^2} \right] ,$$ (2.19)

where D is the diffusion coefficient. The diffusive behavior is evident in the denominator in Eq. (2.19) and will be referred to as the diffusion pole.

To make the connection to Eq. (2.17), we expand the operator ρ in Eq. (2.18) in terms of the exact eigenstates.[22] Restricting ourselves to $T=0$ for simplicity, for $\omega>0$ only one ordering of the commutator is nonvanishing and Eq. (2.18) becomes

$$A(q,\omega)=\sum_{m,n}\int d\vec{r}\,d\vec{r}\,'e^{i\vec{q}\cdot(\vec{r}-\vec{r}\,')}\langle\psi_m(\vec{r})\psi_n^*(\vec{r})\psi_m^*(\vec{r}\,')\psi_n(\vec{r}\,')\delta(E_n-E_m-\omega)\rangle_{\mathrm{av}}\,,\qquad(2.20)$$

where the sum is restricted to n occupied and m unoccupied. We convert the sum into an energy intergration and compare with Eq. (2.17). We have

$$A(q,\omega)=\int_0^\infty dE\int_{-\infty}^0 dE'F(E,E',\vec{r},\vec{r}\,')e^{i\vec{q}\cdot(\vec{r}-\vec{r}\,')}\delta(E-E'-\omega)\,.\qquad(2.21)$$

We expect $F(E,E';\vec{r},\vec{r}\,')$ to be dependent on $E-E'$ and $\vec{r}-\vec{r}\,'$ (the latter because we have translational invariance after impurity averaging). Then $F(E,E';\vec{r},\vec{r}\,')=F(\omega,\vec{r}-\vec{r}\,')$ and Eq. (2.21) becomes

$$A(q,\omega)=\omega\int F(\omega,\vec{r})e^{i\vec{q}\cdot\vec{r}}d\vec{r}\,.\qquad(2.22)$$

Comparing with Eq. (2.19), we have

$$\int F(\omega,\vec{r})e^{i\vec{q}\cdot\vec{r}}d\vec{r}=\frac{\partial n}{\partial\mu}\frac{Dq^2}{\omega^2+(Dq^2)^2}\,.\qquad(2.23)$$

Equation (2.23) is remarkable because it is divergent in the limit $q,\omega\rightarrow0$. Going back to the definition $F(\omega,\vec{r})$ we see that the seemingly innocent assumption of diffusion implies a certain correlation between wave-function overlap for eigenstates that are nearby in energy. We are now in a position to calculate the self-energy using Eqs. (2.16) and (2.23)

$$\widetilde{\Sigma}_E=\frac{\partial n}{\partial\mu}\int_{-\infty}^0 dE'$$

$$\times\int d\vec{q}\,\frac{Dq^2}{(E-E')^2+(Dq^2)^2}v(\vec{q})\,,\qquad(2.24)$$

where $v(\vec{q})$ is the Fourier transform of $v(\vec{r})$ and d the dimensionality. According to Eq. (2.24) the exchange interaction between the added electron with energy E and the electrons in the Fermi sea depends strongly on the energy separation. As a result the self-energy is also dependent on E. This will give rise to a change in the density of states, given by

$$\delta N=\frac{\delta\widetilde{\Sigma}_E}{\partial E}=\int\frac{d\vec{q}}{(2\pi)^d}\frac{Dq^2v(q)}{E^2+(Dq^2)^2}\,.\qquad(2.25)$$

The integrand in Eq. (2.25) is singular in the limit $q,\omega\rightarrow0$. We can therefore replace $v(q)$ by $v(0)$. Just from power counting we immediately see that δN is logarithmically divergent in 2D and goes like $\sqrt{E}$ in 3D. Equation (2.25) agrees with results given elsewhere based on summation of diagrams.[1]

The above model example is particularly simple in that for a static interaction, $\widetilde{\Sigma}_E$ is purely real and independent of ω. This means that $\widetilde{\gamma}=0$ and $\widetilde{Z}=1$. If the dynamically screened interaction [Eq. (2.2)] is used, $\widetilde{\Sigma}$ is complex and depends on ω. If we compute $\widetilde{\Sigma}$ using Eq. (2.15) with a complex v given by Eq. (2.1) or Eq. (2.2), we obtain a decay rate $\widetilde{\Gamma}_E$ which varies as $E^{d/2}$. However, the simple argument described here is not sufficient to yield the full ω dependence. This is done with the more powerful method described in the next section. Nevertheless, the simple method does produce the correct answer for the δN_Q defined in Eq. (2.12). Specific-heat measurements are insensitive to the quasiparticle fraction Z and are controlled by N_Q, as can be checked by direct computation. On the other hand, the single-particle density of states, as measured by tunneling for instance, is given by

$$N(\omega)=\pi^{-1}\sum_m\mathrm{Im}G_{mm}(\omega)\qquad(2.26)$$

and is sensitive to both Z and $\partial\widetilde{\Sigma}_E/\partial E$. At $d=2$, Z has logarithmic corrections that diverge at the Fermi surface. This in itself signals the failure of Fermi-liquid theory in disordered 2D metals. Furthermore, we shall find in the next section that the decay rate $\widetilde{\Gamma}$ has a logarithmic singular behavior in ω at finite T. This singularity will be cured by including the real part of the self-energy in the determination of $\widetilde{E}_m$ as shown in Eq. (2.7). The result in decay rate for a quasiparticle is thus proportional to $T\ln T$ which exceeds the width of the thermal distribution at sufficiently low temperatures.

III. CALCULATION OF COULOMB LIFETIME

We begin with a description of a method for the calculation of the impurity configuration average of *any* diagonal operator in the disordered energy-state representation. The procedure is to write an expression for the desired matrix element as a func-

tion of the disordered eigenstate m. Since upon configuration averaging, the eigenenergy E_m is not fixed, an appropriate quantity to study is the operator averaged over states m at energy E. At this point, everthing can be written in coordinate representation and impurity averaged in the conventional manner.[23]

We apply the method to the self-energy of state m which in the lowest order is given by the dia-

gram of the figure where the wavy line is the Coulomb propagator $V_c(q,\Omega)$ of Eq. (2.1) and the solid line is the Matsubara Green's function for an intermediate state n. The steps described above leading up to the impurity averaging are in the Appendix. From Eq. (A6) we have the energy averaged self-energy Σ_E; it is ready to be impurity averaged:

$$\widetilde{\Sigma}_E(\omega)=\frac{-kT}{(2\pi i)^2 N_0}\sum_{\Omega,q}V_c(q,\Omega)\int dv\frac{1}{v-i(\omega-\Omega)}\int d\vec{r}\,d\vec{r}'e^{i\vec{q}\cdot(\vec{r}-\vec{r}')}\langle(G^R-G^A)_{r,r';v}(G^R-G^A)_{r',r;E}\rangle_{\rm av}\,. \quad (3.1)$$

The angular brackets indicate the impurity average; G^R, G^A are retarded and advanced Green's functions of the disordered Hamiltonian at coordinates $\vec{r}$ and $\vec{r}'$ and at frequency v or E. As discussed in the Appendix, we have already separately averaged the Coulomb interaction to obtain V_c. Here, the remaining impurity average makes the product of the G's translationally invariant; it is then convenient to proceed in momentum space. The average in Eq. (3.1) becomes

$$\langle\sum_k(G^R-G^A)_{k+q,v}(G^R-G^A)_{k,E}\rangle_{\rm av}\,. \quad (3.2)$$

We look for the terms which are large for small q, where V_c is large. These come from $\langle G^R(v)G^A(E)\rangle_{\rm av}$ and vice versa.[24] In fact, for example,

$$\langle\sum_k G^R_{k+q}(v)G^A_k(E)\rangle_{\rm av}=-\frac{i/\tau}{v-E+iDq^2}\,. \quad (3.3)$$

Equation (3.3) and its complex conjugate go into Eq. (3.1). The result is

$$\widetilde{\Sigma}_E(\omega)=kT\sum_{\Omega,q}\frac{V_c(q,\Omega)}{i(\omega-\Omega)-E+iDq^2{\rm sgn}(\omega-\Omega)}\,. \quad (3.4)$$

The Ω sum may be performed in the standard manner after introducing Eq. (2.1) for V_c. We continue the Matsubara frequency $i\omega$ to the real axis and take the imaginary part, ${\rm Im}\widetilde{\Sigma}_E(i\omega\to\omega+i\delta)=\widetilde{\Gamma}(\omega,E)$. The result is

$$\widetilde{\Gamma}(\omega,E)=-e^2\sum_q D^2\kappa q^2\int dy\left[\coth\left[\frac{y}{2kT}\right]-\tanh\frac{y-\omega}{2kT}\right]\frac{y}{y^2+\epsilon Dq^2}\frac{1}{(y-\omega+E)^2+D^2q^4}\,, \quad (3.5)$$

where $\epsilon=D\kappa^2$. A complete evaluation of Eq. (3.5) is not feasible so we investigate separately the regions $T\gg\omega,E$ and $T=0$.

For large T, the square bracket in Eq. (3.5) may be replaced by $2kT/y$ and the integral cut off for $|y|=O(kT)$. The last factor is rapidly varying so
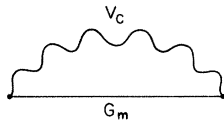

FIG. 1. Lowest order self-energy diagram for an electron in a state n scattered to an intermediate state m (straight line propagator G_m) by the Coulomb interaction V_c (wavy line).

that elsewhere y may be replaced by $\omega-E$. For $|\omega-E|\ll kT$ and $Dq^2<kT$, the y integral simply gives a factor π/Dq^2. Thus we cutoff the q integral at $Dq^2\approx kT$. With these simplifications we find

$$\Gamma=\frac{-e^2\kappa}{2}\left[\frac{kT}{\epsilon}\right]\ln(\omega-E)^2/kT\epsilon\,. \quad (3.6)$$

If we pass now to the energy shell to find the quasiparticle decay rate, we need to cure the logarithmic singularity at $\omega=E$. This can be taken care of in a higher-order calculation which includes the first-order shift of the quasiparticle pole Δ which is evidently

$$\Delta\approx\frac{e^2\kappa}{2}(kT/\epsilon)\,. \quad (3.7)$$

The final result is then

$$\Gamma = -e^2 \kappa \frac{kT}{\epsilon} \ln T/T_1 \ ,\tag{3.8}$$

where $kT_1 = D\epsilon^2/e^4$.

At $T = 0$, the square bracket in Eq. (3.5) restricts the y integral to the range $(0,\omega)$. The integrals are most easily approximated when $E - \omega > 0$. Then, as at finite T, we may replace y in the denominator of the second to last factor by its value where the rapidly varying last factor is largest, in this case zero. With this simplification the integrals can be done with the result

$$\Gamma = \frac{e^2 \kappa}{4} \frac{1}{\epsilon} [\omega + (E - \omega)\ln(E - \omega)/E] \ .\tag{3.9}$$

On the energy shell $\omega = E$ we recover the result of Schmid.[17]

We assume the quasiparticle renormalization Z to be unity, so that the decay rate Γ is the needed quasiparticle width $\tilde{\gamma}_E$ [Eq. (2.13)]. The Coulomb correction to Z is of first order in the screened interaction, though logarithmically dependent on T and ω. Thus in an approximation where only the leading term in interaction is retained, $\tilde{\Gamma}_E = \tilde{\gamma}_E$.

IV. DISCUSSION

Recent detailed measurements of negative and positive magnetoresistance in 2D Si inversion layers,[12] when combined with theoretical analysis,[2] lead to accurate numbers for $1/\tau_{in}$. Typically, for films with resistances in the range $200 - 2000 \ \Omega$ per square, $1/\tau_{in} \simeq (10^{10}$ to $10^{11})T \sec^{-1}$.

Our results for 2D systems may be expressed, from Eq. (3.8), as

$$-\tau/\tau_{in} = (kT/E_F)\ln T/T_1 \ .\tag{4.1}$$

This agrees in order of magnitude, dependence on T and concentration (E_F) with the data[12-14] quoted above. For thicker 2D systems, when $k_F t > 1$, the inelastic rate is suppressed by a factor $\pi/k_F t$.

In the case of thin wires, two factors of $1/k_F t$ will suppress the mechanism further and give a temperature dependence of $1/\tau_{in} \propto T^{1/2}$. This leads to disagreement in magnitude with the recent direct experimental determination of the 1D inelastic rate at one particular temperature.[25] Furthermore, the $T^{1/2}$ dependence of τ_{in}^{-1} leads to a $T^{-1/4}$ behavior for resistivity based on the scaling theory, in disagreement with the $T^{-1/2}$ behavior measured experimentally.[13] The explanation may be either that the inelastic scattering rate is dominated by other processes or that one is observing interaction effects. Further experiments, possibly magnetoresistance measurements similar to those performed in 2D, should clarify the situation.

ACKNOWLEDGMENTS

APPENDIX

We begin with a formal derivation of the starting equation (3.1). The method here[22] is appropriate for the calculation of any operator in the disordered energy-state representation.

For a given configuration of impurities, the self-energy of state m in lowest order is given in the figure where the wavy line is the Coulomb propagator $V_c(q,\Omega)$ of (2.1) and the solid line is the Matsubara Green's function for the eigenstate n:

$$\Sigma_m(\omega) = kT \sum_n \sum_{q,\Omega} V_c(q,\Omega) G_n(\omega - \Omega) L_{mnnm}(q) \ .\tag{A1}$$

The factor $L_{mnnm}(q)$ is given by

$$L_{mnnm}(q) = \int d\vec{r} \int d\vec{r}' e^{i\vec{q}\cdot(\vec{r}-\vec{r}')} \psi_m^*(\vec{r})\psi_n(\vec{r})\psi_n^*(\vec{r}')\psi_m(\vec{r}')\tag{A2}$$

and arises because the Coulomb interaction couples to the density which in this representation is determined by the disordered one-electron eigenfunctions $\psi_n(r)$. Notice that we have already configuration averaged the Coulomb propagator to obtain Eq. (2.1). Performing this average independently neglects terms smaller then the ones we keep by a factor at least $1/k_F l$ where l is the elastic mean free path. Thus we work always in the weak-scattering limit $k_F l >> 1$ and we shall use averaging procedures which are well known for this domain. These methods are formulated for

Green's functions and vertices in the coordinate representation so it is convenient to transform Eq. (3.1) by using the fact that

$$\sum_n \psi_n(r) G_n(\omega-\Omega) \psi_n^*(r') = G(r,r',\omega-\Omega) \; .$$
(A3)

Since the energy E_m is not fixed upon averaging, an appropriate quantity to study is the self-energy averaged over states m at energy E. This energy

average is obtained from

$$\Sigma_E(\omega) = \frac{1}{N_0} \sum_m \delta(E-E_m) \Sigma_m(\omega) \; ,$$
(A4)

where N_0 is the one-spin density of states. Since

$$\delta(E-E_m) = -[G_m^R(E) - G_m^A(E)]/2\pi i \; ,$$

we may use Eq. (A3) again and obtain

$$\Sigma_E(\omega) = -\frac{kT}{2\pi i N_0} \sum_{q,\Omega} V_c(q,\Omega) \int d\vec{r} \, d\vec{r}' e^{i\vec{q}\cdot(\vec{r}-\vec{r}')} G(\vec{r},\vec{r}';\omega-\Omega) [G^R(\vec{r}',\vec{r};E) - G^A(\vec{r}',\vec{r};E)] \; .$$
(A5)

Here A,R denote the advanced and retarded components of G. The spectral function $\mathscr{A}$ for G is just given by $(G^R - G^A)$. The remaining G in Eq. (A5) can be written as an integral over its spectral function, and we finally obtain

$$\Sigma_E(\omega) = -\frac{kT}{2\pi i N_0} \sum_{\Omega,q} V_c(q,\Omega) \frac{1}{2\pi i} \int dv \frac{1}{v-i(\omega-\Omega)} \int d\vec{r} \, d\vec{r}' e^{i\vec{q}\cdot(\vec{r}-\vec{r}')} \mathscr{A}(\vec{r},\vec{r}';v) \mathscr{A}(\vec{r}',\vec{r};E) \; .$$
(A6)

To average over impurity configurations it is only necessary to consider the product of $\mathscr{A}$'s in (A6). The averaging of this quantity is described in Sec. III.

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