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Static susceptibilities of the hydrodynamic order-parameter variables of ^{3}He-A

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The $1/k^2$ divergence of the static susceptibilities of the hydrodynamic order-parameter variables of ^{3}He-A is calculated by microscopic means. It is shown that the self-consistent gap equation plays a crucial role in discriminating the hydrodynamic and microscopic variables. The Bardeen-Cooper-Schrieffer approximation and the Landau correction of the superfluid densities in spin and real space and of $c_\perp$, $c_{\parallel}$, K_B, K_T, and K_s are determined. The results for the superfluid densities are exact within the mean-field approximation, and we find that $\rho^s_{\parallel} = -2c_{\parallel}$. As far as $c_\perp$, K_B, K_T, and K_s are concerned, the Landau parameters up to $l=3$ have been included. It is shown that K_s is only influenced by F^s_3. The results are compared with those of other work and their effect on experimentally accessible quantities is discussed.

I. INTRODUCTION

The pioneering work of Landau[1] on liquid ^{3}He has given the possibility of describing all essential features of this strong-coupling system by a set of parameters F^s_l and F^a_l. Unfortunately, only F^s_0, F^s_1, and F^a_0, which enter the compressibility, the effective mass, and the spin susceptibility, are accessible to experiments. All other Landau parameters are unknown and often assumed to vanish as far as theoretical work is concerned. It is a different matter if ^{3}He turns superfluid. The existence of a p-wave order parameter gives rise to a new set of static susceptibilities which enter the static and dynamic behavior of the system under investigation. The symmetry of the order parameter allows $F^{s,a}_1$ and $F^{s,a}_3$ to influence macroscopic measurable susceptibilities. Up to now, all theoretical work on ^{3}He-A neglects the Landau parameters altogether or takes into account only $F^{s,a}_1$ and neglects the higher ones.

As far as the B phase is concerned we have shown in a previous paper[2] that the static suceptibilities of $\vec{\theta}$ (the hydrodynamic order-parameter variable in spin space) is uniquely determined by F^a_1 and F^a_3. This result is rigorous in the whole temperature regime. In real space, the B phase has a very simple structure. The only additional hydrodynamic variable is the phase ϕ. Thus, the real-space dynamics of ^{3}He-B is isomorphic to a neutral s-wave superconductor and it is not surprising to find that F^s_1 is the only Landau parameter entering the static susceptibility of ϕ in real space. The A phase, on the other hand, has a much more complicated symmetry in real space; therefore, one can hope to get more information on the higher symmetric Landau parameters from this phase.

In this paper we calculate the static suceptibilities of the hydrodynamic order-parameter variables of ^{3}He-A in spin and real space, respectively. We derive the mean-field result containing all Landau parameters for the superfluid densities in spin and real space, and we show that at intermediate temperatures all Landau parameters contribute to the static susceptibilities. At very low temperatures, only $F^{s,a}_1$ survives in the case of the superfluid densities. As far as the elastic constants of $\vec{1}$ are concerned all Landau parameters must be taken into account due to the anisotropy of the gap.

In order to give explicit results, we have to assume that the higher Landau parameters ($l \geq 5$) can be neglected. It will be shown, that one of them is only influenced by F^s_3, not by F^s_1. The inclusion of higher Landau corrections generates five new temperature-dependent functions, generalizations of $\rho^s_\perp$ and $\rho^s_{\parallel}$. The influence of $F^{s,a}_3$ on the static susceptibilities of the hydrodynamic order parameter variables are shown to be non-negligible, although smaller than that of $F^{s,a}_1$. A definition of new renormalized Landau parameters which describe the temperature dependence of the static susceptibilities as the variation of an effective Landau parameter, equivalent to the B phase is not possible.

Some of the aforementioned results can be compared with previous results of Ambegaokar, de Gennes, and Rainer,[3] who calculated the gradient part of the free energy $F_{\rm grad}$ in the Ginzburg-Landau regime. We reproduce their results if we restrict ourselves to the lowest order in $1-T/T_c$. More closely related to our work is that of Cross,[4] who also determined $F_{\rm grad}$. His results are valid in the whole temperature regime and include the first Landau parameters F_1^s and F_1^a. We arrive, although on a quite different path, at results identical to his if we neglect all corrections due to Landau parameters $F_l^{s,a}$ with $l>1$.

The method applied is a generalization of a procedure used by Leggett[5] to investigate the static and dynamic properties of the s-wave superconductor. We extend it to the case of triplet pairing and then calculate the linear response function of the order-parameter variables.

A detailed discussion of the method is given in Ref. 2, and we restrict ourselves to a summary stressing only those points that are peculiar to the A phase. In particular, we point out that care has to be taken that the statistical ensemble is well defined in order to avoid the occurrence of unphysical hydrodynamic modes. This is done in Sec. II. In the following section we calculate the order-parameter susceptibilities in the weak-coupling limit. Section IV is devoted to the Fermi-liquid corrections and contains the central results of this paper. In the last section we discuss the results, in particular, the experimental implication, and make a comparison with other work.

II. GENERAL REMARKS

The order parameter A_{ik} of the axial A phase consists of a real unit vector in spin space and a complex vector in real space. Thus, two continuous symmetries are broken in spin space and three in real space. To each of the broken symmetries belongs an additional hydrodynamic—that is, slow—variable. Because the Hamiltonian is invariant against global rotations and phase transformations, it costs no energy to rotate the ground state homogeneously, and the smaller the deviation from the equilibrium, the more time the system will require to restore it. In other terms, the inverse static susceptibilities of the slow order-parameter variables measuring the restoring forces must vanish if the perturbation becomes uniform and their static susceptibilities are assumed to behave as $1/k^2$ for $\vec{k}\to 0$.[6] In the following we will calculate them by microscopic means.

The complex order parameter of superfluid ^{3}He is defined as $A_{ij}^0=\langle \hat{A}_{ij}\rangle$, where

$$\hat{A}_{ij}=\int d^3x\,F^*(\vec{x})\hat{\psi}_\alpha(\vec{r}-\tfrac{1}{2}\vec{x})\hat{\psi}_\beta(\vec{r}+\tfrac{1}{2}\vec{x})\times\frac{x_j}{|\vec{x}|}(\sigma_i\sigma_2)_{\alpha\beta}\,. \tag{2.1}$$

If not indicated otherwise, the caret denotes that the quantity under investigation is an operator. Its expectation value is denoted by a superscript 0 or $\langle\cdots\rangle$. Sums over repeated indices are always implied. Greek indices take the values 1,2; italic indices take 1,2,3. The unit vector $x_j/|\vec{x}|$ selects the p-wave part of the anomalous expectation value, whereas $(\vec{\sigma}\sigma_2)$ gives the projection onto the triplet spin axes. $F^*(\vec{x})$ is chosen in such a way to ensure always that

$$A_{ij}A_{ij}^*=1\,. \tag{2.2}$$

In the A phase A_{ij} factorizes into two vectors:

$$A_{ij}=n_i\Delta_j\,. \tag{2.3}$$

$\vec{n}$ is a real unit vector $\vec{n}^2=1$, Δ_j a complex vector with $\vec{\Delta}\cdot\vec{\Delta}=0$ and $\vec{\Delta}\cdot\vec{\Delta}^*=1$. A small deviation from the equilibrium can be parametrized by

$$\delta\Delta_j(\vec{x})=i\Delta_j^0\delta\phi(\vec{x})-l_j^0\Delta_k^0\delta l_k(\vec{x})\,, \tag{2.4}$$

where $\vec{l}=i\vec{\Delta}\times\vec{\Delta}^*$. The definitions of $\vec{\Delta}$ and $\vec{l}$ imply that $\vec{l}^0\delta\vec{l}(\vec{x})=0$ ($\vec{l}$ breaks the rotational symmetry in real space, $\vec{n}$ the rotational symmetry in spin space).

The small dipole-dipole coupling of spin and real space will be neglected in the following, and we may choose $\vec{n}^0$ parallel to the 3-axis, the real part of $\vec{\Delta}$ parallel to the 1-axis, and the imaginary part parallel to the 2-axis. The equilibrium value of the order parameter A_{ij}^0 takes the form

$$A_{ij}^0=\frac{1}{\sqrt{2}}\begin{bmatrix}0&0&0\\0&0&0\\1&i&0\end{bmatrix}, \tag{2.5a}$$

and $\vec{l}^0$ is parallel to the 3-axis. There are five hydrodynamic variables, δn_1 and δn_2 in spin space and $\delta\phi$, δl_1, and δl_2 in real space, which give rise to a large number of static susceptibilities. The $1/k^2$ dependence is common to all of them. It can be shown by symmetry arguments[7] that there is no coupling between real- and spin-space variables. Thus, susceptibilities of the type $\chi_{\delta n_1,\delta\phi}$ or $\chi_{\delta n_1,\delta l_1}$ vanish. Within the real space all couplings between $\delta\phi$, δl_1, and δl_2 exist, but the system can be further simplified if we restrict the wave vector $\vec{k}$—without loss of generality—to the 1-3 plane. Then δl_1 becomes decoupled from $(\delta\phi,\delta l_2)$ and can be dealt with independently. In the appropriate coordinate system the operators $\delta\hat{n}_i$, $\delta\hat{l}_i$, and $\delta\hat{\phi}$ can be built up from linear combinations of $\hat{A}_{ij}(\vec{x})$ and can be written

$$\begin{aligned}
\delta\hat{n}_1 &= \frac{1}{\sqrt{8}}[(\hat{A}_{11}+\hat{A}_{11}^\dagger)+i(\hat{A}_{11}^\dagger-\hat{A}_{11})]\ ,\\
\delta\hat{n}_2 &= \frac{1}{\sqrt{8}}[(\hat{A}_{21}+\hat{A}_{21}^\dagger)+i(\hat{A}_{21}^\dagger-\hat{A}_{21})]\ ,\\
\delta\hat{\phi} &= -\frac{1}{\sqrt{8}}[(\hat{A}_{32}+\hat{A}_{32}^\dagger)-i(\hat{A}_{31}^\dagger-\hat{A}_{31})]\ ,\\
\delta\hat{l}_1 &= \frac{1}{\sqrt{8}}(\hat{A}_{33}+\hat{A}_{33}^\dagger)\ ,\\
\delta\hat{l}_2 &= \frac{i}{\sqrt{8}}(\hat{A}_{33}^\dagger-\hat{A}_{33})\ .
\end{aligned} \tag{2.5b}$$

Owing to the axial symmetry of the A phase nine different parameters are involved. We get only two superfluid densities ($M_\perp$ and $M_{||}$) in spin space because of the degeneration of the variables δn_1 and δn_2. The static susceptibility of $\delta\phi$ contains[8] two superfluid densities $\rho_\perp^s$ and $\rho_{||}^s$, whereas the static susceptibilities of vector field $\vec{1}$ are described by the Franck constants K_1, K_2, K_3. From the coupling between $\delta\phi$ and $\delta l_2 (\chi_{\delta\phi,\delta l_2})$ we get $c_\perp - c_{||}$. The constants $c_\perp$ and $c_{||}$ describe the contribution of curl $\vec{1}$ to the number current. If we restrict our attention exclusively to the hydrodynamic order-parameter variables, we cannot separate the two constants. $c_\perp$ can be obtained calculating the static susceptibility of δl_1 and g_2—the momentum perpendicular to $\vec{1}^0$ and $\vec{k}$.

In order to identify the nine parameters we compare our microscopic results with the results of the phenomenological theory[7] based on symmetry arguments which predict for the static susceptibility in spin space:

$$\chi_{\delta n_1,\delta n_1}^{-1}(\vec{k}) = \chi_{\delta n_2,\delta n_2}^{-1}(\vec{k}) = \frac{1}{4m^2}(M_\perp k_\perp^2 + M_{||}k_{||}^2) \tag{2.6}$$

It will prove to be more convenient to identify the constants involved, making use of inverse static susceptibilities. Although the inverse static susceptibilities of the hydrodynamic order-parameter variables contain parts due to the coupling to the momentum density, the static susceptibilities themselves to not depend on $\vec{g}$, i.e., they remain unchanged if the normal fluid velocity is locked. In this paper, we are exclusively interested in the static susceptibilities of $\delta\vec{1}$ and $\delta\phi$. Therefore, we take the inverse of a submatrix of static susceptibilities, which contains only the contributions of the hydrodynamic order-parameter variables, *not* the inverse of the entire matrix of static susceptibilities. $\chi_{\delta l_1,\delta l_1}(\vec{k})$ proves to be independent from the rest, and the phenomenological theory states

$$[\chi_{\delta l_1,\delta l_1}(\vec{k})]^{-1} = \frac{1}{4m^2}(K_1 k_\perp^2 + K_3 k_{||}^2)\ . \tag{2.7}$$

[The right-hand side of Eq. (2.7) is the inverse of a single element of the susceptibility matrix and not an element of the inverted matrix.] The second group consists of $\chi_{\delta\phi,\delta\phi}(\vec{k})$, $\chi_{\delta\phi,\delta l_2}(\vec{k})$, and $\chi_{\delta l_2,\delta l_2}(\vec{k})$. We build up a matrix of them and take its inverse:

$$\begin{bmatrix} \chi_{\delta\phi,\delta\phi}(\vec{k}) & \chi_{\delta\phi,\delta l_2}(\vec{k}) \\ \chi_{\delta l_2,\delta\phi}(\vec{k}) & \chi_{\delta l_2,\delta l_2}(\vec{k}) \end{bmatrix}^{-1} = \frac{1}{4m^2} \begin{bmatrix} \rho_{||}^s k_{||}^2 + \rho_\perp^s k_\perp^2 & (c_{||}-c_\perp)k_{||}k_\perp \\ (c_{||}-c_\perp)k_{||}k_\perp & K_2 k_\perp^2 + K_3 k_{||}^2 \end{bmatrix}\ . \tag{2.8}$$

In the case of locked normal velocity, the expression in Eqs. (2.7) and (2.8) coincide with the inverse susceptibility matrix used in the Mori formalism. With $\vec{v}_n \neq 0$ expressions (2.7) and (2.8) have no physical meaning—only the static susceptibilities themselves have—but these expressions are still useful to determine the nine constants involved.

A first task in a microscopic theory is to determine the various properties predicted by the phenomenological theory—the decoupling of spin and real space and the decoupling of δl_1 from the other variables in an appropriate system of coordinates. This will be done in the next section within the Bardeen-Cooper-Schrieffer (BCS) approximation. It is very easy to prove that these features remain unchanged if we include Landau corrections. Then we may identify the static susceptibilities in a second step.

Before turning to the actual calculations of the response functions, we have to add a cautionary remark in regard to the statistical ensemble used. It is well known[6] that one has to pass from the grand canonical ensemble to a restricted ensemble if the symmetry of the ground state is lower than that of the Hamiltonian. In the case of superfluid ^{3}He it is convenient[7] to use

$$\hat{\rho}_\eta = \tfrac{1}{2}\exp\left[-\beta\left[\mathscr{H}_0 - \eta\int d^3x[\lambda_{ij}^*(\vec{x})\hat{A}_{ij}(\vec{x}) + \lambda_{ij}(\vec{x})\hat{A}_{ij}^\dagger(\vec{x})]\right]\right]$$

with

$$\hat{\mathscr{H}}_0=\hat{H}-\mu m\hat{N}-\vec{v}^{(n)}\cdot\hat{P}-\gamma\vec{h}\cdot\hat{S} . \quad (2.9)$$

$\hat{H}$ is the Hamiltonian, $\hat{N}$ the number operator, $\hat{P}$ the operator of the momentum and $\hat{S}$ the spin operator. μ, $\vec{v}^{(n)}$, $\vec{h}$, $\eta\lambda_{ij}$, and β are Lagrange parameters which are fixed by the expectation values of the associate variables. Not all matrix elements λ_{ij} differ from zero. In order to see this we have to look for the commutation relations of $\hat{A}_{ij}$ with the generators of the continuous symmetries $\hat{N}$, $\hat{L}$, and $\hat{S}$. The number operator $\hat{N}$ is the generator of phase transformations, $\hat{L}$ generates rotations in real space, and $\hat{S}$ generates rotations in spin space. Performing the commutations, we obtain

$$\langle[\hat{N},\hat{A}_{ij}]\rangle=-2A^0_{ij} , \quad (2.10)$$

$$\langle[\hat{L}_l,\hat{A}_{ij}]\rangle=i\,2\epsilon_{ljk}A^0_{ik} , \quad (2.11)$$

$$\langle[\hat{S}_l,\hat{A}_{ij}]\rangle=i\,2\epsilon_{lik}A^0_{kj} . \quad (2.12)$$

These formulas are valid for any superfluid phase of ^{3}He, not only ^{3}He-A. If we choose the coordinate system indicated above, only the matrix elements A^0_{31} and A^0_{32} and their complex conjugates do not vanish.

Equations (2.10)–(2.12) show that there are four elements, $\hat{A}_{23}$, $\hat{A}_{13}$, $\hat{A}^\dagger_{23}$, and $A^\dagger_{13}$, which commute with all three generators and have vanishing expectation values. Thus, the ground state of the A phase is not broken with respect to these four variables and the corresponding Lagrange parameters in Eq. (2.9) are zero.

If we want to calculate the static susceptibilities we must be aware that they are determined by dynamical processes on the time scales of the conserved variables. Thus, the procedure we exploit must ensure the conservation laws. On the other hand, the long-time relaxation processes are mainly influenced by quasiparticle scattering processes. A procedure that takes into account both aspects was first given by Leggett[5] for calculating the static and dynamic susceptibilities of the neutral s-wave superconductor.

III. THE BCS APPROXIMATION

A detailed description of the procedure has been given in a previous paper.[2] Therefore we will only outline the line of thought and stress only those points which are peculiar to the A phase.

The static susceptibilities of any microscopic variable can be calculated from the correlation function for imaginary times:

$$\langle\langle\hat{A}_{ik}(1)\hat{A}_{jl}(2)\rangle\rangle=\langle\hat{A}_{ik}(1)\hat{A}_{jl}(2)\rangle-\langle\hat{A}_{ik}(1)\rangle\langle\hat{A}_{jl}(2)\rangle .$$

The time ordering is always implied by the notation. This expression is related to the linear response function $\mathscr{L}$ for imaginary times:

$$\mathscr{L}(12,1'2')=G_2(12,1'2')-G(1-1')G(2-2') .$$

G_2 denotes the two-particle Green's function and G the one-particle Green's functions.

$\mathscr{L}$ is a 4×4 matrix in particle-hole space, containing all possible types of propagations (particle-particle, hole-particle, etc.). Each element of the particle-hole matrix $\mathscr{L}$ consists of 16 elements which describe the various spin orientations of the excitation. The lower right 2×2 particle-hole submatrix (the normal part of $\mathscr{L}$) contains the fluctuations of variables, which are also common in the normal fluid phase, such as density-density fluctuations. The upper left particle-hole submatrix—we denote it $\mathscr{L}^a$ (the anomalous part of $\mathscr{L}$)—is peculiar to superfluid systems. Here we find the fluctuations of the hydrodynamic *and* microscopic order-parameter variables. The nondiagonal submatrices of $\mathscr{L}$ describe the coupling of the order-parameter variables with the conserved ones.

We are only interested in the static susceptibilities of the order-parameter variables that are contained in the anomalous part $\mathscr{L}^a$:

$$\langle\langle\hat{A}^{(\dagger)}_{ik}\hat{A}^{(\dagger)}_{jl}\rangle\rangle=\int\frac{d^3q}{(2\pi)^3}\int\frac{d^3q'}{(2\pi)^3}\gamma(q)\hat{q}_k(\sigma_i\sigma_2)_{\alpha\beta}\frac{1}{\beta^2}\sum_{\omega_l,\omega_{l'}}\mathscr{L}^a_{\alpha\alpha',\beta\beta'}(\vec{q}\omega_l,\vec{q}'\omega_{l'};\vec{k}\omega_n)\gamma(\hat{q}')\hat{q}'_l(\sigma_j\sigma_2)_{\alpha'\beta'} , \quad (3.1)$$

where

$$i\gamma(q)\hat{q}_k=\int d^3x\,F(\vec{x})\frac{x_k}{|\vec{x}|}e^{i\vec{q}\cdot\vec{x}}$$

and $\hat{q}$ is the unit wave vector. The symbol $(\dagger)$ is meant to imply that both $\hat{A}_{ik}$ and $\hat{A}^\dagger_{ik}$ may be inserted.

In the following, we restrict ourselves to the leading order in T_c/T_F. In this case, $\mathscr{L}$ can be taken from the Bethe-Salpeter equation:

$$\mathscr{L}=L(1-\Gamma^{\omega}L)^{-1}\ , \tag{3.2}$$

where L is the bare quasiparticle response function and Γ^{ω} the quasiparticle irreducible vertex part. Thus, we have included the BCS approximation and the Landau corrections, but we neglect corrections of the order of $(T_c/T_F)^3$. The integration with respect to the quasiparticle excitation energy and the projection of $\mathscr{L}$ onto the different spin axes by means of the Pauli matrices $(\sigma_i\sigma_2)_{\alpha\beta}$ can be carried out immediately.

In the case of the A phase, L splits into three independent 4×4 particle-hole submatrices. Each submatrix describes the propagation of a single spin orientation which cannot change its direction during the propagation. The susceptibilities of the real-space variables $(\delta\phi,\delta l_1,\delta l_2)$ are determined by the symmetric linear combination of spin directions $|\downarrow\downarrow\rangle+|\uparrow\uparrow\rangle$. The static susceptibilities of the spin-space variables must be taken from the other 4×4 submatrices that belong to the antisymmetric linear combination of spins $|\uparrow\uparrow\rangle-|\downarrow\downarrow\rangle$. The 2×2 anomalous part of the symmetric linear combination $|\downarrow\downarrow\rangle+|\uparrow\uparrow\rangle$ is coupled to the spin-symmetric part of the Landau corrections (Γ^s), whereas $|\uparrow\uparrow\rangle-|\downarrow\downarrow\rangle$ is coupled to the spin-antisymmetric Landau corrections (Γ^a).

We can establish an integral equation for the spin-symmetric and spin-antisymmetric part of $\mathscr{L}^a$, summing all possible couplings to the normal channels and redefining the bare anomalous response function $L^a\to\widetilde{L}^a$:

$$\mathscr{L}^a=\widetilde{L}^a(1-\Gamma^{\phi}\widetilde{L}^a)^{-1}\ . \tag{3.3}$$

$\widetilde{L}^a$ is a 2×2 matrix in particle-hole space. (In the following, we will drop the superfluous index a.) For the spin-symmetric case, the elements of $\widetilde{L}$ read

$$\begin{aligned}\widetilde{L}^{11}&=-G^-G-\tfrac{1}{2}FF(\hat{q}_x^2-\hat{q}_y^2)+2\hat{q}_xGF\Gamma^R\hat{q}_xGF\ ,\\ \widetilde{L}^{22}&=-G^-G+\tfrac{1}{2}FF(\hat{q}_x^2-\hat{q}_y^2)+2\hat{q}_yGF\Gamma^R\hat{q}_yGF\ ,\\ \widetilde{L}^{12}&=-iFF\hat{q}_x\hat{q}_y+2i\hat{q}_xGF\Gamma^R\hat{q}_yGF\ ,\\ \widetilde{L}^{21}&=iFF\hat{q}_x\hat{q}_y-2i\hat{q}_yGF\Gamma^R\hat{q}_xGF\ .\end{aligned} \tag{3.4}$$

(Performing a rotation in particle-hole space, it can be shown[9] that only one element of the normal channels contributes to the anomalous part of $\mathscr{L}$.) The ξ-integrated bare response functions read

$$G^-G+|\vec{d}\,|^2FF=a^2v(0)\int d\xi\frac{1}{\beta}\sum_{\omega_n}\frac{1}{\omega_n^2+\xi^2+|\Delta|^2}-\tfrac{1}{2}v_F^2\frac{(\hat{q}\cdot\vec{k})^2}{|\Delta|^2}\lambda(\hat{q},T)\ , \tag{3.5}$$

$$|\vec{d}\,|^2FF=\lambda(\hat{q},T)+\tfrac{1}{6}v_F^2(\hat{q}\cdot\vec{k})^2\frac{\partial}{\partial|\Delta|^2}\frac{1}{|\vec{d}\,|^2}\times\lambda(\hat{q},T)\ , \tag{3.6}$$

$$GF=\tfrac{1}{2}v_F^2\frac{\hat{q}\cdot\vec{k}}{\Delta_0|\vec{d}\,|^2}\lambda(\hat{q},T)\ , \tag{3.7}$$

$$GG+|\vec{d}\,|^2FF=-a^2v(0)Y(\hat{q},T)\ . \tag{3.8}$$

$v(0)$ is the density of state at the Fermi surface, v_F the Fermi velocity, $d_i=A_{ij}^0\hat{q}_j$, and $|\Delta|^2=\Delta_0^2|\vec{d}\,|^2$. $Y(\hat{q},T)$ is the anisotropic Yoshida function

$$Y(\hat{q},T)=\int_0^{\infty}d\xi\frac{\beta}{2}\operatorname{sech}^2\frac{\beta}{2}(\xi^2+|\Delta|^2)^{1/2} \tag{3.9}$$

and

$$\lambda(\hat{q},T)=\tfrac{1}{2}a^2v(0)[1-Y(\hat{q},T)]\ .$$

Γ^R is defined by the integral equation

$$\Gamma^R=\Gamma^s\{1-\Gamma^s[GG+\tfrac{1}{2}(\hat{q}_x^2+\hat{q}_y^2)FF]\}^{-1}\ . \tag{3.10}$$

The last part of each element in Eq. (3.4) is due to the coupling to the normal channels and contains all Landau corrections, which will be denoted by $\widetilde{L}^{\alpha\beta}|_{\text{Landau}}$, whereas the remainder of $\widetilde{L}^{\alpha\beta}$ is denoted by $\widetilde{L}^{\alpha\beta}|_{\text{BCS}}$.

In order to get the response function due to the spin-antisymmetric part, we have to exchange $\hat{q}_x\leftrightarrow\hat{q}_y$, $i\to-i$, and $\Gamma^s\to\Gamma^a$ in Eqs. (3.4) and (3.10). In the following, we will restrict ourselves to the spin-symmetric part (i.e., to the real-space susceptibilities) and only mention how the spin-space susceptibilities can be obtained at the end of this section. Furthermore, in our presentation, we will drop the Landau corrections altogether and only deal with the BCS part of Eq. (3.4). The generalization necessary to include all Landau parameters will be introduced later.

The interaction vertex $\Gamma^{\phi}(\hat{q}\cdot\hat{q}')$ describes the attractive pair interaction. In the case of p-wave pairing, it can be taken as

$$\Gamma^{\phi}(\hat{q}\cdot\hat{q}')\to\Gamma^{\phi}\hat{q}_i\hat{q}_i'\ , \tag{3.11}$$

and $\widetilde{L}^{\alpha\beta}$ is transformed into

$$\widetilde{L}_{ij}^{\alpha\beta}=\int\frac{d\Omega}{4\pi}\hat{q}_i\hat{q}_j\widetilde{L}^{\alpha\beta}(\hat{q},\vec{k})|_{\text{BCS}}=\widetilde{L}_{ij}^{\alpha\beta}|_{\text{BCS}}\ . \tag{3.12}$$

Γ^{ϕ} is now a constant describing the strength of the attractive pair interaction. Inserting Eqs. (3.11) and (3.12) into Eq. (3.3), the system of integral equations (3.3) has become a system of algebraic equations. Making use of Eq. (3.1) and taking into consideration all transformations we have performed in the meantime, we can establish an expression for the static susceptibilities of $\langle\langle \hat{A}_{3i}\hat{A}_{3j}\rangle\rangle$:

$$\begin{bmatrix} \langle\langle(\hat{A}^\dagger_{3i}-A_{3i})(\hat{A}^\dagger_{3j}-\hat{A}_{3j})\rangle\rangle & \langle\langle(\hat{A}^\dagger_{3i}-\hat{A}_{3i})(\hat{A}^\dagger_{3j}+\hat{A}_{3j})\rangle\rangle \\ \langle\langle(\hat{A}^\dagger_{3i}+A_{3i})(\hat{A}^\dagger_{3j}-\hat{A}_{3j})\rangle\rangle & \langle\langle(\hat{A}^\dagger_{3i}+\hat{A}_{3i})(\hat{A}^\dagger_{3j}+A_{3j})\rangle\rangle \end{bmatrix} = \gamma^2\left[\frac{1}{\Gamma^\phi}(\underline{1}-\Gamma^\phi\underline{\tilde{L}})^{-1}_{ij}-\frac{1}{\Gamma^\phi}\right]. \tag{3.13}$$

The additional term $-1/\Gamma^\phi$ on the right-hand side of Eq. (3.13) can be neglected because it does not contribute to the divergence in $\vec{k}$ of the hydrodynamic order-parameter variables. The fixed spin index relfects the fact that the spin symmetric channels are decoupled from the spin-antisymmetric channels. Apparently, the singular behavior of some elements of the susceptibility matrix (3.13) must be due to a singularity of $(\underline{1}-\Gamma^\phi\underline{\tilde{L}})^{-1}_{ij}$ in the limit $k=0$.

$\Xi^{\alpha\beta}_{ij}=(\underline{1}-\Gamma^\phi\underline{\tilde{L}})^{\alpha\beta}_{ij}$ is a 6×6 matrix, because each element of the 2×2 particle-hole matrix has two real-space indices. In the case $k=0$, $\Xi^{\alpha\beta}_{ij}$ splits into four independent submatrices:

$$\begin{bmatrix} \Xi^{11}_{11} & \Xi^{12}_{12} \\ \Xi^{21}_{21} & \Xi^{22}_{22} \end{bmatrix}_{k=0}, \quad \begin{bmatrix} \Xi^{11}_{22} & \Xi^{12}_{21} \\ \Xi^{21}_{12} & \Xi^{22}_{11} \end{bmatrix}_{k=0},$$

$$\Xi^{11}_{33}\,|_{k=0},\quad \Xi^{22}_{33}\,|_{k=0}\,. \tag{3.14}$$

Making use of the gap equation,

$$1=-\Gamma^\phi\int\frac{d\Omega}{4\pi}\hat{q}_x^2(G^-G+|\vec{d}\,|^2FF)\Bigg|_{k=0}, \tag{3.15}$$

(because of the axial symmetry of the gap, $\hat{q}_x^2$ can be replaced by $\hat{q}_y^2$), one can show that the first matrix is regular whereas the second is singular. By means of the orthogonal transformation

$$\begin{bmatrix} 1 & i \\ 1 & -i \end{bmatrix}$$

one can separate the singular and the regular parts of the second matrix. It is easy to convince oneself that the variable that corresponds to the singular part is simply

$$(\hat{A}^\dagger_{32}+\hat{A}_{32})-i(\hat{A}^\dagger_{31}-\hat{A}_{31})\,, \tag{3.16}$$

which is proportional to the phase $\delta\hat{\phi}$ [compare (2.5b)]. The two other independent submatrices of $\underline{\Xi}$ (Ξ^{11}_{33} and Ξ^{22}_{33}) are likewise singular in the limit $k=0$. This can be seen by making use of an alternative form of the gap equation (see Appendix A):

$$1=-\Gamma^\phi\int\frac{d\Omega}{4\pi}\hat{q}_z^2(G^-G)\,|_{k=0}\,. \tag{3.17}$$

The corresponding variables are $\hat{A}^\dagger_{33}+\hat{A}_{33}$ and $\hat{A}^\dagger_{33}-\hat{A}_{33}$, which are proportional to $\delta\hat{l}_1$ and $\delta\hat{l}_2$.

For $k=0$, the microscopic susceptibilities, described by the regular part of $\underline{\Xi}$, and the hydrodynamic susceptibilities, described by the singular part of $\underline{\Xi}$, are completely decoupled. This does not change for small but finite $\vec{k}$, because all coupling elements between the regular and singular parts are proportional to k^2, and hence the regular part contributes only in the order k^4 to the inverse of the singular part, which is proportional to k^{-2}. Thus we may drop the regular part of $\underline{\Xi}$ altogether, and we are left with a 3×3 submatrix of $\underline{\Xi}$, whose inverse is the static susceptibilities of $\delta\phi$, δl_1, and δl_2. All its elements vanish if $k=0$ and Eqs. (3.4)–(3.10) ensure that only terms proportional to k^2 occur in the lowest nonvanishing order.

Taking into account the correct normalization of $\delta\phi$, δl_1, and δl_2, the static susceptibilities of the real-space hydrodynamic order-parameter variables read

$$\begin{bmatrix} \chi_{\delta\phi,\delta\phi} & \chi_{\delta l_2,\delta\phi} & \chi_{\delta l_1,\delta\phi} \\ \chi_{\delta\phi,\delta l_2} & \chi_{\delta l_2,\delta l_2} & \chi_{\delta l_1,\delta l_2} \\ \chi_{\delta\phi,\delta l_1} & \chi_{\delta l_2,\delta l_1} & \chi_{\delta l_1,\delta l_1} \end{bmatrix}^{-1} = 4\frac{\Gamma^\phi}{\gamma^2} \begin{bmatrix} 2-\Gamma^\phi(\tilde{L}_{11}^{11}+i\tilde{L}_{21}^{21}-i\tilde{L}_{12}^{12}+\tilde{L}_{22}^{22}) & -\Gamma^\phi(\tilde{L}_{13}^{11}+i\tilde{L}_{23}^{21}) & -\Gamma^\phi(\tilde{L}_{13}^{12}+\tilde{L}_{23}^{22}) \\ -\Gamma^\phi(\tilde{L}_{31}^{11}-i\tilde{L}_{32}^{12}) & 1-\Gamma^\phi\tilde{L}_{33}^{11} & -\Gamma^\phi\tilde{L}_{33}^{12} \\ -\Gamma^\phi(\tilde{L}_{31}^{21}-i\tilde{L}_{32}^{22}) & -\Gamma^\phi\tilde{L}_{33}^{21} & 1-\Gamma^\phi\tilde{L}_{33}^{22} \end{bmatrix} . \tag{3.18}$$

Equation (3.18) is correct up to the order $\vec{k}^2$. Each function $\tilde{L}_{ij}^{\alpha\beta}$ depends only on the wave vector $\vec{k}$. Without loss of generality, we are free to fix the plane in which $\vec{k}$ lies:

$$\vec{k}=\hat{e}_3 k_{\parallel}+\hat{e}_1 k_{\perp} .$$

$k_{\parallel}$ is the projection of $\vec{k}$ onto the $\vec{1}^0$ axis ($\vec{1}^0 \parallel \hat{e}_3$).

For simplicity, we have presented here all steps leading to Eq. (3.18) within the BCS approximation. Taking Landau corrections into account, $\Gamma^R \neq 0$, all steps may be repeated. The result (3.18) is reobtained, however, with the replacement

$$\tilde{L}_{ij}^{\alpha\beta} \to \tilde{L}_{ij}^{\alpha\beta}|_{\mathrm{BCS}}+\tilde{L}_{ij}^{\alpha\beta}|_{\mathrm{Landau}} ,$$

where

$$\tilde{L}_{ij}^{\alpha\beta}|_{\mathrm{Landau}}=\int \frac{d\Omega}{4\pi}\int \frac{d\Omega'}{4\pi}\hat{q}_i\hat{q}_j' \times \tilde{L}^{\alpha\beta}(\hat{q},\hat{q}';\vec{k})|_{\mathrm{Landau}} . \tag{3.19}$$

This result will be further evaluated in Sec. IV. Here, we first proceed within the BCS approximation.

It is straightforward to convince oneself that in the frame of coordinates chosen above the elements Ξ_{33}^{21} and $\Xi_{31}^{21}-\Xi_{32}^{22}$ of Eq. (3.18) vanish identically. Thus, the conjecture of Sec. II is confirmed. The two sets of variables $\{\delta\phi,\delta l_2\}$ and $\{\delta l_1\}$ are completely decoupled. We may point out that the comparitively simple structure of the matrix of static susceptibilities is due to the proper choice of the frame of coordinates: Any other choice would not yield two uncoupled sets of variables. Now we compare our results with Eqs. (2.7) and (2.8), which offer the most convenient way to identify the six susceptibilities involved. $(\chi_{\delta l_1,\delta l_1})^{-1}$ is determined by

$$[\chi_{\delta l_1,\delta l_1}(\vec{k})]^{-1}=-\frac{\Gamma^\phi}{\gamma^2}4\Xi_{33}^{22} . \tag{3.20}$$

Inserting G^-G and FF we find for the right-hand side of Eq. (3.20)

$$\frac{(\Gamma^\phi)^2 4v_F^2}{\gamma^2\Delta_0^2}\int \frac{d\Omega}{4\pi}\lambda(\hat{q}_\perp,T)\frac{\hat{q}_z^2}{\hat{q}_\perp^2}(\hat{q}_x^2 k_\perp^2+\hat{q}_z^2 k_{\parallel}^2)+\tfrac{2}{3}\hat{q}_z^2\hat{q}_x^2(\hat{q}_x^2 k_\perp^2+\hat{q}_z^2 k_{\parallel}^2)\frac{\partial}{\partial\hat{q}_\perp^2}\left[\frac{1}{\hat{q}_\perp^2}\lambda(\hat{q}_\perp^2,T)\right] . \tag{3.21}$$

The terms proportional to $k_\perp^2$ and $k_{\parallel}^2$ are collected, and after some integration by parts we arrive at

$$[\chi_{\delta l_1,\delta l_1}(\vec{k})]^{-1}=\frac{\rho}{4m^2}[k_\perp^2\tfrac{1}{4}\rho_\perp^{0s}+k_{\parallel}^2(\tfrac{1}{2}\rho_{\parallel}^{0s}+\tfrac{1}{3}\tau)] , \tag{3.22}$$

where Eq. (2.2) and the gap equation are exploited in order to cancel the microscopic prefactors in Eq. (3.21). The three superfluid densities are defined by

$$\rho_\perp^{0s}=\tfrac{3}{2}\int \frac{d\Omega}{4\pi}\hat{q}_\perp^2\Phi(\hat{q}_\perp,T),\quad \rho_{\parallel}^{0s}=3\int \frac{d\Omega}{4\pi}\hat{q}_{\parallel}^2\Phi(\hat{q}_\perp,T),\quad \tau=6\int \frac{d\Omega}{4\pi}\hat{q}_{\parallel}^4\hat{q}_\perp^{-2}\Phi(\hat{q}_\perp,T) . \tag{3.23}$$

ρ is the mass density of ^{3}He and $\Phi(\hat{q},T)=[1-Y(\hat{q}_\perp,T]$. The calculation of the static susceptibilities of δl_2 and $\delta\phi$ follows a similar procedure, and we obtain

$$\begin{bmatrix} \chi_{\delta\phi,\delta\phi} & \chi_{\delta\phi,\delta l_2} \\ \chi_{\delta l_2,\delta\phi} & \chi_{\delta l_2,\delta l_2} \end{bmatrix}^{-1} = \frac{\rho}{4m^2}\begin{bmatrix} k_\perp^2\rho_\perp^{0s}+k_{\parallel}^2\rho_{\parallel}^{0s} & k_\perp k_{\parallel}\rho_{\parallel}^{0s} \\ k_\perp k_{\parallel}\rho_{\parallel}^{0s} & k_\perp^2(\tfrac{1}{12}\rho_\perp^{0s}+\tfrac{1}{3}\rho_{\parallel}^{0s})+k_{\parallel}^2(\tfrac{1}{2}\rho_{\parallel}^{0s}+\tfrac{1}{3}\tau) \end{bmatrix} . \tag{3.24}$$

Therefore the static susceptibilities, defined in the phenomenological theory, are in the BCS approximation

$$\rho_{||}^{s}/\rho=\rho_{||}^{0s},\ \rho_{\perp}^{s}/\rho=\rho_{\perp}^{0s},\ (c_{||}-c_{\perp})/\rho=-\rho_{||}^{0s}\ ,$$
$$K_1/\rho=\tfrac{1}{4}\rho_{\perp}^{0s},\ K_2/\rho=\tfrac{1}{12}\rho_{\perp}^{0s}+\tfrac{1}{3}\rho_{||}^{0s}\ , \quad (3.25)$$
$$K_3/\rho=\tfrac{1}{2}\rho_{||}^{0s}+\tfrac{1}{3}\tau\ ,$$

which is a well known result.[4]

Until now, we have discussed the propagation of spin-symmetric excitations, which determines the susceptibilities of real space. In order to get the static susceptibilities of the hydrodynamic order-parameter variable in spin space $\delta\vec{n}$, we have to evaluate the propagations of spin-antisymmetric excitations. After the transformation indicated above [cf. Eq. (3.10) and below], all arguments can be repeated. There is, however, one remark to add. In the Introduction, we pointed out that the four elements $\hat{A}_{12}$, $\hat{A}_{13}^{\dagger}$, $\hat{A}_{23}$, and $\hat{A}_{23}^{\dagger}$ of the order-parameter matrix $\hat{A}_{ij}$ do not belong to any broken symmetry and do not enter the restricted ensemble. Thus all equilibrium expectation values containing any of these elements vanish identically.

δn_1 and δn_2 are degenerate with respect to rotations around the $\vec{n}^0$ axis. Therefore we can restrict ourselves to the calculation of δn_2. After some algebra, we find that $\chi_{\delta n_2,\delta n_2}$ is identical to $\chi_{\delta\phi,\delta\phi}$ if we replace the Landau parameters F_l^a by F_l^s. Within the BCS approximation, we get

$$[\chi_{\delta n_1,\delta n_1}(\vec{k})]^{-1}=[\chi_{\delta n_2,\delta n_2}(\vec{n})]^{-1}$$
$$=\frac{\rho}{4m^2}(\rho_{\perp}^{0s}k_{\perp}^2+\rho_{||}^{0s}k_{||}^2)\ . \quad (3.26)$$

In this case, the directions $\perp$ and $||$ refer to $\vec{n}^0$, which is, however, parallel to $\vec{l}^0$ by assumption.

IV. LANDAU CORRECTIONS

It is well known, that the BCS model of superfluid ^{3}He is only of academic interest because of the importance of the quasiparticle scattering for the pairing interaction. A first step towards a more realistic description must include the Landau corrections, which are equivalent to the mean-field approximation. In the following, we will discuss their influence on the static susceptibilities, and in particular, the effects of F_3^s. As far as $\chi_{\delta\phi,\delta\phi}$ is concerned a simple expression is presented that includes all Landau parameters. In the case of $\chi_{\delta l_2,\delta l_2}$ and $\chi_{\delta l_2,\delta\phi}$ we restrict ourselves to F_1^s and F_3^s. The inclusion of further Landau parameters is straightforward, but the expressions become rather clumsy.

The higher-order Landau corrections are coupled to the static susceptibilities by two different mechanisms. The p-wave structure of the order parameter, as well as the expansion of GF in terms of $\vec{k}$ introduce three vectors $\hat{q}$ under the solid-angle integrals. They can always be expressed in terms of associated Legendre polynomials and select the corresponding parts of the interaction vertex Γ^R. The anisotropy of the Yoshida function, however, destroys the spherical symmetry of the problem and provides a coupling to all Landau parameters.

The Landau interaction Γ^R is determined by the integral equation (3.8):

$$\Gamma^R(\hat{q};\hat{q}')+\int\frac{d\Omega''}{4\pi}\Gamma^s(\hat{q}\cdot\hat{q}'')Y(\hat{q}_{\perp}'';T)\Gamma^R(\hat{q}'';\hat{q}')=\Gamma^s(\hat{q}\cdot\hat{q}')\ , \quad (4.1)$$

where Eq. (3.8) has been inserted and a factor $a^2v(0)$ is included in Γ^R and Γ^s. $Y(\hat{q}_{\perp};T)$ denotes the anisotropic Yoshida function. Γ^s is parametrized in terms of the symmetric Landau parameters F_l^s:

$$\Gamma^s(\cos\theta)=\sum_l F_l^s P_l(\cos\theta)\ . \quad (4.2)$$

Owing to the axial symmetry of $Y(\hat{q}_{\perp})$ the most general ansatz of $\Gamma^R(\hat{q};\hat{q}')$ reads

$$\Gamma^R(\hat{q};\hat{q}')=\sum_{m,l,l'}C_{ll'}^m P_l^m(\cos\theta)P_{l'}^m(\cos\theta')\times\cos m(\varphi-\varphi')\ . \quad (4.3)$$

The coefficients C_{ik}^m can be obtained from

$$C_{ll'}^m+F_l^s\sum_k Y_{lk}^m C_{kl'}^m=2\frac{(l-m)!}{(l+m)!}F_l^s\delta_{ll'}\ , \quad (4.4)$$

which follows after inserting Eq. (4.3) in (4.1) and using the addition theorem for spherical harmonics. The factor 2 on the right-hand side of Eq. (4.4) has to be omitted in the case where $m=0$. The function Y_{lk}^m is defined by

$$Y_{lk}^m=\frac{(l-m)!}{(l+m)!}\int\frac{d\Omega}{4\pi}P_l^m P_k^m Y(q_{\perp};T)\ . \quad (4.5)$$

Apparently, the system (4.4) is decoupled with respect to even and odd l.

The Landau corrections to the static susceptibilities are brought in by inserting the complete expressions of Eq. (3.4) into Eq. (3.18). With the collection of the different $\widetilde{L}_{ij}^{\alpha\beta}$ in Eq. (3.18) and insertion of GF from Eq. (3.7), the additional contributions from the Landau corrections to the static susceptibilities take the following form. $\chi_{\delta\phi,\delta\phi}^{-1}(\vec{k})$ is supplemented by

$$3\int \frac{d\Omega}{4\pi}(\vec{q}\cdot\vec{k})\Phi(\hat{q}_\perp) \times \int \frac{d\Omega'}{4\pi}(\vec{q}'\cdot\vec{k})\Phi(\hat{q}_\perp')\Gamma^R(\hat{q};\hat{q}') , \tag{4.6}$$

$\chi_{\delta l_2,\delta\phi}^{-1}(\vec{k})$ yields

$$3\int \frac{d\Omega}{4\pi}\hat{q}_\perp^{-2}\hat{q}_z\hat{q}_x(\vec{q}\cdot\vec{k})\Phi(\hat{q}_\perp) \times \int \frac{d\Omega'}{4\pi}\Phi(\hat{q}_\perp')(\vec{q}'\cdot\vec{k})\Gamma^R(\hat{q};\hat{q}') , \tag{4.7}$$

and $\chi_{\delta l_1,\delta l_1}^{-1}(\vec{k})$ yields

$$3\int \frac{d\Omega}{4\pi}\hat{q}_\perp^{-2}\hat{q}_z\hat{q}_y(\vec{q}\cdot\vec{k})\Phi(\hat{q}_\perp) \times \int \frac{d\Omega}{4\pi}\hat{q}_\perp'^{-2}\hat{q}_z'\hat{q}_y'(\vec{q}'\cdot\vec{k})\Phi\Gamma^R(\hat{q};\hat{q}') . \tag{4.8}$$

Replacing $\hat{q}_y$ by $\hat{q}_x$ in Eq. (4.8), we get the contribution to $\chi_{\delta l_2,\delta l_2}^{-1}$. Before we can proceed and identify the Landau corrected parameters according to Eqs. (3.22) and (3.23), we must convince ourselves that the decoupling of δl_1 and $(\delta l_2,\delta\phi)$ occurring in the BCS approximation is not destroyed by the Landau corrections; otherwise, it would be impossible to perform the identification on the level of the inverted static susceptibilities. The crucial terms are, for $\chi_{\delta l_1,\delta\phi}$,

$$3\int \frac{d\Omega}{4\pi}\hat{q}_\perp^{-2}\hat{q}_z\hat{q}_y(\vec{q}\cdot\vec{k})\Phi \times \int \frac{d\Omega'}{4\pi}(\vec{q}'\cdot\vec{k})\Phi\Gamma^R(\hat{q};\hat{q}') \tag{4.9}$$

and for $\chi_{\delta l_1,\delta l_2}$,

$$3\int \frac{d\Omega}{4\pi}\hat{q}_\perp^{-2}\hat{q}_z\hat{q}_y(\vec{q}\cdot\vec{k})\Phi \times \int \frac{d\Omega'}{4\pi}\hat{q}_\perp'^{-2}\hat{q}_z'\hat{q}_x'(\vec{q}'\cdot\vec{k})\Phi\Gamma^R(\hat{q};\hat{q}') . \tag{4.10}$$

Γ^R provides the coupling between the solid-angle integrals. A typical term of Γ^R reads [cf. Eq. (4.3)]

$$P_l^m(\cos\theta)P_{l'}^m(\cos\theta')(\cos m\varphi\cos m\varphi'+\sin m\varphi\sin m\varphi') .$$

However, the left-hand side solid-angle integrals of either Eq. (4.9) or (4.10) depend azimuthally only on $\sin\varphi$ and $\sin 2\varphi$, whereas on the right-hand side 1, $\cos\varphi'$, and $\cos 2\varphi'$ dependence occurs. Thus, it is never possible that both integrals yield nonvanishing values simultaneously, and the decoupling of the two sets of variables δl_1 and $(\delta l_2,\delta\phi)$ is ensured.

We observe that only odd numbers of $\hat{q}_i$ occur under each integral in Eqs. (4.6) and (4.7). The integral equation of Γ^R [Eq. (4.1)], on the other hand, is decoupled with respect to even and odd l's, and, therefore, only the odd Landau parameters contribute to the static susceptibilities of the order parameter.

To begin with, we examine the Landau corrections of the static susceptibility of the phase $\delta\phi$. Owing to the axial symmetry expression (4.6) splits into two independent parts:

$$3k_\parallel^2\Gamma_{11}^0+3k_\perp^2\Gamma_{11}^1 , \tag{4.11}$$

where

$$\Gamma_{ll'}^m=\int \frac{d\Omega}{4\pi}P_l^m(\hat{q}_\parallel)\cos m\varphi\Phi(\hat{q}_\perp) \times \int \frac{d\Omega'}{4\pi}P_{l'}^m(\hat{q}_\parallel')\cos m\varphi'\Phi(\hat{q}_\perp')\Gamma^R(\hat{q};\hat{q}') . \tag{4.12}$$

It is possible to reformulate Eq. (4.1) in terms of Eq. (4.12) exploiting the spherical symmetry of Γ^s and the additional theorem for spherical harmonics:

$$(\Gamma_{ll'}^m+\tfrac{1}{2}\Phi_{ll'}^m)-\sum_k \frac{(k-m)!}{(k+m)!}\Phi_{lk}^m(\Gamma_{kl'}^m+\tfrac{1}{2}\Phi_{kl'}^m) \times \frac{F_k^s}{1+1/(2k+1)F_k^s}=\tfrac{1}{2}\Phi_{ll'}^m , \tag{4.13}$$

where

$$\Phi_{ll'}^m=\int \frac{d\Omega}{4\pi}P_l^m(\cos\theta)P_{l'}^m(\cos\theta)\Phi(\hat{q}_\perp;T) . \tag{4.14}$$

If $m=0$, the factors $\frac{1}{2}$ in Eq. (4.13) must be omitted. Making use of Kramer's rule, Eq. (4.11), and the BCS result (3.25), we get from Eq. (4.13) the longitudinal and transversal superfluid densities $\rho_\parallel^s$ and $\rho_\perp^s$:

$$\frac{\rho_{\parallel}^{s}}{\rho}=3(\Phi_{11}^{0}+\Gamma_{11}^{0})=3(1+\tfrac{1}{3}F_1^s)\frac{\begin{vmatrix}\Phi_{11}^{0} & F_3^s Y_{13}^{0} & \cdots\\ -Y_{13}^{0} & \delta_{lm}+F_m^s Y_{lm}^{0} & \cdots\\ \vdots & \vdots & \end{vmatrix}}{|\,\delta_{ik}+F_k^s Y_{ik}^{0}\,|}\frac{m}{m^*}\,, \tag{4.15}$$

where $|\ |$ denotes the determinant and $l,m \geq 3$, $ik \geq 1$. The indices l,m,i,k run over all odd positive integers.

$\rho_{\perp}^{s}$ is obtained by replacing Φ_{11}^{0} by $\frac{1}{2}\Phi_{11}^{1}$ and Y_{ik}^{0} by Y_{ik}^{1}. The result is exact within the mean-field approximation. We note that $\rho_{\parallel}^{s}$ and $\rho_{\perp}^{s}$ are influenced by all Landau parameters. The coupling to higher Landau parameters $l \geq 3$ is due to the anisotropy of the gap. In the case of very high and very low temperatures, when the Yoshida function or the gap becomes very small, we get approximately

$$\frac{\rho_{\parallel}^{s}}{\rho}=(1+\tfrac{1}{3}F_1^s)\rho_{\parallel}^{0s}\frac{m}{m^*}\,. \tag{4.16}$$

As far as the other static susceptibilities of the order-parameter variables are concerned, it is not possible to obtain a form of comparable simplicity as in the case of the superfluid densities. The angle dependence in expressions (4.7) and (4.8) cannot be expanded in a finite series of polynomials $P_l^m(\cos\theta)\cos m\varphi$. Thus, all Landau parameters contribute, even at $T=0$, and we are forced to assume that the higher Landau parameters may be neglected.

If the explicit dependence of the integrand of (4.7) and (4.8) on $\hat{q}_{\perp}$ is disregarded for a moment, then the remaining $\hat{q}_i$ would introduce only F_1^s and F_3^s. These two parameters enter the problem because of p-wave pairing (the p-wave pairing introduces a factor $\hat{q}_i\hat{q}_j$ into the solid-angle integrals), whereas the higher-order parameters enter through the dependence on $\hat{q}_{\perp}$, i.e., by the anisotropy of the Yoshida function. Hence F_1^s and F_3^s are of equal significance and must be both taken into account. The higher-order Landau parameters are neglected in the following for simplicity.

We note that only those solid-angle integrals which contain an even number of $\hat{q}_z$, $\hat{q}_x$, and $\hat{q}_y$ do not vanish. In order to ascertain which parts of Γ^R contribute to the Landau corrections, we represent the spherical harmonics in terms of unit vectors $\hat{q}_i$ and note which of them pair with each unpaired unit vector in Eqs. (4.7) and (4.8). (The spherical harmonics expressed in terms of $\hat{q}_i$ are listed in Appendix B.) The condition that on either side of (4.7) and (4.8) the same number m must appear [cf. Eq. (4.3)] brings about a further reduction of possible terms. After a lengthy but simple calculation, the Landau correction to $\chi_{\delta\bar{l}_2,\delta\phi}^{-1}$ takes the form

$$\begin{aligned}3k_{\parallel}k_{\perp}[\,&\tfrac{1}{18}(\rho_{\parallel}^{0s})^2C_{11}^{0}+\tfrac{1}{12}(\rho_{\parallel,\parallel}^{0s}-\rho_{\parallel}^{0s})\rho_{\parallel}^{0s}(C_{31}^{0}+C_{13}^{0})+\tfrac{1}{8}(\rho_{\parallel,\parallel}^{0s}-\rho_{\parallel}^{0s})C_{33}^{0}\\&+\tfrac{1}{18}(\rho_{\parallel}^{0s})(\rho_{\perp}^{0s})C_{11}^{1}-\tfrac{1}{12}(3\rho_{\parallel,\parallel}^{0s}-\rho_{\parallel}^{0s})\rho_{\perp}^{0s}C_{31}^{1}-\tfrac{1}{12}\rho_{\parallel}^{0s}(\rho_{\parallel,\perp}^{0s}-\rho_{\perp}^{0s})C_{13}^{1}\\&+\tfrac{1}{8}(\rho_{\parallel,\perp}^{0s}-\rho_{\perp}^{0s})(3\rho_{\parallel,\parallel}^{0s}-\rho_{\parallel}^{0s})C_{33}^{1}\,]\,.\end{aligned} \tag{4.17}$$

The different superfluid densities are defined in Appendix C, whereas $C_{ll'}^{m}$ is determined by Eq. (4.4).

We should point out that Eq. (4.17) determines only $c_{\perp}-c_{\parallel}$ [compare Eq. (2.8)]. In order to specify $c_{\perp}$ and $c_{\parallel}$ separately, one has to calculate the static susceptibility $\chi_{\delta\bar{l}_1,g_2}^{-1}$ from which $c_{\perp}$ can be obtained.[7] The line of thought is similar to the one pursued to calculate the order-parameter susceptibilities; thus we refrain from presenting the technical details. The calculation shows that the Landau corrections to $c_{\perp}$ are contained in the last four terms of expression (4.17)—i.e., $c_{\perp}$ is corrected by the constants C_{ik}^{1}, whereas $c_{\parallel}$ is corrected by the constants C_{ik}^{0}. (A small contribution due to the violation of particle-hole symmetry has been neglected in calculating $\chi_{\delta\bar{l}_1,g_2}^{-1}$. It just cancels in $c_{\parallel}-c_{\perp}$.) Using this result it is now easy to show that $c_{\parallel}=-\frac{1}{2}\rho_{\parallel}^{s}$ for arbitrary Landau parameters. To prove this result it is only necessary to observe that it holds in the BCS approximation, and that the parameters C_{ik}^{0} which make up the Landau correction to $c_{\parallel}$ are all obtained from (4.7) by replacing $\vec{k}$ by $\vec{k}_{\parallel}$, which is equal to (4.6) apart from a factor of $\frac{1}{2}$.

With regard to $\chi_{\delta\bar{l}_2,\delta l_2}^{-1}$ and $\chi_{\delta\bar{l}_1,\delta l_1}^{-1}$, the Landau corrections to K_1,K_2,K_3 read

$$\frac{K_1}{\rho}\frac{m^*}{m}=\frac{K_1}{\rho}\bigg|_{\mathrm{BCS}}+\tfrac{1}{4}(\rho^{0^s}_{||,\perp})^2C^2_{33}\ , \tag{4.18}$$

$$\frac{K_2}{\rho}\frac{m^*}{m}=\frac{K_2}{\rho}\bigg|_{\mathrm{BCS}}+\tfrac{1}{36}(\rho^{0^s}_{||})^2C^0_{11} +\tfrac{1}{24}\rho^{0^s}_{||}(\rho^{0^s}_{||,||}-\rho^{0^s}_{||})(C^0_{31}+C^0_{13}) +\tfrac{1}{16}C^0_{33}(\rho^{0^s}_{||,||}-\rho^{0^s}_{||})^2+\tfrac{1}{4}(\rho^{0^s}_{||,\perp})^2C^2_{33}\ , \tag{4.19}$$

$$\frac{K_3}{\rho}\frac{m^*}{m}=\frac{K_3}{\rho}\bigg|_{\mathrm{BCS}}+\tfrac{1}{36}(\rho^{0^s}_{||})^2C^1_{11} -\tfrac{1}{24}\rho^{0^s}_{||}(3\rho^{0^s}_{||,||}-\rho^{0^s}_{||})(C^1_{31}+C^1_{13}) +\tfrac{1}{16}C^1_{33}(3\rho^{0^s}_{||,||}-\rho^{0^s}_{||})^2\ . \tag{4.20}$$

In order to give explicit expressions of the various constants, we restrict ourselves to a simplified model where only F^s_1 and F^s_3 are retained. Then the static susceptibilities of the order-parameter variables including the Landau corrections up to the order $l=3$ read

$$\frac{\rho^s_{||}}{\rho}\frac{m^*}{m}=\rho^{0^s}_{||}\left[1+\tfrac{1}{3}F^s_1(1+\tfrac{1}{7}F^s_3\rho^{0^n}_{||,||,||})D_0\rho^{0^s}_{||}-\tfrac{1}{2}(\rho^{0^n}_{||,||}-\rho^{0^n}_{||})(\rho^{0^s}_{||,||}-\rho^{0^s}_{||})F^s_1F^s_3D_0 +\tfrac{3}{4}(\rho^{0^s}_{||,||}-\rho^{0^s}_{||})^2\frac{1}{\rho^{0^s}_{||}}F^s_3(1+\tfrac{1}{3}F^s_1\rho^{0^n}_{||})D_0\right]\ , \tag{4.21}$$

$$\frac{\rho^s_{\perp}}{\rho}\frac{m^*}{m}=\rho^{0^s}_{\perp}\left[1+\tfrac{1}{3}F^s_1(1+\tfrac{1}{7}F^s_3\rho^{0^n}_{||,||,\perp})D_1\rho^{0^s}_{\perp}-\tfrac{1}{12}(\rho^{0^s}_{||,\perp}-\rho^{0^s}_{\perp})(\rho^{0^n}_{||,\perp}-\rho^{0^n}_{\perp})F^s_1F^s_3D_1 +\tfrac{1}{8}(\rho^{0^s}_{||,\perp}-\rho^{0^s}_{\perp})^2\frac{1}{\rho^{0^s}_{\perp}}F^s_3(1+\tfrac{1}{3}F^s_1\rho^{0^n}_{\perp})D_1\right]\ . \tag{4.22}$$

These expressions have been taken from the exact relation (4.15) taking $F^s_l\equiv 0$ for $l>3$. Alternating F^s_l into F^a_l we obtain the corresponding expressions with regard to $M_\perp/\rho$ and $M_{||}/\rho$. In order to get $c_{||},c_\perp,K_1,K_2,K_3$, we solve Eq. (4.4) with $F_l\equiv 0$ for $l\geq 5$ and insert the various C^m_{ik} into Eqs. (4.17)–(4.20).

$C_{||}/\rho$ and $c_\perp/\rho$ take the form

$$c_{||}=-\tfrac{1}{2}\rho^s_{||} \tag{4.23}$$

and

$$\frac{c_\perp}{\rho}\frac{m^*}{m}=\tfrac{1}{2}\rho^{0^s}_{||}\left[1+\tfrac{1}{3}F^s_1(1+\tfrac{1}{7}F^s_3\rho^{0^n}_{||,||,\perp})D_1\rho^{0^s}_{\perp}-\tfrac{1}{24}(3\rho^{0^s}_{||,||}-rh^{0^s}_{||})(\rho^{0^n}_{||,\perp}-\rho^{0^n}_{\perp})\frac{\rho^{0^s}_{\perp}}{\rho^{0^s}_{||}}F^s_1F^s_3D_1 -\tfrac{1}{24}(\rho^{0^s}_{||,\perp}-\rho^{0^s}_{\perp})(\rho^{0^n}_{||,\perp}-\rho^{0^n}_{\perp})F^s_1F^s_3D_1+\tfrac{1}{8}(3\rho^{0^s}_{||,||}-\rho^{0^s}_{||})(\rho^{0^s}_{||,\perp}-\rho^{0^s}_{\perp})(\rho^{0^s}_{||})^{-1}F^3_3(1+\tfrac{1}{3}F^s_1\rho^{0^n}_{\perp})D_1\right]\ .$$

The three Franck constants given by Eqs. (4.19)–(4.21) read

$$\frac{K_1}{\rho}\frac{m^*}{m}=\frac{1}{4}\left[\rho^{0^s}_{\perp}+\frac{\tfrac{1}{5}F^s_3}{1+\tfrac{1}{7}F^s_3\rho^{0^n}_{||,\perp,\perp}}(\tfrac{1}{2}\rho^{0^s}_{||,\perp})^2\right]\ , \tag{4.25}$$

$$\frac{K_2}{\rho}\frac{m^*}{m}=\tfrac{1}{12}\rho^{0^s}_{\perp}+\tfrac{1}{3}\rho^{0^s}_{||}+\tfrac{1}{3}F^s_1(1+\tfrac{1}{7}F^s_3\rho^{0^n}_{||,||,||})D_0(\tfrac{1}{2}\rho^{0^s}_{||})^2-\tfrac{1}{8}\rho^{0^s}_{||}(\rho^{0^s}_{||,||}-\rho^{0^s}_{||})(\rho^{0^n}_{||,||}-\rho^{0^n}_{||})F^s_1F^s_3D_0 +\tfrac{3}{16}(\rho^{0^s}_{||,||}-\rho^{0^s}_{||})^2F^s_3(1+\tfrac{1}{3}F^s_1\rho^{0^n}_{||})D_0+\tfrac{1}{4}\frac{\tfrac{1}{5}F^s_3}{1+\tfrac{1}{7}F^s_3\rho^{0^n}_{||,\perp,\perp}}(\tfrac{1}{2}\rho^{0^s}_{||,\perp})^2\ , \tag{4.26}$$

$$\frac{K_3}{\rho}\frac{m^*}{m}=\tfrac{1}{2}\rho_{||}^{0^s}+\tfrac{1}{3}\tau+\tfrac{1}{3}F_1^s(1+\tfrac{1}{7})F_3^s\rho_{||,||,\perp}^{0^n})D_1(\tfrac{1}{2}\rho_{||}^{0^s})^2-\tfrac{1}{48}\rho_{||}^{0^s}(3\rho_{||,||}^{0^s}-\rho_{||}^{0^s})(\rho_{||,\perp}^{0^n}-\rho_{\perp}^{0^n})F_1^sF_3^sD_1$$
$$+\tfrac{1}{32}(3\rho_{||,||}^{0^s}-\rho_{||}^{0^s})^2F_3^s(1+\tfrac{1}{3}F_1^s\rho_{\perp}^{0^n})D_1\ . \tag{4.27}$$

D_0 is given by

$$D_0^{-1}=1+\tfrac{1}{3}F_1^s\rho_{||}^{0^n}+\tfrac{1}{7}F_3^s\rho_{||,||,||}^{0^n}+\tfrac{1}{21}F_1^sF_3^s[\rho_{||}^{0^n}\rho_{||,||,||}^{0^n}-\tfrac{21}{4}(\rho_{||,||}^{0^n}-\rho_{||}^{0^n})^2]\ . \tag{4.28}$$

D_1 can be obtained from Eq. (4.28) replacing one $||$ index of each normal fluid density by $\perp$.

V. DISCUSSION

In the previous sections we derived the static susceptibilities from the linear reponse functions of the order-parameter matrix $\hat{A}_{ij}$. It was shown how the self-consistent gap equation selects a small number of slow variables whose static susceptibilities vary as $1/k^2$. With regard to the phenomenological theory in Ref. 7 the nine independent parameters defined therein could be identified.

These parameters have been calculated previously by Ambegaokar, de Gennes, and Rainer,[3] Cross,[4] and Serene and Rainer[10] within various approximations. They all have in common that they look for the gradient part F_{grad} of the superfluid free energy. Ambegaokar, de Gennes, and Rainer restricted themselves to the Ginzburg-Landau regime and calculated

$$F_{\text{grad}}=\frac{1}{8m^2}\int d^3r(\tilde{K}_1\partial_iA_{\mu i}^*\partial_jA_{\mu j}+\tilde{K}_2\partial_iA_{\mu j}^*\partial_iA_{\mu j}+\tilde{K}_3\partial_iA_{\mu j}^*\partial_jA_{\mu i})\ . \tag{5.1}$$

This parametrization of F_{grad} is valid only near T_c. At lower temperatures one has to consider the general expression[13,20]

$$F_{\text{grad}}=\frac{1}{4m^2}\int d^3\pi\,\{\tfrac{1}{2}\rho_\perp^s(\nabla\phi)^2-\tfrac{1}{2}(\rho_\perp^s-\rho_{||}^s)(\vec{1}\cdot\nabla\phi)^2+c_\perp(\nabla\phi)(\nabla\times\vec{1})$$
$$-(c_\perp-c_{||})(\vec{1}\cdot\nabla\phi)(\vec{1}\cdot\nabla\times\vec{1})+\tfrac{1}{2}K_S(\nabla\vec{1})^2+\tfrac{1}{2}K_T(\vec{1}\cdot\nabla\times\vec{1})^2$$
$$+\tfrac{1}{2}K_B[\vec{1}\times(\nabla\times\vec{1})]^2+\tfrac{1}{2}M_{||}(\vec{1}\cdot\nabla)n_i(\vec{1}\cdot\nabla)n_i+\tfrac{1}{2}M_\perp(\vec{1}\times\nabla)_kn_i(\vec{1}\times\nabla)_kn_i\}\ . \tag{5.2}$$

Contact with the notation of the previous sections is made in Eq. (2.5) if we rename $K_S\equiv K_1$, $K_T\equiv K_2$, and $K_B\equiv K_3$. The parametrization (5.1) predicts $K_S=K_T=\tilde{K}_2$ and $K_B=\tilde{K}_1+\tilde{K}_2+\tilde{K}_3$. It is easy to convince oneself that this holds only in the order t ($t=1-T/T_c$). The Landau corrections, however, belong to the order $O(t^2)$ and may not be included in (5.1). Serene and Rainer, on the other hand, calculated the strong-coupling corrections of Eq. (5.1) [corrections of the order $(T_c/T_F)^3$ in the free energy functional $\tilde{\Phi}$]. Our results, as well as those of Cross, consider only the lowest order of $\tilde{\Phi}$ [i.e., $(T_c/T_F)^2$] but apply to the whole temperature regime. In the lowest order of t and T_c/T_F our results coincide with those of Ambegaokar, de Gennes, Rainer and Serene.[3,10]

Cross[4] calculated the free energy (5.2) and the resulting supercurrents, generalizing a procedure of Werthamer[11] to the case of triplet pairing. The central point is to perform a gradient expansion of the one-particle Green's function and then to calculate the superfluid current. From there, one can deduce the free energy. The basic assumptions—i.e., the restriction to the mean-field approximation—are the same in both his work and ours. If we neglect the higher-order Landau parameters ($l>1$), our results coincide with those of Cross.

Our results for the static susceptibilities of $M_\perp$, $M_{||}$ (the superfluid densities in spin space) and $\rho_\perp^s$, $\rho_{||}^s$ and $c_{||}$ [cf. (4.14)] are exact within the mean-field approximation. As long as $\Phi(\hat{q}_\perp;T)$ is neither ≈ 1 nor ≈ 0, all odd Landau parameters contribute to $\rho_\perp^s$, $\rho_{||}^s$, $c_{||}$, $M_{||}$, and $M_\perp$. If we perform an expansion of $\Phi(\hat{q}_\perp)$ in terms of $\Delta_0^2\hat{q}_\perp^2$, then the determinants in Eq. (4.16) become tridiagonal, pentadiagonal, etc., according to the order of Δ_0^2 we include. We will not follow this line of thought because such an expansion makes sense only near T_c where the influence of the Landau corrections becomes very small. Thus, we are forced to assume that $F_l^{s,a}$ vanishes for l greater than a fixed number. For simplicity we take into account finite values of $F_1^{s,a}$ and $F_3^{s,a}$ but neglect the higher ones.

Figure 1 shows the dependence of $M_\perp$ and $M_{||}$ on F_1^a and F_3^a for a fixed temperature ($T/T_c=0.73$). Each point on the $M_\perp$-$M_{||}$ plane denotes a special choice of the parameters (F_1^a,F_3^a). We have drawn the lines where F_1^a and F_3^a are fixed. The liquid becomes unstable if $F_1^a=-3$ (it is the same instability of the Fermi surface we get in the normal fluid ^{3}He.) $M_{||}$ is much more influenced by F_3^a than is $M_\perp$, whereas the influence of F_1^a is approximately the same on either parameter. The influence of F_3^a is diminished at lower temperatures and vanishes totally at $T=0$ (i.e., the line $F_1^a\equiv 0$ shortens if we continue lowering the temperature). Figure 2 shows more closely the influence of temperature. Here, we plot $\rho_{||}^s$ against $\rho_\perp^s$ (we remind our readers that the expressions obtained in cases of $\rho_{||}^s$,$\rho_\perp^s$ and $M_\perp$,$M_{||}$ are the same if we substitute F_l^a by F_l^s). We have taken a small region of $F_1^s,F_3^s\in[3,15]$, and show how it is deformed when the temperature is lowered. At temperatures close to T_c the effect of Landau corrections is small, and we get the BCS approximated values (as long as strong-coupling corrections are neglected). At an intermediate stage, the influence of F_3^s becomes rather strong although smaller than that of F_1^s. At $T=0$ the region is deformed into a line. We would like to point out that the effects of Landau corrections are very strong because of the large values of F_l^s. Thus, at least with respect to the real-space variables, one may assume that at intermediate temperatures the effects of Landau corrections are a good deal larger than those of strong-coupling corrections. With regard to $M_\perp$, $M_{||}$, $\rho_\perp^s$, and $\rho_{||}^s$, the effects of even higher-order Landau parameters are qualitatively the same as that of F_3^s.

The bending coefficients of $\vec{l}$ offer a different behavior. K_S, K_T, and K_B contain all Landau parameters, even at $T=0$ due to the denominator $\hat{q}_\perp^{-2}$ in the Landau parts of the static susceptibilities. In order to discuss the results we must assume once more that F_l^s vanishes if $l\geq 5$. Cross's calculations yield the remarkable results that, although F_1^s is included, K_S maintains its weak-coupling value. We recover this behavior and find that K_S [in the notation of the previous sections K_1, cf. Eq. (4.25)] depends only on F_3^s and, in principle, higher Landau corrections.

In Fig. 3, we have plotted the dependence of K_S and K_T at a fixed temperature ($T/T_c=0.28$) on F_1^s and F_3^s. The vertical lines of constant F_3^s display the fact that K_S does not depend on F_1^s. The dependence of K_T on F_1^s is much stronger than on F_3^s (about a factor of 5). The influence of F_3^s is approximately the same on both K_S and K_T. Figure 4 illustrates the influence of the temperature on the

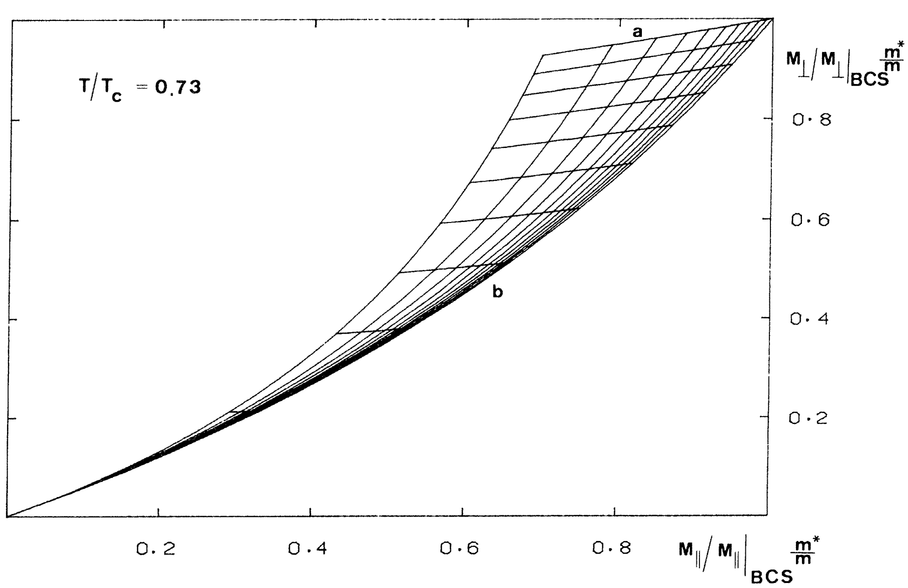

FIG. 1. $M_{||}$ vs $M_\perp$. F_1^a is fixed on lines parallel to a, F_3^a fixed on lines of type b. At a and b F_1^a and F_3^a vanish, respectively. $F_1^a\in[-3,0]$ and $F_3^a\in[-7,0]$. The distance between two lines is 0.3 and 0.7 for F_1^a and F_3^a, respectively.

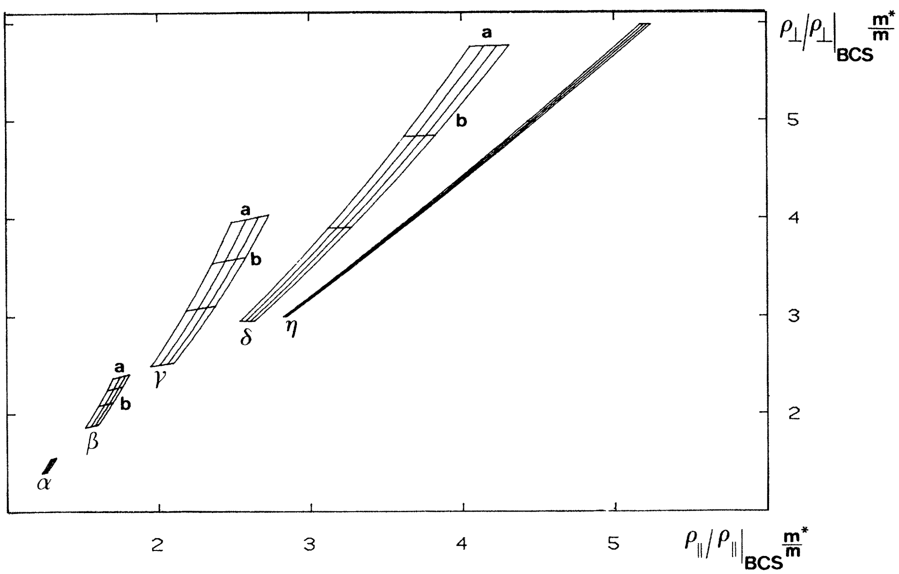

FIG. 2. Dependence of $\rho_\perp^s$ and $\rho_\parallel^s$ on both temperature and Landau parameters. *a*: $F_1^s \equiv 15$. *b*: $F_3^s \equiv 15$. F_1^s and F_3^s are taken from the interval [6,15]. The distance between two lines is 3. Temperature $\tilde{t} = T/T_c$ is chosen to be for α, $\tilde{t}=0.73$; β, $\tilde{t}=0.46$; γ, $\tilde{t}=0.28$; δ, $\tilde{t}=0.19$; η, $\tilde{t}=0.1$.

Landau correction. Only the border line of the region in Fig. 3 is drawn. The Landau corrections vanish in the Ginzburg-Landau region, but at intermediate temperatures they already cause substantial deviations from the BCS value.

We refrain from displaying $c_\perp$, $c_\parallel$, and K_B for they do not offer new interesting points. In Cross's approximation ($F_l^{s,a}=0$ for $l>1$) a relation between $c_\perp$ and $\rho_\perp$ can be established:

$$\frac{c_\perp}{c_\perp|_{\rm BCS}} = \frac{\rho_\perp^s}{\rho_\perp^s|_{\rm BCS}} \,. \tag{5.3}$$

Relation (5.3) does not remain valid if F_3^s is included, whereas the corresponding relation for $c_\parallel$ is valid for arbitrary Landau parameters. It is amusing that the relation $\rho_\parallel^s = -2c_\parallel$ is the only accidental BCS symmetry in the calculations of Serene and

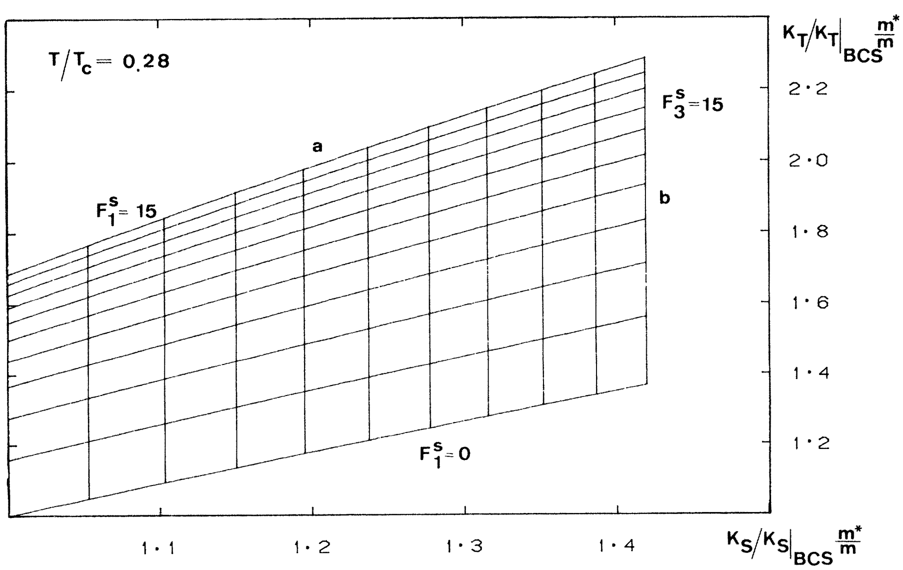

FIG. 3. Dependence of K_S and K_T on F_1^s and F_3^s. On line *a*, F_1^s is fixed by $F_1^s \equiv 15$, on *b* $F_3^s \equiv 15$. Both F_1^s and F_3^s are taken from [0,15]. The distance between two lines is 1.5.

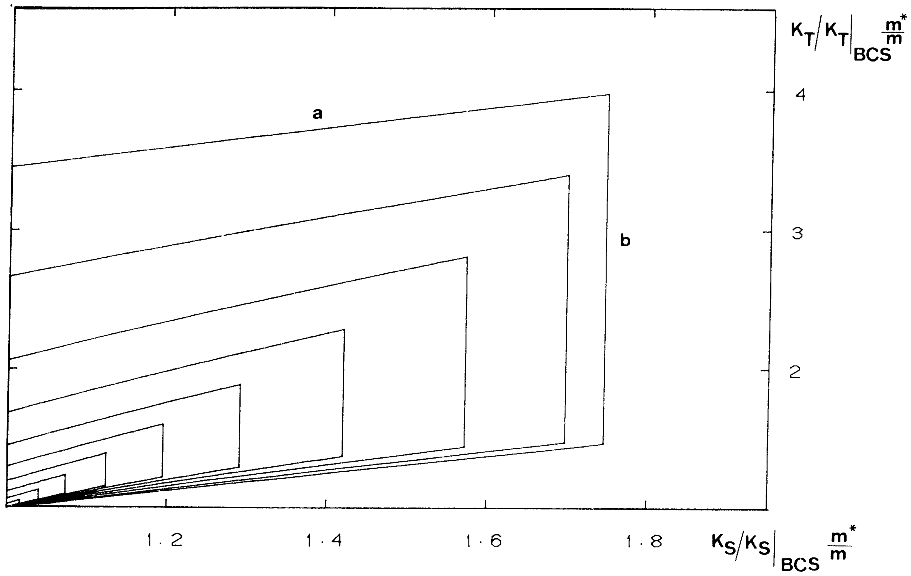

FIG. 4. Dependence of K_S and K_T on the Landau corrections and the temperature. Lines a and b are the same as in Fig. 3. $\tilde{t}=T/T_c$ is linearly increased by 0.09 starting from a where $\tilde{t}=0.1$.

Rainer[10] which survives if strong-coupling corrections are taken into account. On the other hand, I am not aware of any hydrodynamic argument that this symmetry should be exact. K_B depends in a complicated way on both F_1^s and F_3^s. The divergence of $K_B \sim \ln T$ for $T \rightarrow 0$ cannot be removed by any Landau corrections.

The coefficients of the gradient part of the free energy enter both the linearized hydrodynamics and the textures. As far as the linearized hydrodynamics is concerned, the velocities of the various Goldstone modes are intimately related with the static susceptibilities of the hydrodynamic order-parameter variables[12,13]: The spin waves in ^{3}He-A propagate with the frequency

$$\omega^2=c^2k^2\,, \tag{5.4}$$

where

$$c^2=\left[\frac{\gamma}{2m}\right]^2(M_\perp \hat{k}_\perp^2+M_{||}\hat{k}_{||}^2)\chi_\perp^{-1}\,,$$

and $\chi_\perp$ is the transverse part of the magnetic susceptibility. Hence, information on $M_\perp$ and $M_{||}$ can be gathered from NMR experiments. $\rho_\perp^s$ and $\rho_{||}^s$ are accessible by the second and fourth sound waves, whereas the bending coefficients of $\vec{l}$ influence the orbit waves. We refrain from presenting the com- K_S, and $c_\perp - c_{||}$ from the data.

A much more promising field is offered by the textures we always find in superfluid ^{3}He. The calculations described above assume an infinite extended volume in order to ensure the equilibrium state to be homogeneous. The boundary conditions encountered in the actual experiment force the superfluid system to exhibit textures. Furthermore, the topology of the underlying not simply connected group renders it possible to find textures even in the limit of infinite volume. They cannot be deformed homotopically into the homogeneous state and, hence, give rise to a finite frequency response if they are distorted by a small amount. It is only natural that the quantitative behavior (structure, response to small disturbances, etc.) is essentially influenced by K_S, K_T, and K_B. Much work has been done in the last few years to clarify the features of textures and solitons in ^{3}He-A.[14-18] Unfortunately, most of the authors start from the gradient energy shown in Eq. (5.1). Hence, there is no way to extend their results to temperatures outside the Ginzburg-Landau regime; nor does it make sense to include Fermi-liquid effects.

It is a different matter as far as the stability of the superflow $\vec{v}_s$ is concerned.[19-28] Bhattacharyya, Ho, and Mermin[20] pointed out that a uni-

(we have already inserted $2c_{\|}=-\rho_{\|}^{s}$) and if the uniform $\hat{l}$ texture undergoes a phase transition to a helical state. Fetter[21] examined the dynamical stability of the helical structure and demonstrated that there is a temperature regime where Eq. (5.5) is not satisfied and the helical structure becomes stable. Making use of Cross's results,[4] he showed that the Landau correction stabilizes the helical phase. In contrast to the region of stability, the apex angle of the helical structure depends sensitively on both K_T and K_B. The textural transition of ^{3}He-A in a thin slab was examined by Hu,[24,25] demonstrating that in this case the texture is nonplanar and K_S, K_T, and K_B determine the stability of the helical phase. The role of the bending coefficients become even more intriguing by applying a strong magnetic field[25] or rotating the container.[22] Thus, the textural behavior depends critically on the static susceptibilities we calculated above, and one can hope that the experimental investigation of textural phase transition will deliver some estimates of the higher Landau parameters.

However, one must bear in mind that the strong-coupling correction may lead to a misinterpretation of the results. Serene and Rainer[10] calculated that the rate of change of the static susceptibilities due to the strong-coupling corrections is small (less than 10%), and there are arguments[29] that they may be compensated for by just redefining the superfluid densities. A further step along the lines of this work should include the next order of $\tilde{\Phi}$ in terms of T_c/T_F, which is possible in principle, but requires a substantial amount of algebraic work.

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I would like to thank Professor Graham for critical and useful discussions.

APPENDIX A

We prove that

$$1-\Gamma^{\phi}\int\frac{d\Omega}{4\pi}\hat{q}_z^2(-G^{-}G)\bigg|_{k=0}=0\ . \tag{A1}$$

An extended version of the gap equation (3.16) reads

$$1=-\Gamma^{\phi}a^2v(0)\int\frac{d\Omega}{4\pi}\int d\xi\sum_{\omega_n}\frac{\hat{q}_x^2}{\omega_n^2+\xi^2+\frac{1}{2}\Delta_0^2(\hat{q}_x^2+\hat{q}_y^2)}\ . \tag{A2}$$

Expressing the $\hat{q}_i$ in spherical coordinates, the relevant part of (A2) takes the form

$$\int\frac{d\Omega}{4\pi}\frac{\sin^2\theta\cos^2\phi}{\omega_n^2+\xi^2+d^2\sin^2\theta}$$

$$\to\frac{1}{4}\int_0^{\pi}d\theta\sin\theta\frac{\sin^2\theta}{\omega_n^2+\xi^2+d^2\sin\theta}\ , \tag{A3}$$

where $d^2=\frac{1}{2}\Delta_0^2$. By means of integration by parts we arrive at

$$\frac{1}{2}\int_0^{\pi}d\theta\sin\theta\cos^2\theta\frac{\omega_n^2+\xi^2}{(\omega_n^2+\xi^2+d^2\sin^2\theta)^2}\ . \tag{A4}$$

This can be rewritten in terms of $\hat{q}_i$,

$$\int\frac{d\Omega}{4\pi}\hat{q}_z^2\frac{\omega_n^2+\xi^2}{(\omega_n^2+\xi^2+|\Delta|^2)^2}\ , \tag{A5}$$

which is exactly the expression we desire.

APPENDIX B

We list the spherical harmonics with $l=3$ in terms of unit vectors:

$$P_1=\hat{q}_3,\quad P_1^1\cos\phi=\hat{q}_1,\quad P_1^1\sin\phi=\hat{q}_2\ ,$$

$$P_3=\frac{1}{2}\hat{q}_3(5\hat{q}_3^2-3)\ ,$$

$$P_3^1\sin\phi=-\frac{15}{2}\hat{q}_2(\hat{q}_3^2-\frac{1}{5})\ ,$$

$$P_3^1\cos\phi=-\frac{15}{2}\hat{q}_1(\hat{q}_3^2-\frac{1}{5})\ ,$$

$$P_3^2\sin2\phi=30\hat{q}_3\hat{q}_2\hat{q}_1\ ,$$

$$P_3^2\cos2\phi=15\hat{q}_3(\hat{q}_1^2-\hat{q}_2^2)\ ,$$

$$P_3^3 \sin 2\phi = 15\hat{q}_2(\hat{q}_2^2 - 3\hat{q}_1^2) ,$$
$$P_3^3 \cos 3\phi = -15\hat{q}_1(\hat{q}_1^2 - 3\hat{q}_2^2) .$$

APPENDIX C

The inclusion of F_3^s in the static susceptibilities introduces a set of new normal and superfluid densities. $\rho_{\parallel}^{0^n}$ and $\rho_{\perp}^{0^n}$ already appear in the BCS approximation:

$$\rho_{\parallel}^{0^n} = 3 \int \frac{d\Omega}{4\pi} \hat{q}_z^2 Y(\hat{q}_\perp) ,$$
$$\rho_{\perp}^{0^n} = \frac{3}{2} \int \frac{d\Omega}{4\pi} \hat{q}_\perp^2 Y(\hat{q}_\perp) .$$

Besides these expressions, we obtain

$$\rho_{\parallel,\parallel}^{0^n} = 5 \int \frac{d\Omega}{4\pi} \hat{q}_z^4 Y(\hat{q}_\perp) ,$$
$$\rho_{\parallel,\perp}^{0^n} = \frac{15}{2} \int \frac{d\Omega}{4\pi} \hat{q}_\perp^2 \hat{q}_z^2 Y(\hat{q}_\perp) ,$$
$$\rho_{\perp,\perp}^{0^n} = \frac{15}{8} \int \frac{d\Omega}{4\pi} \hat{q}_\perp^4 Y(\hat{q}_\perp) .$$

They are related to Y_{lk}^m [cf. Eqs. (4.15) and (4.13)] by

$$Y_{11}^0 = \frac{1}{3}\rho_{\parallel}^{0^n}, \quad Y_{11}^1 = \frac{1}{3}\rho_{\perp}^{0^n} ,$$
$$Y_{13}^0 = Y_{31}^0 = \frac{1}{2}(\rho_{\parallel,\parallel}^{0^n} - \rho_{\parallel}^{0^n}) ,$$
$$Y_{13}^1 = -\frac{1}{12}(\rho_{\parallel,\perp}^{0^n} - \rho_{\perp}^{0^n}) = \frac{1}{6} Y_{31}^1 .$$

Then we define

$$\frac{1}{7}\rho_{\parallel,\parallel,\parallel}^{0^n} = Y_{33}^0, \quad \frac{1}{7}\rho_{\parallel,\parallel,\perp}^{0^n} = Y_{33}^1 ,$$
$$\frac{1}{7}\rho_{\parallel,\perp,\perp}^{0^n} = Y_{33}^2, \quad \frac{1}{7}\rho_{\perp,\perp,\perp}^{0^n} = Y_{33}^3 .$$

The corresponding function $\rho_{\parallel}^{0^s}$, etc., is obtained replacing $Y(\hat{q}_\perp, T)$ by $\Phi(\hat{q}_\perp, T)$.

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