Predicted precritical second-sound damping in superfluid ^{4}He: "High-temperature" expansion

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A "high-temperature" expansion in powers of $|t|^{-2/3}$, where t is the reduced temperature, is employed to predict the first rise above background in the second-sound damping as the λ point is approached from below. We show that this precritical rise is determined by the corresponding precritical rise in the thermal conductivity as the λ point is approached from above.

Because of the high experimental precision that can be attained, the λ transition in liquid ^{4}He provides a proving ground *par excellence* for testing theories of critical dynamics. Dynamic scaling theory[1,2] predicted a $|t|^{-1/3}$ divergence for the normal-state thermal conductivity, $\lambda(t)$, and for the superfluid state second-sound damping coefficient, $D_2(|t|)[t=(T-T_\lambda)/T_\lambda$ is the reduced temperature]. Experimentally these expectations have been only qualitatively realized. But recently we[3-5] have pointed out that the theory above the λ point can be reconciled quantitatively with experiment by recognizing that the measurements are by necessity *not* carried out in the asymptotic scaling region. Therefore noncritical and nonuniversal background effects are important. Furthermore, the theory predicts that the entropy relaxation rate γ_S is at least one order of magnitude larger than the order-parameter relaxation rate γ_ψ in the experimentally unattainable asymptotic region. This requires a steep rise in γ_S in the observable region as the λ point is approached. The corresponding effective critical exponent for $\lambda(t)$ is therefore about 20% bigger than its dynamic scaling value, in good accord with the trend noted by Ahlers,[6] and as confirmed by further contributions[7-9] to the theory along this line. The purpose of this short note is to point out that the situation is less clear in regard to second-sound damping, the other important test of the theory. We present here a new calculation of $D_2(|t|)$ over the temperature range $10^{-4} < t < 10^{-1}$. In the distant region the measured values of D_2 deviate from our theoretical prediction, suggesting the need for further experimental study.

In order to make our theoretical prediction as compelling and unequivocal as possible we first consider relatively large values of $|t|$, corresponding to temperatures well below T_λ. In this region the same background parameters dominate as in the normal region well above T_λ. Furthermore, when the λ point is approached the onset of criticality is described by a "high-temperature" expansion[4,5] in powers of the correlation length $\kappa^{-1} \propto |t|^{-2/3}$. The predicted behavior of $D_2(|t|)$ below the λ point can thereby be connected to the known behavior of $\lambda(t)$ above, with very little freedom for the theory to go astray.

The background second-sound damping coefficient $D_2^B(|t|)$ is dominated by the background kinetic coefficient for order-parameter relaxation,[4,5] B_ψ, and the background thermal conductivity,[4,5] λ_B. Identifying the half-width at half-maximum of the second-sound resonance with $(D_2/2)k^2$, we have from the standard theory of second-sound damping the total background damping coefficient

$$D_2^B = \frac{\gamma_\psi}{k^2} + \frac{\gamma_S}{k^2} + \frac{4}{3}\frac{\rho_s}{\rho_n}\tilde{\eta} = B_\psi + \frac{\lambda_B}{C_P'} + \frac{4}{3}\frac{\rho_s}{\rho_n}\tilde{\eta} \quad . \tag{1}$$

Critical temperature dependence is introduced by the specific heat in the second term and by the super to normal fluid density ratio in the third term. $\tilde{\eta}$, the kinetic viscosity, has a very weak critical variation. The temperature dependence of the individual terms in Eq. (1) is plotted in Fig. 1 versus $T < T_\lambda$ over a wide temperature range. The dashed curve adds to λ_B the precritical rise explained below. Light scattering provides a means of distinguishing experimentally the separate components. Thermal diffusion pro-

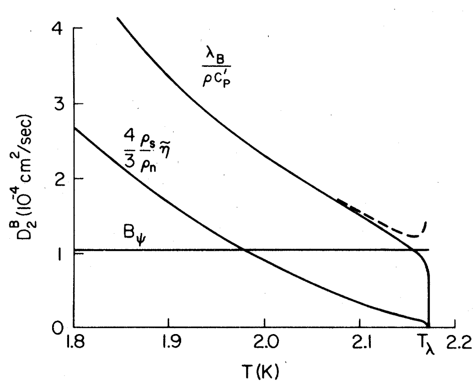

FIG. 1. Temperature dependence of the background contributions to the second-sound damping coefficient.

duces an absence of scattering intensity at the center of the spectrum, while order parameter relaxation tends to fill in this valley. While Tarvin, Vidal, and Greytak[10] infer that the latter is twice as strong as the former in the interval $10^{-3} \leq |t| \leq 10^{-2}$, Fig. 1 indicates roughly equal strength. The temperature dependence of the total background predicted from Eq. (1), i.e., the sum over all three of the components shown in Fig. 1, is shown by the dot-dash curve in Fig. 2.

The dotted curve of Fig. 2 shows the prediction of our theory when the leading terms of the "high-temperature" expansion are included in Eq. (1), producing the "precritical" rise. We calculate the precritical rise below the λ point in close analogy with the calculation[4,5] of the precritical rise above the λ point, which we now briefly review. The precritical thermal conductivity in excess of λ_B for $t > 0$ is given by elementary kinetic theory[11] as

$$\Delta\lambda = \frac{2}{3}\frac{g^2}{8\pi^3}\int \frac{d^3p\, p^2}{\gamma_\psi(p,\kappa)} g^2(p,\kappa) \quad . \tag{2}$$

Here g is the standard coupling constant, while the order-parameter correlation function has the Ornstein-Zernike[12] form $g(p,\kappa) = (p^2+\kappa^2)^{-1}$. In the background region the relaxation rate is expressed by a Van Hove-type formula in which the critical slowing down results entirely from the weakening thermodynamic force

$$\gamma_\psi(p,\kappa) = B_\psi g^{-1}(p,\kappa) = B_\psi(p^2+\kappa^2).$$

Substituted into Eq. (2) this yields $\Delta\lambda = (g^2/3\pi^2) \times (I/\kappa B_\psi)$, where the definite integral is

$$I = \int_0^\infty du\, u^4(u^2+1)^{-3} = 3\pi/16.$$

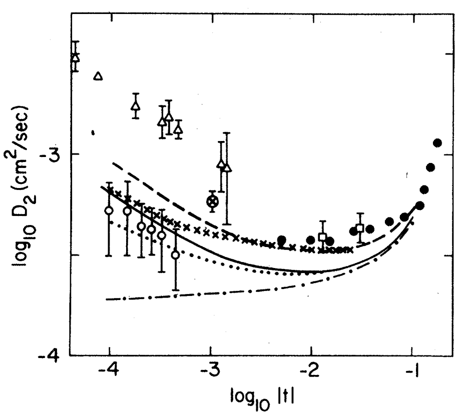

FIG. 2. Predicted background (dot-dash curve) and total (solid curve) second-sound damping vs reduced temperature. The dotted curve includes background and precritical rise while the dashed curve shows the addition of unmodified relaxation. The crosses indicate the theory of Dohm and Folk (Ref. 8). Data references: ○-16, ●-17, □-18, ⊗-19, and Δ-20.

By substituting $\kappa = \kappa_0 t^{2/3}$, where κ_0 is chosen as 0.63×10^8 cm^{-1}, we obtain a predicted "high-temperature" behavior of $\Delta\lambda$ proportional to $t^{-2/3}$, with the coefficient of proportionality equal to 2.2×10^{-1} erg cm sec^{-2} K$^{-1}/B_\psi$. Figure 3 confronts the above precritical theory with the measurements by Ahlers[6] of $\lambda(t)$ at saturated vapor pressure (SVP). It is seen from this plot of $\lambda(t)$ vs $t^{-2/3}$ that the data can be well fitted by a parabola, corresponding to an effective three-term high-temperature expansion. The first term of this expansion is the vertical intercept $\lambda_B = 0.153$ mW/K. The dashed straight line represents the two-term high-temperature expansion. Its slope corresponds to $B_\psi = 1.05 \times 10^{-4}$ cm^2 sec^{-1}. These are the background parameters which have been employed in plotting Figs. 1 and 2. The higher terms of the expansion will be discussed in a forthcoming report.[13]

The corresponding calculation below the λ point can be carried out in a completely analogous way. We know, however, from ultrasonic attenuation,[14] that we must use the different length scale $\kappa_0' = 1.50 \times 10^8$ cm^{-1}. Thus, $\kappa_0/\kappa_0' = 0.42$. A further modification of the theory is that the intermediate lines in the "single-bubble" expression of Eq. (2) are no longer identical. One of the lines belongs to the transverse order parameter, with $\kappa = 0$. Consequently, below the λ point, the integral corresponding to Eq. (2) becomes

$$\Delta\lambda' = \frac{4}{3B_\psi}\frac{g^2}{8\pi^3}\int \frac{d^3p\, p^2 g(p,0) g(p,\kappa')}{g^{-1}(p,0)+g^{-1}(p,\kappa')}$$
$$= \frac{g^2}{3\pi^2}\frac{I'}{B_\psi \kappa'} \quad . \tag{3}$$

The ratio of the two integrals is $I'/I = 8(2-\sqrt{2})/3 = 1.56$. The above results enable us to compare very directly the predicted precritical rise for $t < 0$ with the

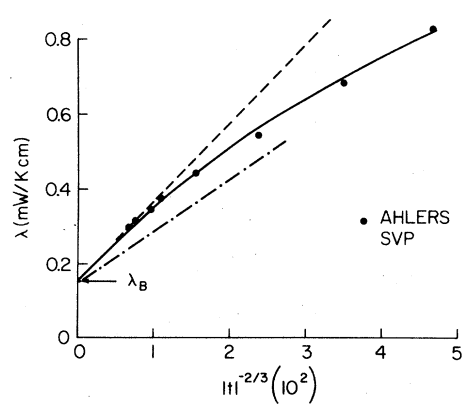

FIG. 3. Thermal conductivity vs correlation length. The dashed and solid curves are the two- and three-term high-temperature fits, respectively, to Ahler's saturated vapor pressure data (Ref. 6). The dot-dash curve is the two-term "high-temperature" prediction for λ' below the λ point.

known precritical rise for $t > 0$. For equal values of $|t|$ we have $\Delta\lambda'(|t|)/\Delta\lambda(t) = (\kappa_0/\kappa_0')(I'/I) = 0.66$. This predicts the precritical behavior of $\lambda' = \lambda_B + \Delta\lambda'$ shown by the dot-dash straight line in Fig. 3 and which yields a two-term "high-temperature" expression for the entropy kinetic coefficient $D'_{S'} = \lambda'/C'_P$. As the λ point is approached, D'_S replaces its background value $B'_S = \lambda_B/C'_P$ in Eq. (1).

It is also necessary to replace B_ψ in Eq. (1) by $D'_\psi = B_\psi + \Delta D'_\psi$, the two-term "high-temperature" expression for the order-parameter kinetic coefficient. The precritical single-loop expression for $C'_P \Delta D'_\psi$ is of the same final form as Eq. (3) where in place of I' the definite integral is

$$J(w') = \frac{3}{2}\int_0^\infty \frac{du\, u^2/(u^2+1)}{(1+w'^{-1})u^2+1} = \frac{3\pi}{4} w' \left[1 - \left(\frac{w'}{1+w'}\right)^{1/2}\right] . \quad (4)$$

The background ratio of the kinetic coefficients is $w' = B_\psi/B'_S = 0.8$. The ratio of Eq. (4) to the earlier integral is $J/I = 1.07$. The precritical rise that has to be added to Eq. (1) is therefore

$$\Delta D'_\psi + \frac{\Delta\lambda'}{C'_P} = 1.69\frac{\Delta\lambda'}{C'_P} = 1.11\frac{\Delta\lambda}{C'_P} \simeq \frac{\Delta\lambda}{C'_P} . \quad (5)$$

The 11% error incurred in the final form of Eq. (5) is within the accuracy of the truncated "high-temperature" expansion. The addition of Eq. (5) to Eq. (1) yields the remarkably simple result

$$D_2(|-t|) = B_\psi + \frac{\lambda_{\text{expt}}(t)}{C'_{P\,\text{expt}}(-t)} + \frac{4}{3}\left(\frac{\rho_s}{\rho_n}\tilde{\eta}\right)_{\text{expt}} , \quad (6)$$

where the subscripts emphasize that the quantities involved are *experimentally determined.* [As explained above, the background constant B_ψ is found from the derivative of $\lambda_{\text{expt}}(t)$ with respect to $t^{-2/3}$.] Equation (6) is plotted versus $\log_{10}|t|$ as the dotted curve in Fig. 2.

The above theoretical treatment is not complete. The three fluctuating fields in the problem are the entropy and the transverse and longitudinal components of the order parameter. Each type of fluctuation contributes in principle to the damping of second sound. We are therefore obliged to consider the longitudinal relaxation of the order parameter. This, in fact, is the sole contribution to the precritical damping[14] of low-frequency first sound for $t < 0$. The relaxation at rate τ^{-1} causes a thermal lag which introduces a dependence of C'_P on the frequency ω. Introducing this ω dependence into the second-sound velocity via its explicit dependence on C'_P would result in the total damping shown by the dashed curve in Fig. 2. But the situation is more complicated, because of some compensating effects. Khalatnikov's theory,[15] without the approximation that he makes, results in a much more moderate relaxational contribution. This yields the total damping shown by the solid curve in Fig. 2, in satisfactory agreement with the recent measurements reported by Ahlers.[16] It lies, however, about a factor of 2 below the measurements of Hanson and Pellam,[17] of Notarys,[18] and of Greytak[19] and co-workers. The latter point is more closely in accord with the earlier measurements of Tyson.[20] This is a puzzling discrepancy which has already been encountered in connection with the light scattering from pressurized superfluid He.[21] It would clearly be desirable to have a single experiment cover the entire precritical and background region. In addition, an experimental check of the pressure dependence predicted by Eq. (6) would be useful. In the interval $10^{-4} \leq |t| \leq 10^{-3}$, $\lambda_{\text{expt}}(t)$ drops by a factor of 2 in going from SVP to 22 bar.[6] $C'_{P\,\text{expt}}$ has a much weaker pressure dependence. We can therefore make the following prediction[22]: *at the top of the λ line the precritical rise in $D_2(|t|)$ has one-half of the strength found by Ahlers at SVP.*

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[21]Left-hand side of Fig. 7 of Ref. 5. The precritical rise calculated in the present paper reduces the discrepancy shown in this figure by approximately 50%.
[22]This is consistent with the weaker pressure dependence reported by Greytak (Ref. 19) at $|t| = 10^{-3}$. At this temperature the relatively pressure insensitive background (dot-dash curve of Fig. 2) constitutes a significant fraction of D_2.