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Interface roughening and random-field instabilities in Ising systems in three or less dimensions

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The ferromagnetically ordered state of random-field Ising systems for dimensionalities $d \leq 3$ is shown to be unstable with respect to domain formation, if the field-induced domain interface roughness is taken into account. For $d = 3$, the interface exhibits a "wiggly" structure on all length scales intermediate between the correlation length and the domain size. This instability, which follows from the assumed random-field-induced roughening, makes the lower critical dimension, $d_l = 3$, consistent with the change of the effective dimensionality by 2 for the random-field model. Experimental consequences of these results are indicated.

The influence of a random field,[1,2] which couples linearly to the order parameter, on a regular, second-order phase transition is extremely marked. The critical behavior is drastically modified and the changes in the critical exponents are much larger than those caused by most symmetry-breaking pertubations studied so far.[3] In particular, ϵ expansions in $d = 6 - \epsilon$ dimensions[4–7] show, that the critical behavior of a d-dimensional model with quenched random fields that have only short-range correlations is identical with that of a $(d-2)$-dimensional "pure" model, to all orders in ϵ. Recently, Parisi and Sourlas[8] have elegantly formulated the above proof. One is led to consider the interesting possibility that the rule $d \rightarrow d-2$ might apply even beyond where the diagrammatic expansions are applicable for Ising-like models. (For spin dimensionality $n > 1$ the correspondence works only in the range $4 < d < 6$.)

Various physical systems can be described by the random-field model: Random antiferromagnets,[2,5,9,10] or Mattis[11,12] spin-glasses in uniform magnetic fields, charge density (as well as spin density, etc.) waves coupled to impurities,[13] mixtures of competing anisotropy magnets,[14] and polymers.[15] Also the problem of the influence of impurities on *first-order* transitions[16] has similarities to the random-field model. The phase diagrams of random-field systems as functions of the amplitude of the field exhibit a rich structure.[17–19]

For the case of a random field with an extremely small amplitude, an outstanding question is what is the "lower critical dimension," d_l, below which the vanishingly small random field disrupts the ordering of the usual order parameter. Using magnetic language, where the latter ordering is ferromagnetic and the field is a random magnetic field, such an instability can be demonstrated, for example, by showing that the system is unstable against domain formation.

In the simplest picture, compact domain formation on a scale L is determined by the competition between the bulk random-field gain, which is of order $L^{d/2}$ per domain, and the surface energy loss.[2] The latter is of the order L^{d-2} for models with continuous symmetry, where the thickness of the domain wall is of order L, while for Ising-like systems the surface energy is of order L^{d-1}. These estimates lead to an instability of the ordered state for $d \leq 4$ in continuous symmetry systems and for $d \leq 2$ in the Ising-like systems. From this it strictly follows that $d_l \geq 2$ for Ising-like systems. $d_l = 2$ would follow *only if* no further, stronger instability mechanism exists. Since the reduction of the effective dimensionality by two[4–8] would imply $d_l = 3$ (d_l for the pure Ising model is 1), such a stronger instability is desirable, for $d \leq 3$, to avoid a contradiction for the value of d_l. Here, we shall show that the random-field induced roughness[20,21] of the Ising interface below and at $d = 3$, can be used to construct such an instability. This provides a physical picture for the random-field phase at $d \leq 3$. The domain sizes will be estimated and ensuing phase diagrams indicated. We emphasize that d_l is therefore just the physical dimension $d = 3$. The rough Ising interface is an intermediate case between the sharp Ising interface for sufficiently high d and the extremely diffuse domain wall for continuous symmetry systems.

We consider for definiteness the usual ferromagnetic model,

$$\mathcal{H} = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j - \sum_i h_i \sigma_i \ ,$$

with

$$J > 0, \quad \langle h_i \rangle = 0, \quad \langle h_i h_j \rangle = h^2 \delta_{ij} \ .$$

We now consider the interface between up and down compact domains of linear size L, with $L >> \xi_T$, where ξ_T is the usual Ising correlation length. This interface for the pure case $h = 0$ is known to have a rough phase below $d = 3$, in the sense that the characteristic domain-wall thickness $w_d(L)$ diverges with L like[20]

$$w_d(L)/\xi_T \sim O((L/\xi)^{(3-d)/2}), \quad d<3 \quad . \tag{1}$$

The interface (for $T>0$) is always structured on the microscopic scale ($<<\xi_T$), but it is smooth, with large fluctuations in the rough phase, see Fig. 1, on the macroscopic ($>>\xi_T$) scale. However, the interface energy even then is of order L^{d-1}, in the usual, $h=0$ case.[20] One may now attempt to distort the actual interface, so that a maximal random-field negative surface energy is gained. However, one can only gain an energy on the order of hL^{d-1} out of this, which cannot compete with the exchange energy, for $h<<J$.

We note, however, that if the expected reduction of the effective d by 2 were valid, then one should expect a similar reduction for the roughening problem, with appropriate changes in d in Eq. (1) which should *raise the critical dimension for roughening from three to five.* This is in fact borne out by calculations done by Pytte, Imry, and Mukamel[21] using the replica method. Independently of these calculations we establish here a particular instability by physical considerations, *assuming* the appropriate dimensional change in Eq. (1), and showing that it yields self-consistently the expected instability. We therefore assume that in the presence of the random field h, Eq. (1) is replaced in the range of d of interest, $d<5$, by

$$w_d(L) \propto \frac{h}{J} L^{(5-d)/2} \quad . \tag{2}$$

Since for $d \leq 3$, Eq. (2) yields $w_d(L) \geq L$, it is immediately suggested that for $d<3$ the notion of stable domain is meaningless for large enough L since for any $h \neq 0$, however small, the domain wall would sweep a range larger than the domain itself. Thus, the system will split to finite domains. To establish this in more detail and to get an estimate for the finite domain sizes, we note that the effective area of the rough domain wall for $d<3$ with $w_d(L) >> L$ would thus be $L^{d-2}w_d(L)$ (all lengths being measured in some microscopic length unit). Since for each piece of area ξ^{d-1} one can gain an energy of at least of the order $h\xi_T^{d/2}$ by microscopic distortion (Fig. 2) of the interface, one obtains for the total gain E_L:

$$E_L \gtrsim (L/\xi_T)^{d-1} w_d(L) \xi_T^{d/2} (h/L) \xi_T^{-\beta/\nu} \quad , \tag{3}$$

where the last factor represents the reduction of the underlying magnetization near T_c, β and ν are the usual critical exponents. At the same time the interface free energy price is on the order of $JL^{d-1}(\xi_T)^{-(d-1)}$, where the last factor represents the critical vanishing of this energy as $T \rightarrow T_c^-$. We thus obtain a negative total interface free energy, from (2) and (3), provided that

$$L \gtrsim L_0 \equiv \xi_T^{-\gamma/\nu(3-d)} \left(\frac{J}{h} \right)^{4/(3-d)} \quad , \tag{4}$$

where we used the usual scaling relations. L_0 will set

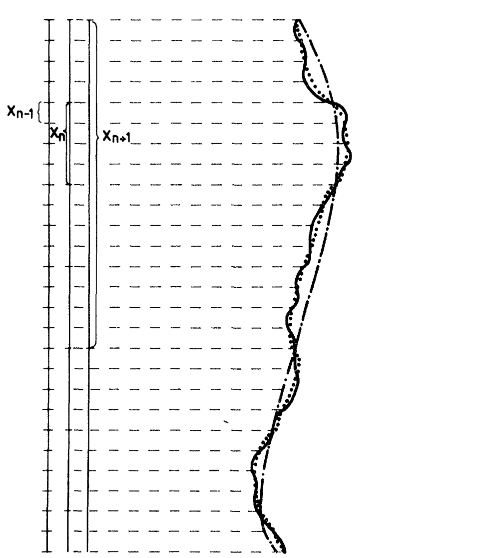

FIG. 1. Domain wall on scales x_n ($\xi_T << x_n << L$) in the rough phase. The "diverging" thickness $w_d(L)$ is due to large wavelength fluctuations. The interface is smooth locally but its fluctuations that are $w_d(x_n)$ on scale x_n accumulate to $O(w_d(L))$ on the scale L.

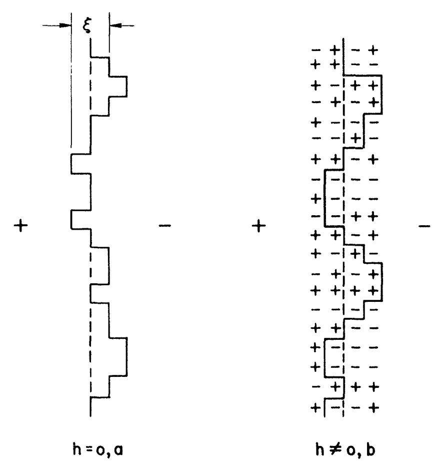

FIG. 2. Typical actual interface between up (+) and down (−) domains, on the scale of several ξ_T. It actually is rough on the scale ξ_T due to thermal fluctuations but will appear smooth on a scale $>>\xi$. Shown are the $h=0$ case (a) and the $h \neq 0$ case (b) where, without changing the structure much, the interface now has microscopically distorted so as to attach more sites with negative h_i to the down domain and likewise for the positive h_i sites.

the scale for the spontaneous domains formed according to this particular mechanism by the random field for $d < 3$. For very small values of h/J these domains will be very large; in fact, the above estimates are valid only when $L_0 \gtrsim \xi_T$. The condition $L_0 \sim \xi_T$ should signify how close one can get towards the $h = 0$ T_c before the correlations in the system become affected by the random field. This suggests that the extent of broadening of the transition by the field, ΔT_c, should be given by

$$L_0 \sim \xi_T, \text{ or } \Delta T_c/T_c \sim \xi_0^{1/\nu}\left(\frac{h}{J}\right)^{4/[\gamma+\nu(3-d)]} . \quad (5)$$

The physical case $d = 3$ is very special. The gain of Eq. (3) per unit area is L independent. Therefore just increasing L cannot make the total interface energy negative. In fact, the exponent in Eq. (4) blows up as $d \to 3$ for every $h < J$. Thus the case $d = 3$ needs a more general calculation. We consider simultaneous adjustment of the interface on many intermediate scales x_n, instead of using only the length scale ξ as above (Fig. 1). The excess of random field in a volume $x_n^{d-1} w_d(x_n)$ (extending over an area x_n^{d-1} of the interface) is practically statistically independent of the excess in a volume $x_{n+1}^{d-1} w_d(x_{n+1})$ if $x_{n+1} >> x_n$. The contribution of scale x_n to the energy of a piece of scale L is then from Eq. (3) of the order of $\xi_T^{-\beta/\nu} h (L/\xi_T)^{d-1} x_n^{-1} w_d(x_n) \xi_T^{d/2}$. The total gain is hence obtained by summing over all scales,

$$E_L = \xi_T^{-\beta/\nu - d/2+1} h L^{d-1} \sum_{n=1}^{n_{\max}} x_n^{-1} w_d(x_n)$$

$$\cong \xi^{-\beta/\nu - d+5/2} \sum_{n=1}^{n_{\max}} c^{(3-d)n/2} \frac{h^2}{J} L^{d-1} ,$$

where, for instance, $x_n = \xi c^{n-1}$, with c being a constant sufficiently larger, but of the order of unity (e.g., $c = 4$ in Fig. 1), and $x_{n_{\max}} = L$. Since $L >> \xi$, $n_{\max} >> 1$; the summation would diverge for $n_{\max} \to \infty$ in the case $d \leqslant 3$. For $d < 3$, it is given essentially by the largest term ($n = n_{\max}$), which will reduce to our previous estimate (3). For $d > 3$, the sum converges and thus $E_L = hO(L^{d-1})$, which cannot compete with the interface energy for $h << J$. For the special case of $d = 3$, all the terms in the sum are equal, and thus

$$E_{L,d=3} = (h^2/J)\xi^{-\beta/\nu - 1/2} L^2 n_{\max}$$

$$\sim \frac{h^2}{J} L^{d-1} \xi^{(-2\beta+\nu)/2\nu} \ln\frac{L}{\xi_T} .$$

Equating this again to the positive surface-free energy $\xi^{-2} J L^2$, we find for the minimum domain size

$$L_0 \sim \xi_T \exp[(J/h)^2 \xi_T^{(2\beta-3\nu)/2\nu}(\text{const})] , \quad (6)$$

and the T_c smearing is again given by $L_0 \sim \xi_T$, or $\Delta T_c/T_c \propto \xi_0^{1/\nu}(h/J)^{4/\gamma}$, which agrees with Eq. (5) at $d = 3$.

The resulting phase diagram for the Ising systems at $d \leqslant 3$ with random fields is indicated in Fig. 3. The ferromagnetic state exists for $h = 0$ only. For $T = 0$ one has weak singularities at all rational values of h/J for $h < h_c$, while for $h_c < h < 2h_c$ a sequence of well isolated singularities occurs.[18] The dashed line physically shows up as a line of smeared phase transitions between a "paramagnetic" phase with short-range ferromagnetic correlations to a rather complicated "domain" phase. [The T_c smearing is given by Eqs. (5) and (6).] In the latter state, the instability of the ferromagnetic state found in this paper will cause an arrangement of domains whose size is larger than L_0. This domain phase implies a sort of spin-glass ordering[18] similar to the one exhibited by the Mattis model.[11,12]

To summarize, we have physically demonstrated an instability of the "rough-interface" phase of Ising-type systems at $d \leqslant 3$ to quenched random ordering fields where we considered self-consistently the interface roughness induced by the random field itself. This instability is stronger than the simpler one demonstrated for Ising-like systems at $d \leqslant 2$ in Ref. 2. The lower critical dimension for the random-field model thus just coincides with the physical dimension $d = 3$. This removes the inconsistency with the $d \to d-2$ mapping. Characteristic domain sizes and T_c broadening were estimated and qualitative phase diagrams indicated. Our work implies that experi-

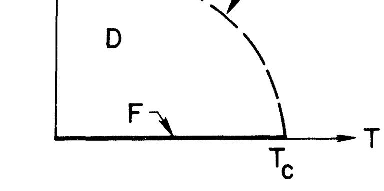

FIG. 3. Schematic phase diagram in the $(h/J,T)$ plane for $d < 3$. The ferromagnetic phase (F) exists only for $h = 0$. A domain state (D) exists at low temperatures, and $h \neq 0$, its transition $h_c(T)$, broken line, to the high-temperature paramagnetic (P) phase is probably "smeared." The broken h/J axis for $h/J > 1$ signifies the singular many-phase structure (Ref. 18).

ments on $d = 3$ random-field systems would be extremely instructive. In fact, the agreement with existing experiments[10,14] is quite encouraging. In particular, the transition found in Ref. 10 for the paramagnetic-dilute antiferromagnetic case is substantially broadened by the uniform field, which is random for this order parameter. For the same sample the paramagnetic spin flopped transition, for which the field is not random, remains sharp, indicating no sizable macroscopic inhomogeneity. The random-field broadening thus appears to be an intrinsic effect. As mentioned before, another implication of these results is that the (rather common) first-order transitions in impure $d = 3$ systems[16] should often exhibit some rounding. The results of this letter are substantiated by expansions in ϵ for $d = 3 + \epsilon$ which can be performed for this model.[21]

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[1] P. Lacour-Gayet and G. Toulouse, J. Phys. (Paris) 35, 425 (1974).
[2] Y. Imry and S.-K. Ma, Phys. Rev. Lett. 35, 1399 (1975).
[3] A. Aharony, J. Magn. Magn. Mater. 7, 198 (1978).
[4] G. Grinstein, Phys. Rev. Lett. 37, 944 (1976).
[5] A. Aharony, Y. Imry, and S.-K. Ma, Phys. Rev. Lett. 37, 1367 (1976).
[6] N. Nakanishi, Prog. Theor. Phys. Suppl. 18, 1 (1961).
[7] A. P. Young, J. Phys. C 10, L257 (1977).
[8] G. Parisi and N. Sourlas, Phys. Rev. Lett. 43, 744 (1979).
[9] S. Fishman and A. Aharony, J. Phys. C 12, L729 (1979).
[10] H. Rohrer, A. Aharony, and S. Fishman, J. Magn. Magn. Mater. 15–18, 396 (1980); H. Rohrer, J. Appl. Phys. 52, 1708 (1981); H. Rohrer and H. J. Scheel, Phys. Rev. Lett. 44, 876 (1980).
[11] D. C. Mattis, Phys. Lett. 56A, 421 (1976).
[12] A. Aharony and Y. Imry, Solid State Commun. 20, 899 (1976).
[13] B. R. Patton and L. J. Sham, Phys. Rev. B 13, 3151 (1976); L. H. Fukuyama and P. A. Lee, *ibid.* 17, 535 (1978); P. A. Lee and T. M. Rice, *ibid.* 19, 3970 (1979).
[14] P. Z. Wong, P. A. Horn, R. J. Birgeneau, C. R. Safinya, and G. Shirane, Phys. Rev. Lett. 45, 1974 (1980).
[15] G. Parisi and N. Sourlas Phys. Rev. Lett. 46, 871 (1981).
[16] Y. Imry and M. Wortis, Phys. Rev. B 19, 2042 (1979).
[17] A. Aharony, Phys. Rev. B 18, 3318, 3328 (1978).
[18] I. Morgenstern, K. Binder, and R. M. Hornreich, Phys. Rev. B 23, 287 (1981).
[19] D. P. Landau, H. H. Lee, and W. Kao, J. Appl. Phys. 49, 1356 (1978).
[20] J. D. Weeks, in *Ordering in Strongly Fluctuating Condensed Matter Systems,* edited by T. Riste (Plenum, New York, 1980), p. 293; see also B. Widom, in *Phase Transitions and Critical Phenomena,* edited by V. Domb and M. S. Green (Academic, New York, 1977), p. 79. The simple equation (1) follows from the interface capillary modes.
[21] E. Pytte, V. Imry and D. Mukamel, Phys. Rev. Lett. 46, 1173 (1981); A. Aharony, Y. Shapir, and J. Rudnick (unpublished); D. Mukamel and E. Pytte (unpublished).