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Physics of the dynamical critical exponent in one dimension

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The value of the dynamical critical exponent z is calculated for various one-dimensional kinetic Ising models using simple physical arguments. The values agree with all known exact results and with the lower bounds obtained by Haake and Thol.

The most important states of the one-dimensional Ising model at low temperatures consist of large domains with lengths of the order of the correlation length ξ, separated by sharp domain walls. The characteristic time for the decay of a domain is proportional to ξ^z where z is the dynamical critical exponent. In this paper z is evaluated for several different kinetic Ising models using simple physical arguments about the motion of domain walls. In each case there is one dominant process governing the decay of domains and the rate for this process can be found from simple random walk results.

Glauber[1] introduced a simple single-spin-flip model with nearest-neighbor interactions. This model can be solved explicitly and yields $z=2$. Deker and Haake[2] and Kimball[3] independently devised a modification of the Glauber model which has $z=4$. The essential difference between this model and the Glauber model is that the motion of domain walls is strongly suppressed at low temperatures. This model has been extended by Haake and Thol[4] to a continuous set of models with z between 2 and 4. Another class of models is the double-spin-flip models in which nearest-neighbor pairs of spins flip together. The most important model of this type is the conserved order parameter model in which only pairs containing opposite spins can flip. Haake and Thol,[4] using a variational principle, obtained a lower-bound inequality $z \geqslant 5$ for this model.[5]

The method of this Communication is to find the time for a domain to decay by finding the fastest way for a domain wall to move one correlation length by random fluctuations. The resulting values of z agree with all the above exact results and with the lower bounds found by Haake and Thol.

The static properties are determined by the equilibrium distribution function

$$P\{\sigma\} = Z^{-1}\exp(-H\{\sigma\}) \ , \tag{1}$$

where H is the usual nearest-neighbor Ising Hamiltonian

$$H\{\sigma\} = -K\sum_i \sigma_i\sigma_{i+1} \ . \tag{2}$$

The correlation length grows exponentially as $T=K^{-1}$ approaches zero:

$$\xi \propto \exp(+2K) \ . \tag{3}$$

The single-spin-flip rate in the Glauber model is

$$W_i = \Gamma[1-\tfrac{1}{2}\gamma\sigma_i(\sigma_{i-1}+\sigma_{i+1})] \ , \tag{4}$$

where $\gamma=\tanh 2K$. Γ^{-1} defines the unit of time. A spin at a domain wall which has one neighbor up and the other down has a flip rate Γ. An up spin in the middle of an up domain has a much slower flip rate, $\Gamma(1-\gamma)\propto\Gamma\xi^{-2}$. The dominant way for a domain to decay is by the random motion of the domain wall which occurs at one step every Γ^{-1} in a random direction. Random-walk arguments show that the wall must move ξ^2 steps on the average to move a distance ξ. The time for this to occur, $\Gamma^{-1}\xi^2$, is the characteristic time for the decay of a domain, and we obtain Glauber's result, $z=2$.

The flip rate in the Glauber model can be modified to slow down the motion of domain walls,[3,4]

$$W_i = \Gamma[1 + (1 - ce^{-2\mu K})\sigma_{i-1}\sigma_{i+1}] \times [1 - \tfrac{1}{2}\gamma\sigma_i(\sigma_{i-1} + \sigma_{i+1})] \quad . \qquad (5)$$

If σ_{i-1} and σ_{i+1} have opposite signs σ_i is on a domain wall and has a flip rate $\Gamma c \exp(-2\mu K)$. The time for the wall to move a distance ξ by random steps is ξ^2 times the time for one step. The characteristic time for decay of the domain is thus $\xi^2\Gamma^{-1}c^{-1}\exp(2\mu K) \propto \xi^{2+\mu}$ which gives $z = 2+\mu$. At low temperatures the Deker-Haake-Kimball model is equivalent to $\mu = 2$.

If $\mu > 2$ a new feature occurs. The motion of domain walls by single steps is so slow that it is faster for the wall to move through an intermediate higher-energy state. First, a spin-one lattice spacing from the wall flips. The rate for this is $\Gamma(1-\gamma) \propto \Gamma\xi^{-2}$ [in W_i the first factor is $O(1)$ rather than $O(\xi^{-\mu})$]. Now the spin at the original boundary has a probability $\frac{1}{2}$ of flipping before the newly flipped spin returns to its original value. The domain wall has moved two steps in a time $\Gamma^{-1}\xi^2$. This must occur ξ^2 times for the wall to move a distance ξ by a random walk, so the time for decay of a domain is $\Gamma^{-1}\xi^4$. Thus, in agreement with the lower bound of Haake and Thol,[4] we find $z = 4$ instead of $2+\mu$ for $\mu > 2$. This illustrates the fact that the correct z will only be obtained if the fastest mode is used.

If magnetization is to be conserved only pairs of spins of opposite sign can flip. The simplest flip rate is

$$W_{i,i+1} = \Gamma(1 - \sigma_i\sigma_{i+1}) \times [1 - \tfrac{1}{2}\gamma(\sigma_{i-1}\sigma_i + \sigma_{i+1}\sigma_{i+2})] \quad . \qquad (6)$$

Domain walls cannot move independently and conserve magnetization so it is simpler to study the motion of spins. The easiest way for a domain to decay is for it to move a distance ξ. This can occur by spins moving through the domain from one side to the other.

In the first step the spins at a domain wall exchange. The large energy required for this gives it a slow rate $\Gamma(1-\gamma) \propto \Gamma\xi^{-2}$. The next step is for the spin to move through the domain and come out on the other side. Only a small fraction of the spins which enter the domain leave on the other side. The probability of this can be found from a random walk problem: What is the probability, P_N, of a random walk starting at 1 arriving at N before it arrives at 0? The answer is found by solving for P_N in terms of P_{N-1}:

$$P_N = \sum_{n=0}^{\infty} \tfrac{1}{2}P_{N-1}[\tfrac{1}{2}(1 - P_{N-1})]^n = \frac{P_{N-1}}{1 + P_{N-1}} \quad . \qquad (7)$$

The solution of this which satisfies the condition $P_1 = 1$ is $P_N = N^{-1}$.

The rate of spins moving through a domain is thus $P_\xi\Gamma\xi^{-2} \propto \Gamma\xi^{-3}$. In this process the whole domain has moved one space in a time $\Gamma^{-1}\xi^3$. To move one correlation length by a random walk this must occur ξ^2 times. The time for the domain to move one correlation length is thus proportional to $\Gamma^{-1}\xi^5$ which means $z = 5$, in agreement with Zwerger.[5]

The method used in this paper allows the value of z to be determined from physical arguments. In each case it is possible to understand the value of z in terms of random-walk arguments once the fastest way for the domain wall to move has been identified. The suppressed domain-wall motion model with $\mu > 2$ illustrates the necessity of finding the correct mode of motion of the domain wall. If a slower mode is chosen (e.g., motion by one step at a time in the case $\mu > 2$) then the value of z found will be too large. In this sense the value of z found by our calculation is an upper bound. The fact that we always agree with the lower bound of Haake and Thol confirms that we have found the fastest modes in each case.

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[2]U. Deker and F. Haake, Z. Phys. B 35, 281 (1979).

[3]J. C. Kimball, J. Stat. Phys. 21, 289 (1979).

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[5]Recently the result $z = 5$ has been proven [see W. Zwerger, Phys. Lett. 84A, 269 (1981).