Universal resistive transition for two-dimensional superconductors

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Vortex fluctuations are shown to imply that the resistance of a two-dimensional superconductor is a universal function of a certain combination of sample parameters. Analysis of experiments gives suggestive evidence for the validity of this universality. In view of the new results presented it is argued that recent resistance predictions based on the "asymptotic" Kosterlitz renormalization-group equations are only valid over a small temperature interval compared to the width of the resistive transition.

I. INTRODUCTION

It has recently been suggested that the broad resistive transition observed in "dirty" two-dimensional (2D) superconductors should be due to thermally excited vortex-antivortex pairs.[1-4] This explanation of the resistive transition is based on the following three ingredients: The Coulomb gas analogy of the vortex fluctuations by Pearl,[5] the Kosterlitz-Thouless charge-unbinding transition,[6,7] and the proportionality between the flux-flow resistance and the number of free vortices.[8] In the present paper it is shown that these three ingredients also lead to the prediction of a "universal" resistive transition. This means that the measured resistance for all samples should fall on the same curve when plotted against a certain combination of sample parameters.

II. UNIVERSALITY

We assume that the vortex-fluctuations are adequately described by the model due to Pearl.[5] The Pearl model is equivalent to a two-dimensional Coulomb gas where the vortices with vorticity ± 1 play the role of Coulomb gas particles with positive and negative charge.[5] This equivalence is summarized in Table I.

"Dirty" superconducting films obey the condition[1] $\xi << L \leq \Lambda_0$ where ξ is the Ginzburg-Landau coherence length, L is the sample dimension, and Λ_0 is the transverse penetration depth (compare Table I). When this condition is satisfied the energy of a neutral configuration of $2N$ particles (vortices) is given by[5,9]

$$H_{2N} = q^2\left[-\frac{1}{2}\sum_{i\neq j} s_i s_j \ln\left(\frac{c_1 r_{ij}}{\xi}\right) + c_2 \sum_i s_i^2\right] , \qquad (1)$$

where q, ξ, and s are defined in Table I, r_{ij} is the distance between particle i and j, and c_1 and c_2 are dimensionless constants. The important point in the following is that c_1 and c_2 are sample independent. The thermodynamics of the vortex fluctuations is determined from the partition function $Z = \mathrm{Tr}\exp(-\tilde{H}_{2N}/\tilde{T})$ where $\tilde{H}_{2N} \equiv H_{2N}/q^2$ and $\tilde{T} \equiv T/q^2$ [the non-neutral configurations can be ignored because their energies are proportional to

TABLE I. Relation between 2D Coulomb gas and 2D superconductor.

2D Coulomb gas	2D Superconductor
Particle	Vortex
Particle dimension, ξ	Ginzburg-Landau coherence length $\xi = \xi(T=0)[(T_c^0 - T)/T_c^0]^{-1/2}$; T_c^0 = Ginzburg-Landau temperature
Sign of charge, $s = \pm 1$	Vorticity
Magnitude of charge, q	$\frac{\emptyset_0}{2\pi}\Lambda_0^{-1/2}$; $\Lambda_0 = \frac{mc^2}{2\pi e^2}\frac{1}{n_s^0}$ = transverse penetration depth; n_s^0 = Ginzburg-Landau areal density of superconducting electrons $= n_s^0(T=0)(T_c^0 - T)/T_c^0$
Chemical potential, μ	Vortex core energy, $\lvert\mu\rvert = -\mu$; $\lvert\mu\rvert \approx \pi\xi^2 d\frac{H_c^2}{8\pi} = \frac{1}{8}q^2$

$\ln(L/\xi)$]. It follows that a dimensionless thermodynamic quantity can only be a function of the dimensionless variable $\tilde{T}$. *Or, in other words, a dimensionless thermodynamic quantity must be a "universal" (i.e., sample independent) function of the variable* $\tilde{T}$.

The flux-flow resistance is given by[8]

$$R/R_N = 2\pi n^F \xi^2 \ , \tag{2}$$

where $R(R_N)$ is the resistance (normal-state resistance) and n^F is the density of free vortices. A possible explicit definition of n^F is given by[9]

$$\lambda^{-2} = \frac{2\pi n^F}{\xi \tilde{T}} \ , \tag{3}$$

where λ is the screening length of a potential outside an infinitesimal test charge. The important point is that $n^F \xi^2$ is a dimensionless thermodynamic quantity. Thus it follows that R/R_N *should be a "universal" function of* $\tilde{T}$.

The 2D Coulomb gas has a Kosterlitz-Thouless transition at the T_c given by[6,7]

$$\tilde{T}_c = T_c/q^2 = 1/4\epsilon(\tilde{T}_c) \ , \tag{4}$$

where $\epsilon(\tilde{T})$ is the dielectric constant of the Coulomb gas.[10] Note that ϵ is a dimensionless thermodynamic quantity and hence it follows that $\epsilon(T_c)$ is a "universal" constant. Thus it also follows that R/R_N should be a "universal" function of the variable $X = \tilde{T}/\tilde{T}_c$. In terms of the superconductor parameters the parameter X is given by (see Table I)

$$X = \frac{T}{T_c^0 - T} \frac{T_c^0 - T_c}{T_c} \ . \tag{5}$$

III. EXPERIMENTAL VERIFICATION

How can the prediction of a "universal" $R/R_N(X)$ curve be tested experimentally? We will illustrate this by measurements done on three samples. The sample parameters, the method used to obtain them, and references to the measurements are given in Table II.[11–15]

Among the potential possibilities to determine T_c^0 are: Extrapolating from below, e.g., measuring the kinetic inductance, L_k, and using $L_k^{-1} \sim T_c^0 - T$ as in Ref. 16 or measuring R in a finite perpendicular magnetic field, B, and using $R^{-1} \sim T_c^0 - T$ as in Fig. 3 of Ref. 13; extrapolating from above T_c^0 using the Aslamozov-Larkin formula as in Ref. 17; fitting to specific-heat data as in Ref. 11.

The Kosterlitz-Thouless temperature, T_c, is signaled by a rapid increase of the number of free vortices. Among the potential possibilities to observe this increase are: A rapid increase in the kinetic inductance L_k (compare Fig. 2 of Ref. 16); break down of the linear relation $R^{-1} \sim (T_c^0 - T)$ when R is measured for a fixed B (compare Fig. 3 of Ref. 13); measuring R as a function of B for fixed T and extrapolating to $B = 0$ and using the approximate condition $R|_{B=0} = 0$ for $T = T_c$ (compare Fig. 11 of Ref. 14).

TABLE II. Sample parameters. (A) Amorphous niobium-germanium sample: Fig. 3 in Ref. 11 and Figs. 5 and 6 in Ref. 12. (B) Granular aluminum sample: Fig. 3 in Ref. 13. (C) Granular aluminum sample: Figs. 10 and 11 in Ref. 14.

	A	B	C
T_c^0 (K)	2.60[a]	2.21[b]	1.99[c]
T_c (K)	<2.47[d]	2.12 ± 0.01[e]	1.67 ± 0.02[f]
$R_N(\Omega/\Box)$	63 ± 10	500	2100

[a]Specific-heat measurement gives $T_c^0 = 2.60$ K (Ref. 12). 2D Aslamazov-Larkin formula gives $T_c^0 = 2.608$ K (Ref. 15).
[b]Fit to formula $R^{-1} \sim \xi^{-2} \sim T_c^0 - T$ for $T < T_c^0$ (compare Fig. 3 in Ref. 13).
[c]2D Aslamazov-Larkin formula (Ref. 14).
[d]T_c smaller than lowest measured point (compare Fig. 3 in Ref. 11).
[e]$T_c \approx$ temperature where measured points deviates from the linear relation $R^{-1} \sim T_c^0 - T$ (compare Fig. 3 in Ref. 13).
[f]The approximate condition $R = 0$ for $T = T_c$ (compare Fig. 11 of Ref. 14).

In an "ideal" experiment T_c^0 and T_c should be determined in several ways in order to make sure that the sample is well characterized with these parameters.

In Fig. 1 we have plotted $y = (T_c^0 - T)/T$ against $\ln(R/R_N)$ for sample A. According to the universality prediction this should be a sample-independent curve up to the scale factor $(T_c^0 - T_c)/T_c$ on the y axis [compare Eq. (5)]. We may test this prediction in the following way: Take a T for sample B, this T corresponds to a measured $\ln(R/R_N)$, the $\ln(R/R_N)$ corresponds to a "universal" X, this X is up to a constant equal to y^{-1} in Fig. 1. The result is shown in

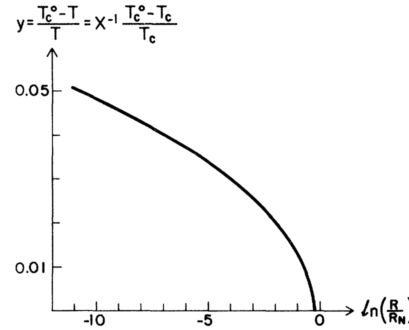

FIG. 1. Plot of resistance data for sample A. The sample parameters are given in Table II.

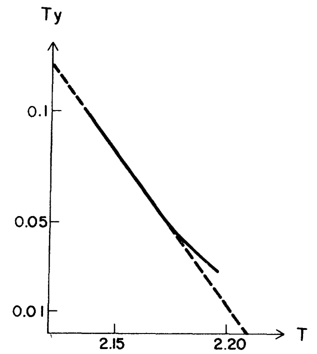

FIG. 2. A universality test applied to data from samples A and B. T refers to data from sample B and y is given in Fig. 1. The full curve results from the construction $T(B) \rightarrow \ln[(R/R_N)(B)] = \ln[(R/R_N)(A)] \rightarrow y \rightarrow T(B)y$ (see text). Universality predicts that $Ty \sim (T_c^0 - T)$. The dashed line shows that this prediction is borne out for a large portion of the resistive transition. The crossing point between the dashed line and the T axis gives T_c^0 for sample B.

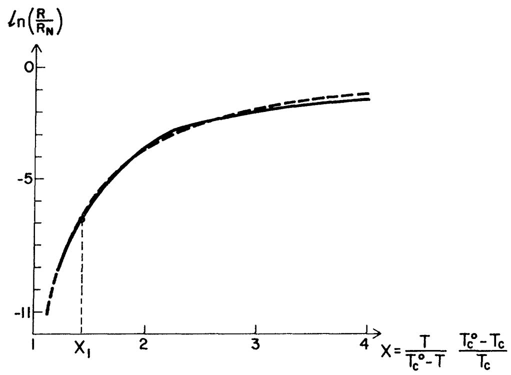

FIG. 3. Plot of $\ln(R/R_N)$ vs X. Dot at X_1: data point for sample C; full curve: data for sample B; dashed curve: data for sample A. Sample parameters are given in Table II except T_c for sample A where the value $T_c = 2.46$ obtained from the universality prediction and sample C was used (see Table III). Universality predicts that all data should fall on a single curve.

Fig. 2 where T for sample B is plotted against yT. According to the universality prediction the curve in Fig. 2 should be a straight line where the crossing point with the T axis is the T_c^0 for sample B [compare Eq. (5)]. As seen in Fig. 2 these predictions are born out to an extraordinary degree. Especially note that the T_c^0 for sample B determined from sample A and the universality prediction in Fig. 2 is identical to the T_c^0 determined by direct measurement on sample B. This and other results obtained from "universality" are given in Table III. Also note in Fig. 2 that as T_c^0 is approached the universality prediction breaks down. This breakdown is expected since the Coulomb gas model of a 2D superconductor only is valid as long as only a small fraction of the sample area consists of vortex cores.

TABLE III. Universality predictions. (The arrows indicate results obtained using the predicted universality.)

	A	B	C
T_c^0 (K)	2.60 $\xrightarrow{a}$	2.21	1.99
T_c (K)	2.46 $\xleftarrow{b}$		
	2.46 $\xleftarrow{c}$	2.12 $\xleftarrow{b}$	1.67

[a]From Fig. 2: Crossing point of dashed line with T axis.
[b]From the X_1 point on the universal curve in Fig. 3: $X_1 = [(T_c^0 - T_c)/T_c][T_1/(T_c^0 - T_1)]$ for all three samples.
[c]From Fig. 2 and the relation "slope of dashed line in Fig. 2" = $\{[T_c^0(A) - T_c(A)]/T_c(A)\}\{T_c(B)/[T_c^0(B) - T_c(B)]\}$.

In Fig. 3 we have plotted $\ln(R/R_N)$ vs X for the three samples. For sample A and C we have used the values T_c^0 and T_c given in Table II. Note that for sample C we only have data for one point (denoted by X_1 in Fig. 3). For sample A we have used T_c^0 from Table II and T_c determined from sample C and the universality prediction (see Table III). The predicted universality of the $R/R_N(X)$ curve is borne out to an extraordinary degree. Especially notice that the curves for sample A (amorphous niobium-germanium) and sample B (granular aluminum) fall almost on top of each other, with *no* adjustable parameter.

IV. VALIDITY RANGE OF ASYMPTOTIC THEORY

In Fig. 4 we have plotted $\{-\ln[(R/R_N)(4/X)]\}^{-1}$ vs X for samples A and B. Plotted in this way the "universal" curve is to good approximation a straight-line (from $X \approx 1.1$ to $X \approx 2$, say). The quantity $(R/R_N)(4/X)$ is in terms of Coulomb gas quantities given by [using Eqs. (2)–(5) and assuming that $\epsilon(\tilde{T}) \approx \epsilon(\tilde{T}_c)$]

$$\lambda^{-2}\xi^2 \approx \frac{R}{R_N}\frac{4}{X} \quad . \tag{6}$$

Thus Fig. 4 implies that

$$\lambda^2\xi^{-2} \approx \exp[A/(X-B)] \quad \text{for } 0.1 < (X-1) < 1 \quad , \tag{7}$$

where A and B are sample independent constants. This result is not inconsistent with theoretical expectations.[9,18] On the other hand Kosterlitz's renormalization-group equations predict[7]

$$\lambda^2\xi^{-2} \approx C\exp[D/(X-1)^{1/2}] \quad \text{for } 0 < (X-1) << 1 \quad , \tag{8}$$

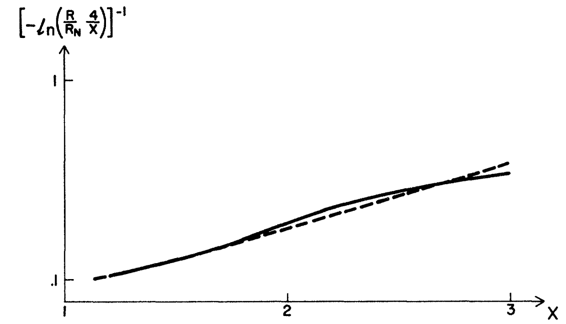

FIG. 4. Plot of $\{-\ln[(R/R_N)(4/X)]\}^{-1}$ vs X for sample A (dashed curve) and sample B (full curve) for the same sample parameters as in Fig. 3.

where C and D are constants of order unity. We will call resistance predictions based on Eq. (8) (Ref. 4) "asymptotic" since they are justified only close to T_c. Equation (7) implies that the "asymptotic" predictions can only be valid within the range $0 < (X-1) < 0.1$. For the three samples in Table III this means within a temperature range $0 < (T-T_c) \lesssim 0.01$ K and $R < 10^{-5} R_N$. Thus the implication is that the "asymptotic" predictions can only explain a tiny portion of the resistive transition. In contrast, vortex fluctuations *per se* as manifested in the universality prediction is a potential explanation of a large portion of the resistive transition.

V. CONCLUSIONS

It is shown that, if vortex fluctuations are responsible for the broad resistive transition of "dirty" thin superconducting films, then this explanation leads to an universality prediction. We tested this universality prediction against available experimental data and found very suggestive evidence for its validity.[19] In order to get conclusive evidence for its validity experiments are needed directly designed for the purpose. It was also argued that predictions for the shape of the resistive transition based on the "asymptotic" Kosterlitz renormalization-group results can only explain a small portion of the resistive transition.

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