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Critical parameters from electrical resistance of nickel

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High-precision resistance measurements (rms relative error below 4×10^{-7} per point) for nickel around T_c are reported. When fitting the singular part of the resistance to the function $R_s = A|t|^{1-\alpha}(1 + D|t|^z)$, we obtain $\alpha = -0.095$, $A/A' = -1.52$, $z = 0.57$, and $D/D' = -0.8$ within the temperature region $1.6 \times 10^{-3} < |t| < 12 \times 10^{-3}$. The cause of the rounding of experimental data close to T_c is discussed.

I. INTRODUCTION

The anomaly in the resistance of a ferromagnetic metal at the Curie temperature T_c is today well understood. It has been shown[1,2] that the singular parts of the temperature dependence of the specific heat and of the temperature derivative of the resistivity are the same. To leading order we have for the derivative of the resistivity

$$\frac{d\rho_s(T)}{dT} = A|t|^{-\alpha} , \tag{1}$$

where the critical exponent α and the amplitude ratio A/A' are the same as for the specific heat (primed quantities refer to temperatures below T_c, unprimed to temperatures above, and t is the reduced temperature $T/T_c - 1$). This implies that the specific-heat critical index and leading amplitude ratio can be determined either from a specific-heat measurement, or from a resistance measurement. A resistance measurement can be performed with a considerably higher precision than a specific-heat measurement. Further, it can be done at constant temperature, in a quasistationary state. On the other hand, since the resistance is less singular at T_c than is the specific heat, a higher accuracy is required in a measurement of the resistance in order to obtain equally good estimates for the critical parameters. Anyway, there is a good possibility that resistance measurements can give more accurate values of the critical parameters. However, published resistance measurements on nickel[3] and iron[4–6] from the last decade have given rather differing results for the critical parameters. In contrast, modern theoretical methods give very accurate predictions of the critical indices.[7] One source of error in previous analysis of experimental data is the use of the simple function (1). It is today realized,[8] that to obtain good results one must not only have good data, but must also know the form of the singular temperature dependence to a corresponding degree of accuracy. This means that it is in general necessary to provide for higher-order corrections, leading to functions with confluent singularities, of the form

$$\frac{d\rho_s(T)}{dT} = A|t|^{-\alpha}(1 + D|t|^z) \tag{2}$$

in order to obtain correct results even for the leading exponent and amplitude ratio. The form of the expression (2) makes the traditional log-log plot method to extract the index α unworkable. Instead, a nonlinear least-squares method has to be applied directly to the expression (2). Such a method of analysis was first applied by Ahlers[9,10] to specific-heat measurements. The first applications to resistance data seem to have been done by Balberg *et al.*[11] and by Malmström and Geldart[12] to the high-precision measurement on the antiferromagnet dysprosium by Rao *et al.*[13]

Stimulated by the good results obtained for dysprosium, we decided to measure the resistance of pure nickel, if possible at still higher precision and under as well defined circumstances as possible, which implies, among other things, doing the measurements according to a fixed schedule, in one single run and at constant time and temperature intervals.

The data and the analysis are presented in this paper. We have succeeded in increasing the accuracy of measurements by two orders of magnitude (as compared to the Dy measurement), and the accuracy of the critical parameters obtained seems to be higher than of any data on Heisenberg ferromagnets published so far. We have also undertaken to study the rounding of experimental data close to T_c, and offer a plausible explanation of this rounding as being the combined effect of structural disorder and critical slowing down. We also show that taking the effect of rounding into account in the fit gives a rather small change in the values of the critical parameters.

The experimental equipment and method is described in Sec. II, and the analysis of our data in Sec. III. Section IV contains a discussion of the rounding of data close to T_c. In Sec. V we discuss our results and make a comparison with results from earlier measurements of the temperature derivative of the resistance of Fe, and from specific-heat measurements on Ni and EuO. We also compare our results with theoretical predictions of the critical parameters. In an appendix, we present a simple model for the influence of critical slowing down on the observed resistance values.

II. EXPERIMENTAL METHOD

The sample was made from 99.999% pure nickel, drawn to a wire of 0.4-mm diameter and annealed in a hydrogen atmosphere at 850 °C. The residual resistivity ratio (RRR) value with no external field present was 540. Current and potential connections (made from the same wire) were welded to a 45-cm sample piece (resistance at T_c about 1 Ω). The sample and a platinum thermometer (130 Ω at 0 °C) were contained in a cylindrical aluminium sample holder, designed to give good thermal contact between sample and thermometer and small temperature gradients within the sample.

Two separate furnaces were used for heating, the outer one fed from a dc constant current supply, and the inner one connected to a temperature regulator and fed only about 10^{-4} of the total heating power. This arrangement gives a very small temperature difference between the sample holder and the inner furnace wall. Furthermore, due to the low power input, the temperature control of the inner furnace can be made very accurate.

We followed closely a fixed procedure for changing the temperature between readings in order to keep the temperature gradients within the sample holder as constant as possible. To shorten the time required for temperature stabilization at the measuring points, a combined manual and automatic control was developed. It enabled us to take readings at 45 min intervals and still keep the temperature constant within a few times 10^{-4} K for 10 to 15 min before taking the readings. The temperature interval between measuring points was kept as close as possible to 0.2 K, and the whole experiment was made in one single run of 54 h duration.

The resistance of the sample and of the platinum thermometer were read alternately on a resistance bridge. The signal from the bridge was fed to a plotter, an arrangement which allowed us to control (and correct for) the temperature drift during measurements. This drift, however, never did exceed 10^{-4} K/min.

The resistance bridge used was the same 8-decade current comparator thermometer bridge as was used for the measurements on Dy (Ref. 13) and Cr (Ref. 14) (a description of the bridge is given in Ref. 13). In these investigations a resolution of about 2 parts in 10^5 was obtained. We have succeeded in increasing the resolution to better than 4 parts in 10^7. At this very high level of precision even very small changes in the resistance of the sample, due to other causes than temperature change, must be taken into account. As a matter of fact we did observe a slight increase in resistance at constant temperature. The most probable explanation of this is surface oxidation due to traces of oxygen in the protective helium atmosphere. The rate of this aging effect was determined before and after the measuring period. The time dependence of the resistance at constant tem-

TABLE I. Measured resistances (in Ω) of the platinum thermometer and nickel sample.

Pt	Ni	Pt	Ni
301.870 49	1.049 297 4	305.179 93	1.083 121 0
301.966 00	1.050 201 6	305.272 40	1.083 829 2
302.052 05	1.051 019 7	305.363 14	1.084 498 1
302.144 02	1.051 896 2	305.460 00	1.085 192 0
302.236 00	1.052 775 6	305.552 19	1.085 836 7
302.327 93	1.053 659 1	305.644 23	1.086 465 7
302.419 95	1.054 546 4	305.734 52	1.087 071 8
302.512 07	1.055 438 4	305.825 55	1.087 672 6
302.603 93	1.056 331 8	305.917 61	1.088 270 1
302.696 00	1.057 231 9	306.009 80	1.088 859 3
302.787 64	1.058 130 5	306.101 57	1.089 437 4
302.880 20	1.059 043 6	306.194 00	1.090 012 6
302.972 68	1.059 959 0	306.285 26	1.090 573 7
303.061 43	1.060 841 6	306.376 25	1.091 126 2
303.158 54	1.061 813 4	306.467 20	1.091 673 0
303.248 68	1.062 718 2	306.559 91	1.092 224 0
303.340 32	1.063 646 2	306.652 80	1.092 770 4
303.432 17	1.064 580 5	306.744 80	1.093 306 2
303.525 64	1.065 535 3	306.834 89	1.093 826 0
303.617 98	1.066 486 3	306.927 81	1.094 357 0
303.708 44	1.067 424 4	307.021 00	1.094 885 2
303.799 74	1.068 376 0	307.114 00	1.095 408 0
303.891 88	1.069 345 0	307.206 00	1.095 920 3
303.983 50	1.070 316 4	307.300 85	1.096 445 3
304.075 96	1.071 305 3	307.388 37	1.096 925 4
304.168 65	1.072 304 4	307.483 07	1.097 440 7
304.260 90	1.073 310 2	307.570 80	1.097 916 0
304.351 55	1.074 310 4	307.663 38	1.098 413 0
304.444 80	1.075 349 4	307.759 61	1.098 926 4
304.536 19	1.076 381 0	307.849 10	1.099 400 7
304.627 80	1.077 428 5	307.940 50	1.099 882 2
304.718 21	1.078 483 0	308.032 30	1.100 362 3
304.810 88	1.079 570 5	308.124 80	1.100 842 5
304.908 20	1.080 679 0	308.216 19	1.101 315 2
304.993 49	1.081 561 0	308.308 09	1.101 787 8
305.091 39	1.082 412 0	308.400 30	1.102 258 5

perature could be fitted to a quadratic polynomial, with errors of fit smaller than the errors of the final fit of the resistance-temperature function. Since our measurements were made at constant time intervals, the only effect of aging is to give a contribution to the part of the resistance which is analytic at T_c (the background). The increase in resistance due to aging was about 10^{-8} Ω/min.

The measured data (in the temperature range 623.8 to 637.9 K) are given in Table I. Before analysis, the resistance of the thermometer has been converted to temperature by use of the expression

$$R(T) = -16.774\,864 + 0.558\,018\,92T - 7.5684 \times 10^{-5} T^2 \ , \tag{3}$$

which is the standard quadratic expression for the Pt thermometer, adjusted by calibration at the freezing points of Sn and Zn. The relation (3) defines our temperature scale.

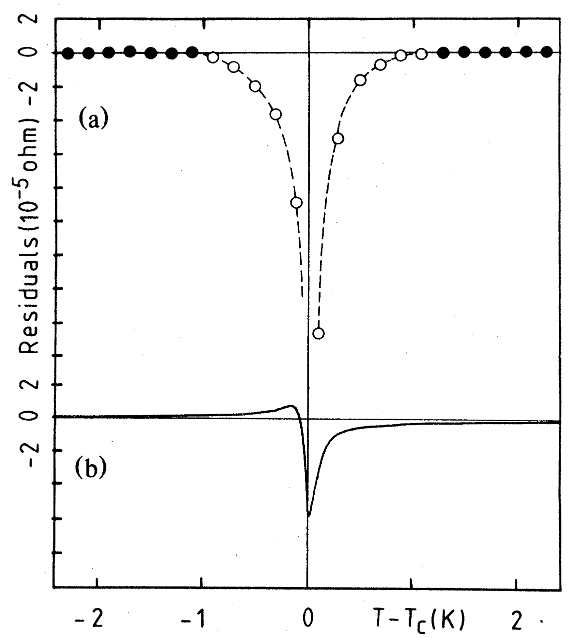

FIG. 1. (a) Residuals from the function f_3, Table II, in the region close to T_c. Open circles are residuals for points not included in the fit. (b) Residuals to be expected from a Gaussian distribution in T_c. The curve shows the difference between the convoluted function $f_3{}^*$ (with $\sigma = 0.1$ K) and the function f_3.

III. ANALYSIS AND RESULTS

We have chosen to work directly with $R(T)$ rather than to do a numerical differentiation of the experimental data. The singular part of the resistance is represented by a function of the form

$$R_s(T) = A\,|t|^{1-\alpha}(1 + D\,|t|^z) \ , \tag{4}$$

which is obtained from (2) by integration [the parameters A and D are not the same in (2) and (4)]. The data of Table I have been fitted to five functions, $f_1(t)$ to $f_4(t)$ and $f_3^*(t)$, all of which are combinations of a singular part according to (4) and a regular part, a linear or quadratic polynomial in t. For all functions we have adopted the scaling relation $\alpha = \alpha'$. The functions and the results of the fits are given in Table II. Good fits could not be obtained unless points close to T_c were omitted, because of the rounding of data in the vicinity of the critical point. The rounding effect is too small to be visible in a R-T plot. It is, however, clearly visible in a residual plot, see Fig. 1(a). This effect is further discussed in Sec. IV.

We have deleted points in a symmetric interval around T_c until residual plots of the best fit did not show the characteristic rounding structure close to T_c.

TABLE II. Fits to the data obtained from Table I by use of Eq. (3), with the points in the interval $629.3 < T < 631.4$ deleted. The functions are $f_1 = C_0 + C_1 t + A\,|t|^{1-\alpha}$, $f_2 = f_1 + C_2 t^2$, $f_3 = C_0 + C_1 t + A\,|t|^{1-\alpha}(1 + D\,|t|^{0.57})$, and $f_4 = f_3 + C_2 t^2$. The function f_3^* has been obtained from f_3 by convolution with a Gaussian according to Eq. (5), with a width $\sigma = 0.1$ K.

	α	A/A'	D/D'	T_c (K)	rms (Ω)
f_1	-0.046 ± 0.001	-1.200 ± 0.005		630.226 ± 0.004	2.366×10^{-6}
f_2	-0.081 ± 0.001	-1.402 ± 0.008		630.266 ± 0.002	4.347×10^{-7}
f_3	-0.096 ± 0.002	-1.522 ± 0.012	-0.86 ± 0.08	630.280 ± 0.003	3.844×10^{-7}
f_4	-0.097 ± 0.017	-1.525 ± 0.120	-0.86 ± 0.13	630.280 ± 0.007	3.844×10^{-7}
f_3^*	-0.095 ± 0.005	-1.512 ± 0.012	-0.79 ± 0.07	630.284 ± 0.003	3.828×10^{-7}

The range in reduced temperature decided on after this procedure was $1.6\times10^{-3}<|t|<12\times10^{-3}$, which is the range used in the fits of Table II. All points were given the same weights in the fit, and the confidence intervals given are 0.95 intervals obtained in the usual way, assuming independent and random errors in the measured resistances.

We now proceed to discuss the results of the fits as shown in Table II. The function f_1 (without correction term) gives a bad fit. As can be seen from Fig. 2(a), there is a pronounced systematic pattern in the residuals both above and below T_c. A substantial improvement of the fit is obtained when a t^2 term is added to the function f_1, giving the function f_2. For this fit a systematic pattern in the residuals is only just visible above T_c.

When the correction-to-scaling term is added, we expect computational difficulties, if all parameters are to be freely varying, due to the presence of two powers with unknown exponents and amplitudes. For the function f_3 (and the stated temperature interval) we have, however, been able to find a minimum at $z=0.57$, but the minimum is so flat as to be virtually without statistical significance. To illustrate this, we have made a number of fits at different (fixed) values of z, as shown in Table III. Evidently, there exists a curve in parameter space, an almost linear relation between the critical indices, along which the minimum is extremely shallow.

A plot of the residuals of the fit to the function f_3 is shown in Fig. 2(b), and as can be seen there is no visible systematic pattern in this residual plot.

When a quadratic term C_2t^2 was added to the function f_3, giving f_4, a fit at constant $z=0.57$ gave $C_2=-6.2\times10^{-2}$, a very small number compared to $C_2=2.38$ which was obtained in the fit to the function f_2. From Table II it can be seen that only small changes occurred in the critical parameters, and the decrease in rms error was only 0.002%. This improvement in the rms error is far too small to motivate the inclusion of the quadratic term.

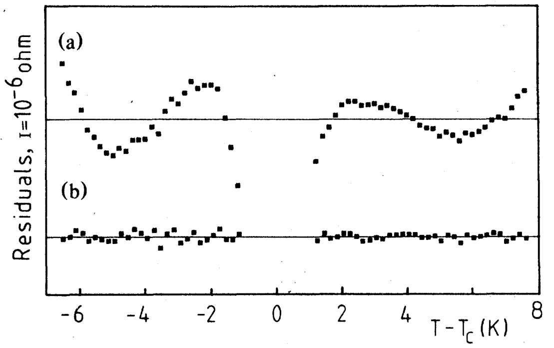

FIG. 2. (a) Residuals from the function f_1, Table II. (b) Residuals for the fit to f_3. The rms error per point for the whole region is 3.84×10^{-7} Ω. For the points below T_c it is 4.8×10^{-7} Ω, and above 2.8×10^{-7} Ω. It seems to be common to this type of measurement that the errors are larger in the ordered phase.

TABLE III. Result of fits to the function $f_3=C_0+C_1t+A|t|^{1-\alpha}(1+D|t|^z)$ for a number of (fixed) values of z. $z=0.57$ corresponds to our minimum and $z=0.55$ is the value predicted by RG theory.

z	α	A/A'	D/D'	rms error normalized
0.75	−0.086	−1.44	−0.67	1.005
0.57	−0.096	−1.52	−0.86	1.000
0.55	−0.098	−1.53	−0.89	1.001
0.37	−0.116	−1.70	−1.32	1.005

Instead of deleting the points close to T_C one can attempt to take rounding into account already in the function to which the data points are fitted. Provided the function f_3 is a good representation of the singular part of the temperature dependence in a completely homogeneous material, we can take into account inhomogeneities in the material, leading to different T_c in different parts of the sample, by using a function of the form

$$f_3^*(T,T_C,\sigma)=\int f_3(T,T_C-x)g_\sigma(x)dx\ , \qquad (5)$$

where $g_\sigma(x)$ is a Gaussian in x of width σ. The integral in (5) was done numerically, using 10-point Hermite integration, and the width of the Gaussian was also a parameter in the fit. The function $f_3{}^*$ gave a very bad fit when fitted to all the points in Table I. The result was $\sigma=0.2$ K, a rms error equal to 25×10^{-7} Ω per point and a pronounced structure in the residuals. Next, we made fits with the points closest to T_C removed one by one. The rms error of the fits decreased rapidly to a final, almost constant, value which was reached when points within ±1.1 K from T_C had been deleted, see Fig. 3. At the same time the width of the Gaussian decreased from 0.2 to approximately 0.1 K, as indicated by the broken line in Fig. 3.

The result of a fit of the function f_3^* to the same data as used for the functions f_1 to f_4 is shown in Table II, from which one can see that the critical parameters obtained from f_3^* and f_3 are almost the same. The value obtained for the width σ of the Gaussian is 0.13 ± 0.12 K. However, the rms error as a function of σ is very flat for $\sigma<0.2$ K, and for larger values of σ it rises sharply. Thus, the estimated confidence interval on σ may be misleading, and we have obtained another estimate by varying the range of the fit. We then get $\sigma=0.1\pm0.05$, in good

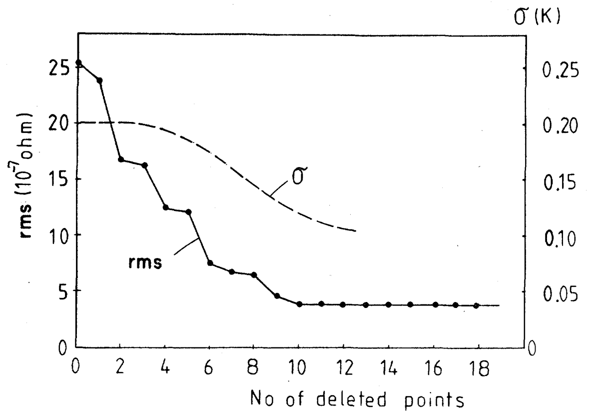

FIG. 3. Improvement in rms error obtained when fitting f_3^* (Table II) with the points closest to T_C successively removed. The broken line indicates the change in the width σ of the Gaussian.

agreement with the value given by Connelly *et al.*[15] for specific-heat data of Ni.

Accepting, on physical grounds, that there exists a rounding of the data due to structural disorder describable by the form (5) with $\sigma = 0.1$ K, we deduce from the results presented in Fig. 3 that most of the rounding in the interval $T_C \pm 1$ K must have other causes, to be discussed in Sec. IV. As an illustration of the effect of rounding as described by (5), we show in Fig. 1(b) the difference between the functions f_3^* and f_3. The difference is of order ten times our rms errors at $T_C \pm 1$ K, and is relatively large in a much wider interval. It is thus not restricted to a narrow region close to T_c.

The confidence intervals given in Table II should be thought of as lower bounds, a more realistic picture of the errors involved may perhaps be gained from the way the fitted parameters change with the range of the fit. When the range $|t|_{\max}$ of the fit was reduced from 12×10^{-3} to 7×10^{-3}, the results changed very little, e.g., the exponent α varied by about 3%. From this, we would like to give the following result for the leading critical parameters: $\alpha = -0.095 \pm 0.005$, $A/A' = -1.52 \pm 0.05$. The value $z = 0.57$ which we have obtained is consistent with the predicted value 0.55 for the isotropic short-range interaction. The next-to-leading-amplitude ratio D/D' is more uncertain, it varies by about 20% when $|t|_{\max}$ is reduced as above, and is rather sensitive to the value of the next to leading exponent z. However, in all cases we find that the amplitude ratio D/D' is negative, and of order -1.

IV. ROUNDING

The rounding of experimental data close to T_C has also been observed by other investigators,[15] and has commonly been attributed to impurities and structural defects (grain boundaries and dislocations) which cause a variation of T_C within the sample. In our analysis in Sec. III we have, however, not been able to give a satisfactory description of the rounding as a result of a distribution in T_C. We propose that the main part of the rounding is due to the fact that close to T_C the relaxation time of the system becomes very long (critical slowing down). Our arguments for this interpretation are based on (i) the observation that the measured resistance of the points very close to T_C was still drifting even though the temperature was well stabilized (readings were nonetheless taken at constant time intervals) and (ii) the steep decrease in rms error and the abrupt change to an almost range-independent fit obtained when points close to T_C were deleted, as discussed in Sec. III and illustrated in Fig. 3.

These facts lead us to conclude that there exists, close to T_C, a disturbance which gives rise to deviations from the function f_3 which cannot be described by the convolution in (5), and further that the influence of this disturbance decays so rapidly as to be undetectable (submerged in the experimental scatter) outside $T_C \pm 1$ K. To investigate the nature of this extra disturbance, we have calculated residuals from a fit to the function f_3^* (with $\sigma = 0.1$) according to Table II. Tests with other values of σ have shown that the precise value of σ is unimportant for the following, semiquantitative, discussion.

When we plot the logarithm of the residuals against temperature, we get approximately straight lines, see Fig. 4. This indicates an exponential (rather than a power-law) dependence of the residuals on t, which we interpret as the result of the exponential approach of the system towards equilibrium.

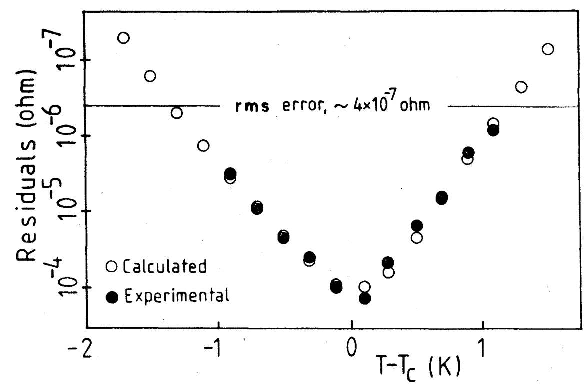

FIG. 4. Comparison between residuals, calculated by means of the model described in the Appendix (open circles) and experimental residuals obtained as the difference between experimental points and the convoluted function f_3^*, with $\sigma = 0.1$ K (filled circles).

We have made a simple model calculation to support this interpretation. The model, which is defined in the Appendix, is based on the assumption that the relaxation towards equilibrium can be described by a single relaxation time, and that this relaxation time diverges at T_C with an exponent Δ. From this model we obtain a recursion formula for the residuals r_n:

$$\ln(r_n) = \ln(a + r_{n-1}) - (t_0/B)|t|^{\Delta} . \quad (6)$$

By variation of the parameters a, B, B', and Δ, we have fitted the numbers r_n of (6) to the actual residuals obtained by using the function f_3^*. The result of the fit is shown in Fig. 4. The value obtained for the exponent Δ of the relaxation time varies between 1 and 1.3 depending on the σ chosen, with essentially the same quality of fit.

V. DISCUSSION

As far as we know, there are no results published for critical parameters for ferromagnets which have been obtained from fits to functions with confluent singularities, like (2). However, experimental data for dR/dT of iron were given in Ref. 4, and we have taken the opportunity to reanalyze these data in the same manner as described in Sec. II. Table IV gives the result of fits to some functions. Note, that a quadratic term in R corresponds to a linear term in dR/dT, and that A/A' must be positive in dR/dT if negative in R. Rounding effects forced us to discard points in the interval 1037 to 1044 K (T_C is approximately 1041 K). This unfortunately leaves us with only 26 points. The best fit is given by the function g_4, which contains the correction to scaling term as well as a term linear in t. This fit gives values for α, A/A' and D/D' which are very close to our corresponding values for Ni, although the confidence intervals are much wider.

In Table V, we have collected our results, the result of analysis of specific-heat data on Ni (Ref. 16) and EuO,[17] and of renormalization-group (RG) predictions for the critical indices and amplitude ratios. The values of z and D/D' in the row marked "RG theory" refer to corrections to scaling for the fully isotropic (and incompressible) Heisenberg ferromagnet, while the confluent singular term from the experimental data may also have other causes, to be discussed below. The table shows good agreement for the leading exponent and amplitude ratio between different experiments. As for most experiments on ferromagnetic systems, our value of α is slightly larger than the value predicted by RG theory.

The same is the case for the leading amplitude ratio A/A', for which our result agrees very well with other experiments as shown in Table V, but not with RG theory prediction. The theoretical result for A/A' is however obtained by truncating the ϵ expansion at second order,[18] and may be rather uncertain.

The RG results given in Table V are calculated for the specific heat at constant volume, while our measurement is of the resistance at constant pressure. To be able to make a proper comparison between our results and the predictions of RG theory, we should convert our values from resistance at constant pressure to resistivity at constant volume. This we have not done, and we do not expect such a transformation to change the leading singularity, neither the exponent nor the amplitude ratio. The next to leading singularity, however, could be changed by spin-lattice coupling, which may explain the inconsistency between our value of $D/D' \simeq -1$ and the RG calculation of Chang and Houghton,[19] which gives $D/D' = 1.13$.

Magnetic fields will influence the measurements close to T_C. Although the wiring of the furnaces was made bifilar, and the magnetic fields from the heating currents were calculated to be smaller than the geomagnetic field, no attempt was made to shield the sample from the geomagnetic field itself. We must thus accept that the field in the sample holder was at least of the same order of magnitude as the geomag-

TABLE IV. Fits to the dR/dT data for iron published by Shacklette (Ref. 4). Because of the small number of data points (26) confidence intervals are not given.

Function	α	A/A'	z(fix)	D/D'	T_c(K)	rms (Ω)
$g_1 = c_0 + A\|t\|^{-\alpha}$	−0.04	1.16			1040.72	2.86×10^{-4}
$g_2 = c_0 + c_1 t + A\|t\|^{-\alpha}$	−0.08	1.30			1040.62	2.16×10^{-4}
$g_3 = c_0 + A\|t\|^{-\alpha}(1 + D\|t\|^{z})$	−0.06	1.24	0.57	0.1	1040.84	2.03×10^{-4}
$g_4 = c_0 + c_1 t + A\|t\|^{-\alpha}(1 + D\|t\|^{z})$	−0.10	1.50	0.57	−0.9	1040.78	2.01×10^{-4}

TABLE V. Critical indices and amplitude ratios for ferromagnets by different methods. The quantity P is defined as $P=(|A/A'|-1)/(-\alpha)$. For the different signs of the amplitude ratio A/A', see text.

Material	α	A/A'	P	z	D/D'	Reference
Ni, R	-0.095 ± 0.005	-1.52 ± 0.05	5.4	0.57	-0.8 ± 0.2	This work
Fe, dR/dT	-0.10	1.50	5.0	0.57[a]	-0.9	This work[b]
Ni, C_p[c]	-0.091 ± 0.002	1.40 ± 0.01	4.4			16
Ni, C_p	-0.10[a]	1.49 ± 0.02	4.9	0.5[a]		16
EuO, C_p	-0.10 ± 0.05	1.51 ± 0.2	5.1	$0.56\ \pm0.20$	2.0	17
RG theory	-0.115 ± 0.009	1.24		0.550 ± 0.005	1.13	7,18,19

[a] Variable held fixed in the fit. [b] Data taken from Ref. 4. [c] A fit without correction term $(D=D'=0)$.

netic field. The effect of a magnetic field on the specific heat is discussed in Ref. 15, and we use the equivalence of the specific heat and the temperature derivative of the resistivity to make the following comments: At an external field $H=1$ G, the reduced field $h=HM_0/(kT_c)$, with M_0 the spontaneous magnetization, is, for nickel, $h=6.6\times10^{-8}$. The corresponding characteristic reduced temperature $t_m=h^{1/\beta\delta}=7.4\times10^{-5}$, which is 20 times smaller than the minimum value of $|t|$ used in the fits in Table II. Judging from Fig. 12 of Ref. 15, we think it very unlikely that a magnetic field of this size (1 G) can have any noticeable influence on our fits.

Very close to T_C the long range, but weak, dipolar forces are expected to be important for the critical behavior. We have not seen any indication of a crossover to dipolar behavior in our fits, and furthermore the crossover temperature in nickel has been estimated[20] to be of order $\hat{t}=3\times10^{-4}$, which is well outside the temperature range we have used.

In summary, we consider our best fits to be of high quality. The reason for this is twofold: First, it is not possible to detect, by visual inspection, any systematic trend in the plot of the residuals [Fig. 2(b)] and second, there is no small scale structure in the residuals in the form of jumps or discontinuities. Such small structure is commonly seen in fits to critical quantities, and greatly increases the difficulty of obtaining good fits. For these reasons, we conclude that the function used (f_3 or f_3^*) gives a good representation of, and exhausts the information contained in our experimental data. We believe that the critical parameters we have given in Table V gives an accurate description of the resistivity of nickel within the temperature region we have used, $1.6\times10^{-3}<|t|<12\times10^{-3}$.

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APPENDIX

Here we define a simple model, used to explain the residuals close to T_C in Sec. IV. We make the following assumptions: The approach to equilibrium follows a simple exponential process, with one relaxation time τ. This relaxation time has a critical temperature dependence given by

$$\tau=B|t|^{-\Delta}\ . \tag{A1}$$

The temperature is increased by constant amounts ΔT at constant time intervals t_0, and the temperature change of the sample is considered to be instantaneous. This is an adequate approximation close to T_C, where the relaxation time of the temperature-control system is short compared to the relaxation time of the sample. Let the equilibrium resistance of the sample in the time interval nt_0 to $(n+1)t_0$ be R_n, and the observed resistance at the end of this time interval be R_n-r_n. We then get the following relation

$$r_n=(R_n-R_{n-1}+r_{n-1})\exp(-t_0/\tau) \tag{A2}$$

or, using (A1):

$$\ln(r_n)=\ln(R_n-R_{n-1}+r_{n-1})-(t_0/B)|t|^{\Delta}\ . \tag{A3}$$

For simplicity we take $R_n-R_{n-1}=a=$const, and arrive at

$$\ln(r_n)=\ln(a+r_{n-1})-(t_0/B)|t|^{\Delta}\ , \tag{A4}$$

which is used in Sec. IV.

[1] D. J. W. Geldart and T. G. Richard, Phys. Rev. B 12, 5175 (1975).

[2] I. Balberg and J. S. Helman, Phys. Rev. B 18, 303 (1978); S. Alexander, J. S. Helman, and I. Balberg, *ibid.* 13, 304 (1976).

[3] F. C. Zumsteg and R. D. Parks, Phys. Rev. Lett. 24, 520 (1970).

[4] L. W. Shacklette, Phys. Rev. B 9, 3789 (1974).

[5] Ya. F. Kraftmakher and T. Yu. Pinegina, Sov. Phys. Solid State, 16, 78 (1974).

[6] M. D. Launchbury and N. H. Saunders, J. Phys. F 6, 1967 (1976).

[7] J. C. Le Guillou and J. Zinn-Justin, Phys. Rev. B 21, 3976 (1980).

[8] G. Ahlers, Rev. Mod. Phys. 52, 489 (1980).

[9] G. Ahlers, Phys. Rev. A 8, 530 (1973).

[10] A. Kornblit and G. Ahlers, Phys. Rev. B 8, 5163 (1973).

[11] I. Balberg and A. Maman, Physica B 96, 54 (1979).

[12] G. Malmström and D. J. W. Geldart, Phys. Rev. B 21, 1133 (1980).

[13] K. V. Rao, Ö. Rapp, Ch. Johannesson, D. J. W. Geldart, and T. G. Richard, J. Phys. C 8, 2135 (1975).

[14] Ö. Rapp, G. Benediktsson, H. U. Åström, S. Arajs, and K. V. Rao, Phys. Rev. B 18, 3665 (1978).

[15] D. L. Connelly, J. S. Loomis, and D. E. Mapother, Phys. Rev. B 3, 924 (1971).

[16] G. Ahlers and A. Kornblit, Phys. Rev. B 12, 1938 (1975).

[17] A. Kornblit and G. Ahlers, Phys. Rev. B 11, 2678 (1975).

[18] C. Bervillier, Phys. Rev. B 11, 4964 (1976).

[19] M.-C. Chang and A. Houghton, Phys. Rev. B 21, 1881 (1980).

[20] M. E. Fisher and A. Aharony, Phys. Rev. Lett. 30, 559 (1973).