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Scaling theory of the Hall effect in disordered electronic systems

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A scaling theory for the Hall effect in disordered electronic systems is developed. It is suggested that a universal scaling function for the Hall conductance exists, and the leading quantum correction to the classical value $(d-2)$ of this function is calculated (d is the dimensionality). It is shown, by means of a scaling argument, that at the mobility edge the zero-temperature Hall conductivity approaches zero with an exponent $t_H = 2t$, where t is the conductivity exponent. This relation between the exponents is supported by a microscopic calculation in $2+\epsilon$ dimensions, which yields $t=1$, $t_H=2$.

I. INTRODUCTION

The purpose of this paper is to develop a scaling hypothesis for the Hall effect in disordered electron systems. Our arguments are based on the scaling theory of electron localization which has recently been developed by Abrahams *et al.*[1,2] The scaling parameter in their theory is the dimensionless conductance $g(L) = G(L)/(e^2/\hbar)$ at length scale L. Here $G(L)$ is the zero-temperature conductance of a hypercubic sample of size L in d dimensions. The conductance at scale bL (b is the scaling factor) is determined by the scaling relation[1]

$$g(bL) = f(b; g(L)) \quad , \tag{1.1}$$

or in continuous terms

$$d\ln g / d\ln L = \beta(g) \quad , \tag{1.2}$$

where, for each d, β is a universal function of g only.

The scaling theory predicts $d=2$ as the lower critical dimensionality in the following sense: The existence of a mobility edge is indicated by a zero of the β function. At $d=2$, $\beta(g) < 0$ for all finite g, and no zero occurs. For $d>2$, $\beta(g)$ has a zero, and the conductivity

$$\sigma(E) = (e^2/\hbar) \lim_{L\to\infty} L^{2-d} g(L;E) \tag{1.3}$$

is finite for Fermi energies E higher than the mobility edge E_c. Near (above) E_c, we have

$$\sigma(E) \sim (E-E_c)^t \quad . \tag{1.4}$$

The conductivity exponent t is related to the correlation length exponent ν by

$$t = (d-2)\nu \quad , \tag{1.5}$$

a result first obtained by Wegner.[3]

We want to extend the scaling arguments of Ref. 1 to the Hall effect. The influence of magnetic field on the localization picture at $d=2$ has been discussed from the microscopic point of view by several authors.[4–6] The calculations in these papers are restricted to the weak scattering limit. Here we shall give a scaling hypothesis which gives the behavior near the mobility edge. While our arguments are given for temperature $T=0$, we expect that at finite T the length scale is set by the temperature-dependent Thouless length[7,8] $L_T \simeq (l_{el} l_{in})^{1/2}$ where the l's are the elastic and (temperature-dependent) inelastic mean free paths.

In Sec. II, we formulate the scaling hypothesis. The hypothesis is supported to some extent by a microscopic calculation in Sec. III. In Sec. IV, we give an explicit calculation in $2+\epsilon$ dimensions where the mobility edge is accessible by perturbation theory. The conclusions are summarized in Sec. V.

II. SCALING HYPOTHESIS FOR THE HALL CONDUCTANCE

The Hall conductance G_H is defined in terms of the transverse Hall voltage U_H, the longitudinal voltage U, and the conductance G by

$$G_H = G U_H / U \quad . \tag{2.1}$$

The Hall conductivity is defined as

$$\sigma_H(E) = \lim_{L\to\infty} L^{2-d} G_H(L;E) \quad . \tag{2.2}$$

For $E < E_c$, i.e., in the insulating region, $\sigma_H = 0$ since there can be no Hall voltage without an Ohmic current. When the mobility edge is approached from above, $\sigma_H(E)$ presumably approaches zero according to

$$\sigma_H(E) \sim (E-E_c)^{t_H} \tag{2.3}$$

which defines the Hall conductivity exponent t_H.

Let us introduce a dimensionless Hall conductance $g_H(L)=G_H(L)/(e^2/\hbar)$ and try to understand how it might scale with the sample size L.

It is instructive to start with the classical transport regime. In this regime macroscopic transport theory is valid, the parameter $g(L)$ is much larger than unity and it scales with L as L^{d-2}. The Hall field $E_H=rBj$, where r is the Hall constant of the material and j is the current density. Since $U_H=E_HL$ and $jL^{d-1}=UG$, we have

$$U_H=(e^2/\hbar)rBL^{-(d-2)}gU \tag{2.4}$$

or, using Eq. (2.1)

$$g_H(L)=(e^2/\hbar)rBL^{-(d-2)}g^2(L) \ . \tag{2.5}$$

In this classical regime $g(bL)=b^{d-2}g(L)$ and thus Eq. (2.5) implies the following scaling relation for $g_H(L)$:

$$\begin{aligned} g_H(bL)&=(e^2/\hbar)rBL^{-(d-2)}b^{d-2}g^2(L)\\ &\equiv h_{\text{cl}}^{-1}(L)b^{d-2}g^2(L) \ . \end{aligned} \tag{2.6}$$

The parameter h_{cl} has the meaning of a conductance L^{d-2}/rB, measured in units $(e^2/\hbar)$. In the classical transport regime both $h_{\text{cl}}(L)$ and the scaling parameter $g(L)$ scale as L^{d-2}, and so does the Hall conductance $g_H(L)$.

We now assume that $g_H(L)$ possesses scaling behavior not only in the classical limit (i.e., for $g\to\infty$) but for any value of the scaling parameter $g(L)$. With Eq. (1.1) in mind, the generalization of Eq. (2.6) suggests the following scaling relation:

$$g_H(bL)=h^{-1}(L)F(b;g(L)) \ , \tag{2.7}$$

where $h(L)$ is a (dimensionless) conductance inversely proportional to the magnetic field B.

According to the universality argument of Ref. 1 the dimensionless conductance $g(L)$ scales classically, i.e., as L^{d-2}, whenever it is large. We now assume that the same is true for $h(L)$, i.e., $h(L)\sim L^{d-2}$ for $h>>1$. For small enough B the condition $h>>1$ will be satisfied whatever the value of g is. In fact our assumption is that even when quantum corrections to $g(L)$ become important, $h(L)$ still scales classically if it is large enough. Fukuyama's microscopic calculation[4] in two dimensions gives some evidence in favor of this assumption as does the $2+\epsilon$ dimension calculation in Sec. III. Thus, if we require h to be large even at some typical microscopic scale L_0, we get a rough criterion for a weak magnetic field. This requirement gives $B<<\hbar L_0^{d-2}/e^2r$. (Estimating $r\simeq 1/enc\simeq L_0^d/ec$, we have $B<<\hbar c/eL_0^2$. If L_0 is the order of interatomic spacing this gives the usual requirement for a "classical" magnetic field in metals.)

If the conjecture about classical scaling of the parameter $h(L)$, for any value of $g(L)$, is correct, we have

$$g_H(bL)\sim L^{-(d-2)}F(b;g(L)) \ . \tag{2.8}$$

It follows from this equation that the Hall-conductivity exponent [Eq. (2.3)]

$$t_H=2(d-2)\nu=2t \ . \tag{2.9}$$

To derive this result we introduce

$$\Delta(L)=[g(L)-g_c]/g_c \tag{2.10}$$

as the basic scaling parameter [rather than $g(L)$ itself as in Ref. 1]. For L of the order of the correlation length ξ this parameter is of order unity.[9] For $L<<\xi$, $\Delta(L)<<1$. In this case $\xi\sim L\Delta^{-\nu}(L)$ and thus, for $\Delta<<1$, the parameter $\Delta(L)$ scales as

$$\Delta(L)=\Delta_0(L/L_0)^{1/\nu} \ , \tag{2.11}$$

where $\Delta_0\sim(E-E_c)/E_c$ is the initial value of the parameter at some scale, e.g., the microscopic scale L_0.

We consider now a sample of size $\mathcal{L}$ and divide it into blocks of size L, i.e., $b=\mathcal{L}/L$. For a large ($\mathcal{L}>>\xi$) sample with E above E_c (i.e., in the metallic region), the Hall conductance $G_H(\mathcal{L})$ must be proportional to $\mathcal{L}^{d-2}$, which via Eq. (2.8) implies

$$g_H(\mathcal{L})\sim L^{-(d-2)}(\mathcal{L}/L)^{d-2}\phi(\Delta(L)) \ . \tag{2.12}$$

Since $\Delta\propto E-E_c$, we require, by Eq. (2.3) that for $\Delta<<1$, $\phi(\Delta)\sim\Delta^{t_H}$. Hence using Eq. (2.11) we have

$$g_H(\mathcal{L})\sim L^{-(d-2)}(\mathcal{L}/L)^{d-2}\Delta_0^{t_H}L^{t_H/\nu} \ . \tag{2.13}$$

Since the block size is arbitrary (the only condition being $L<<\xi$), it must cancel out from Eq. (2.13), which immediately leads to relation (2.9) for the exponents.[10] Clearly, Eq. (1.5) for the conductivity exponent t can also be derived by a similar argument.

The Hall coefficient $r(E)$ is proportional to $\sigma_H(E)/\sigma^2(E)$. Thus it follows from Eq. (2.9) that $r(E)$ approaches some constant value when $E\to E_c$ from above. This constant is presumably of the same order of magnitude as the Hall coefficient in the classical transport regime. Thus we conclude that strong disorder near or at the mobility edge affects the Hall coefficient much less than its affects $\sigma(E)$ or $\sigma_H(E)$. In fact a perturbative calculation in the weak scattering limit shows that leading quantum correction to $\sigma(E)$ and $\sigma_H(E)$ cancel each other in $r(E)$.[4,6] Thus our results for $r(E)$ near and at the mobility edge can be viewed as a generalization to strong disorder of the corresponding results in the weak scattering regime.[4,6]

Taking $b=1+\delta(\delta\to 0)$, the scaling relation (2.8) [or (2.7)] can be cast into differential form

$$d\ln g_H(L)/d\ln L=\gamma(g(L)) \ , \tag{2.14}$$

where $\gamma(g)$ is a universal function of the scaling parameter $g(L)$. For $g \rightarrow \infty$, i.e., in the classical transport regime, $g_H(L) \sim L^{d-2}$ and hence $\gamma = d-2$. On the other hand, for $g \rightarrow 0$, i.e., in the strongly localized regime, $g_H(L)$, as well as $g(L)$, is exponentially small and $\gamma \rightarrow -\infty$. Thus the qualitative behavior of the γ function is similar to that of the β function.[1] However, quantitatively these two functions are different as is shown in the next section. It is this difference which accounts for the difference in the critical behavior of $\sigma(E)$ and $\sigma_H(E)$.

Finally we would like to comment on the following point: In the above arguments, as well as in the following calculations in Secs. III and IV, we ignore the dependence of the scaling parameter $g(L)$ on the magnetic field B. This is justified[6] only for $B \ll \hbar c/4eL^2$, where L is either the sample size or the Thouless length, whichever is smaller. In particular, if $T=0$ and the sample size $L \rightarrow \infty$ (it is under these conditions when the critical exponents can be rigorously defined) any finite field would change $g(L)$ in an essential way, and thus our results for this case refer strictly speaking to an infinitesimal B. At present there seems to be no definite answer to the important question about relevance of the magnetic field to the Anderson transition. Calculations in the weak scattering regime[5,6,11–13] suggest that a weak magnetic field acts as a delocalizing factor and that it represents a relevant perturbation, i.e., it does influence scaling functions and critical exponents of the transition. In particular, it was recently claimed[14] that, at $d=2$, a single parameter scaling theory is not applicable. If this is the case also for $d>2$, then our results for $T=0$, $L \rightarrow \infty$ are indeed restricted to infinitesimal B (i.e., the magnetic field only "probes" the system, but does not affect the transition). On the other hand a recent calculation of Sadovski[15] suggests that a magnetic field, when strong enough, can only increase localization. Such a field might be an irrelevant perturbation for the Anderson transition, i.e., it might only shift the mobility edge without changing the scaling functions and the exponents. If this is the case, our results are valid also for finite B, although in the explicit calculations we neglect the dependence of the scaling parameter g on B.

III. WEAK SCATTERING REGIME

In this section we calculate the leading (i.e., proportional to $1/g$) quantum correction to the classical limit $(d-2)$ of the γ function. Thus we consider the weak scattering regime $kl \gg 1$, where k is the Fermi wave number and l is the electron mean free path.

For the β function this leading correction has been calculated in Refs. 2 and 16. This correction is due to the maximally crossed diagrams, which can be summed explicitly. The sample size L enters the calculation of Ref. 2 via the lower cutoff $1/L$ in some integrals over the momentum space. This leads to an L-dependent conductivity $\sigma(L)$ from which one obtains the conductance as $G(L) = L^{d-2}\,\sigma(L)$. Although the explicit calculation in Ref. 2 has been done for $d=2$, it is trivially generalized to any dimension, with the following result:

$$g(L;E) = \frac{\hbar}{e^2}\sigma_0(E)L^{d-2} - \frac{A}{d-2}\left[\left(\frac{L}{l(E)}\right)^{d-2} - 1\right] . \tag{3.1}$$

Here $A = 2\pi^{-1}(2\pi)^{-d}S_d$, where $S_d = 2\pi^{d/2} \times [\Gamma(d/2)]^{-1}$ is the area of a d-dimensional sphere of unit radius. The first term in Eq. (3.1) represents the classical conductance, with

$$\sigma_0 = 2(e^2/\hbar)d^{-1}(2\pi)^{-d}S_d k^{d-1} l \ . \tag{3.2}$$

The second term in Eq. (3.1) represents the leading quantum correction. It was assumed in the derivation of Eq. (3.1) that the sample size L is bigger than the mean free path l. The factors 2 in the expressions for A and σ_0 account for spin degeneracy.

In the limit $d \rightarrow 2$ Eq. (3.1) takes the form

$$g = (\hbar/e^2)\sigma_0 - (1/\pi^2)\ln(L/l) \ , \tag{3.3}$$

and the result of Ref. 2 is recovered.

For the perturbative calculation above to be valid the second term in Eq. (3.1) must be much smaller than the first one, which, with the help of Eq. (3.2), leads to the following criterion:

$$\alpha(L;E) \equiv \frac{d}{\pi}(kl)^{1-d}\frac{1}{d-2}\left[1 - \left(\frac{l}{L}\right)^{d-2}\right] \ll 1 \ . \tag{3.4}$$

In three dimensions the weak scattering condition $kl \gg 1$ itself ensures the fulfillment of criterion (3.4), for any L (bigger than l). However, near two dimensions, i.e., for $(d-2) \ll 1$, the condition (3.4) is satisfied, for any L, only if a more restrictive requirement on kl is imposed, namely, $kl \gg 1/(d-2)$. (As we shall see below this corresponds to energies much higher than the mobility edge.) Otherwise the condition (3.4) can be satisfied only for not-too-long samples. In particular, at two dimensions the condition (3.4) reduces to $\ln(L/l) \ll kl$, and hence for large enough L the perturbative calculation breaks down, however high is the energy. In terms of the parameter α Eq. (3.1) can be rewritten as

$$g(L;E) = (\hbar/e^2)\sigma_0(E)L^{d-2}[1-\alpha(L;E)] \ . \tag{3.5}$$

The β function calculated from Eq. (3.1) is

$$\beta(g) = d-2-A/g \ . \tag{3.6}$$

Since A is a constant, depending only on dimensionality, the perturbation calculation supports the existence of a universal scaling function $\beta(g)$.

We now discuss quantum corrections to the Hall conductance $g_H(L)$. The contribution of the maximally crossed diagrams to the Hall conductivity in two dimensions has been calculated by Fukuyama[4] and, using a somewhat different technique, by Altshuler *et al.*[6] Again the restriction to $d=2$ in Refs. 4 and 6 is not essential and in fact the value of d is introduced only at the final state of the calculation. Employing the technique of Ref. 6 we find

$$g_H(L;E)=\omega_c\tau\left\{\frac{\hbar}{e^2}\sigma_0L^{d-2}-\frac{2A}{d-2}\left[\left(\frac{L}{l}\right)^{d-2}-1\right]\right\}$$

$$=\omega_c\tau(\hbar/e^2)\sigma_0L^{d-2}[1-2\alpha(L;E)]\ , \quad (3.7)$$

where $\omega_c=eB/mc$ is the cyclotron frequency (m is the electron effective mass). Thus the quantum correction to the Hall conductance, relative to the leading classical term, is twice as big as the correction to the Ohmic conductance [Eq. (3.5)]. This result which for $d=2$ has been derived in Refs. 4 and 6, holds for any dimensionality.

Differentiating Eq. (3.7) and using Eq. (3.1) for g, we obtain

$$\frac{d\ln g_H}{d\ln L}=d-2-\left[\frac{2A}{g}\right]\frac{1-\alpha}{1-2\alpha}\ . \quad (3.8)$$

With the approximation involved in calculation of g and g_H [Eqs. (3.5) and (3.7)] it would be inconsistent to keep the small α terms in Eq. (3.8). Our approximation enables us to derive only the leading quantum correction in the γ function:

$$\gamma(g)=d-2-2A/g=-(d-2)+2\beta(g)\ . \quad (3.9)$$

In order to check if there are indeed no nonuniversal terms of order $\alpha A/g$ in the γ function one needs to calculate both g and g_H to the accuracy α^2. The existence of the universal β function implies that there is no α^2 term (nor higher-order terms in α) in Eq. (3.5). For $d=2$ the absence of the α^2 term has been proven in Ref. 16. On the contrary, if a universal γ function exists, one must expect a term α^2 (and no higher-order terms in α) in the square brackets in Eq. (3.7). This would ensure that Eq. (3.9) holds also to higher order in α.

Integrating Eq. (3.9) we have

$$g_H(L)=g_{H0}(L/L_0)^{-(d-2)}[g(L)/g_0]^2\ , \quad (3.10)$$

where g_0 and g_{H0} are the initial values of $g(L)$ and $g_H(L)$ at some (microscopic) scale L_0. Equation (3.10) implies that the leading quantum correction does not spoil the classical, i.e., as L^{d-2}, scaling of the parameter $h(L)$ [Eqs. (2.7) and (2.8)]. This is in agreement with the assumption we made in Sec. II.

On this basis, we conclude that the Hall conductivity exponent has the value $t_H=2t=2(d-2)\nu$.

IV. ϵ EXPANSION NEAR TWO DIMENSIONS

In this section we shall be interested in the critical behavior of $\sigma(E)$ and $\sigma_H(E)$ near the mobility edge E_c. The only physical dimension of interest in this problem is $d=3$, since at $d=1,2$ all the states are localized and there is no mobility edge at all.[1] Unfortunately at $d=3$ the calculations of the preceding section are not valid near E_c, because the parameter kl there is of order unity, while the weak scattering regime requires $kl\gg1$. However, at $d=2+\epsilon$ with $\epsilon\ll1$, one can establish a connection between the weak scattering regime and the critical regime. This is possible because $E_c\to\infty$ when $\epsilon\to0$, and thus for small ϵ the parameter kl remains large even at the mobility edge. In terms of the scaling parameter g this means that for small ϵ, g_c is large, and therefore the necessary weak scattering condition $g\gg1$ is fulfilled at the mobility edge (while for $d=3$, $g_c\simeq1$).

The calculation is straightforward. The β function for small ϵ is given by

$$\beta(g)\equiv\frac{d\ln g}{d\ln L}=\epsilon-\frac{1}{\pi^2g}\ . \quad (4.1)$$

Small terms of the order ϵ/g are omitted in Eq. (4.1). The zero of the β function is $g_c=1/\pi^2\epsilon\gg1$. Integrating Eq. (4.1), with $g(L_0)=g_0$ as an initial condition, we obtain

$$g(L)=g_c[1+\Delta_0(L/L_0)^\epsilon]\ , \quad (4.2)$$

where $\Delta_0=(g_0-g_c)/g_c$. In fact, this equation, with a properly chosen g_0, is the same as Eq. (3.1). The difference is that Eq. (4.2) is valid in a much larger region than the initial Eq. (3.1). Namely, in Eq. (4.2) there is no restriction on L due to the condition $\alpha\ll1$ [Eq. (3.4)]. This is of course because we are relying on the universal character of the β function. Thus the only condition for validity of Eq. (4.2) is $g\gg1$. This means that for $\Delta_0\geq0$ (i.e., above or at the mobility edge) Eq. (4.2) is valid for any L. On the contrary, for $\Delta_0<0$, i.e., below the mobility edge, Eq. (4.2) is valid for not too large a sample.

The exponents t and ν are immediately obtained from Eq. (4.2). For $\Delta_0>0$ and in the limit $L\to\infty$, $g(L)$ is proportional to Δ_0L^ϵ. Since $\Delta_0\sim E-E_c$, this means that the conductivity exponent $t=1$. It follows then from the scaling relation (1.5) that the localization length exponent $\nu=1/\epsilon$. The values for these exponents have been also obtained by an approach based on a Lagrangean formulation of the localization problem (see Refs. 11–13 and references therein).

The Hall conductivity exponent t_H is obtained in a similar way. Equation (3.9) for the γ function and hence Eq. (3.10) for g_H are valid if $g >> 1$. Thus, for $\epsilon << 1$, Eq. (3.10) is valid for a sample of any size L all the way down to the mobility edge. Substituting expression (4.2) for $g(L)$ into Eq. (3.10) we obtain

$$g_H(L) = (g_{H0}/g_0^2)(L/L_0)^{-\epsilon} g_c^2 [1 + \Delta_0 (L/L_0)^{\epsilon}]^2 \ . \tag{4.3}$$

For fixed L_0, the initial parameters g_0 and g_{H0} depend on energy, i.e., on Δ_0. Since we are interested in the region near the mobility edge ($\Delta_0 << 1$), we can take in Eq. (4.3) the values of these parameters at the mobility edge. It follows from Eq. (4.3) that in the limit $L \to \infty$, $g_H(L)$ is proportional to $\Delta_0^2 L^{\epsilon}$, and thus the exponent $t_H = 2$. This result confirms the scaling relation (2.9) between the exponents.

In addition to the difference in the critical behavior of $\sigma(E)$ and $\sigma_H(E)$ (in an infinite sample), there are some essential differences in the behavior of $g(L;E)$ and $g_H(L;E)$ as functions of sample size L, for fixed Fermi energy E: (i) At $E = E_c$ the conductance $g(L) = g_c$ and it is independent of L. In contrast, the Hall conductance $g_H(L)$ at the mobility edge does depend on L as $L^{-\epsilon}$. (ii) If the energy is fixed slightly above E_c (i.e., $\Delta_0 << 1$), the conductance $g(L)$ is a monotonically increasing function of L. However, the Hall conductance $g_H(L)$ first decreases with L and reaches a minimum at a scale equal to the correlation length $\xi = L_0 \Delta^{-1/\epsilon}$. Only for $L > \xi$ does $g_H(L)$ increase monotonically with L.

These differences between g (or σ) and g_H (or σ_H) arise from the quantitative difference between the scaling functions $\beta(g)$ and $\gamma(g)$, and in particular, from the fact that these functions have zeros at different values of g. For small ϵ, the zeros of β and γ functions are at $1/\pi^2\epsilon$ and $2/\pi^2\epsilon$, respectively. Note that it is only the zero of the β function which is associated with a critical point [the fixed point g_c of the recursion formula (1.2)]. The zero of γ function has no such meaning, since the scaling behavior of g_H is driven by g.

Finally, at finite T and near the mobility edge the relevant length scale for the conductivity is set by the temperature-dependent Thouless length[7,8] L_T rather than by the correlation length ξ (see Ref. 9). Then the conductivity $\sigma(T)$ and the Hall conductivity $\sigma_H(T)$ are expected to be proportional to $L_T^{-\epsilon}$ and $L_T^{-2\epsilon}$, respectively [compare to $\sigma(E) \sim \xi^{-\epsilon}$ and $\sigma_H(E) \sim \xi^{-2\epsilon}$ at $T = 0$]. This statement is not restricted to small ϵ. Thus in three dimensions, at low temperatures, one expects near and at the mobility edge $\sigma(T) \sim L_T^{-1}$ (Ref. 9) while $\sigma_H(T) \sim L_T^{-2}$.

V. CONCLUSIONS

We have developed a scaling hypothesis for the Hall effect in disordered systems. It is suggested that a universal scaling function for the Hall conductance exists, and the leading quantum correction ($\sim 1/g$) to the classical value $(d-2)$ of this function is calculated.

It follows from our theory that the critical exponent t_H for the Hall conductivity is twice the conductivity exponent t. Since the Hall coefficient r is proportional to σ_H/σ^2, we conclude that $r(E)$ approaches a constant value when the mobility edge is approached from above. On the other hand, the Hall mobility $\mu_H = cr\sigma$ approaches zero as $(E - E_c)^t$. The behavior of the Hall coefficient is in agreement with the accepted view[17,18] that in a degenerate electron gas the "classical" expression $1/enc$ for the Hall coefficient (n is the electron concentration) remain approximately valid even in the regime $kl \sim 1$. The agreement with the picture of Refs. 17 and 18 of course fails when it comes to the Hall mobility near E_c, since the minimum metallic conductivity assumed in Refs. 17 and 18 does not occur in the scaling theory.

It is important to note that we have ignored completely the effect of electron-electron interactions. It was shown in Ref. 6 that in the weak scattering regime interaction effects introduce essential changes into the Hall conductivity as compared to the case of noninteracting electrons. Interaction effects, almost certainly, also remain relevant near the mobility edge, although the extent to which the results of this paper will be changed is not clear at present.

In this paper we have considered the Hall coefficient above the mobility edge. Below the mobility edge, i.e., in the insulating phase both $\sigma(E)$ and $\sigma_H(E)$ are zero. However, the Hall coefficient $r(E)$ of the material is still a meaningful quantity defined as

$$r(E) = \lim_{L \to \infty} L^{d-2} U_H(L;E)/BI(L;E) \ , \tag{5.1}$$

where U_H is the Hall voltage and I is the current. For large sample size (i.e., $L >> \xi$) both U_H and I are exponentially small, and it is not clear whether $r(E)$, for $E < E_c$, is finite, infinite, or zero. At finite temperature, when the length scale is set by the Thouless length L_T, the Hall coefficient, as well as the conductivity[9] or the Hall conductivity, will be a continuous function of E at E_c.

Finally, we believe that the phenomenological scaling results of this paper, such as $t_H = 2t$, are valid for any classical magnetic field. On the other hand, the numerical values $t = 1$, $t_H = 2$ of the exponents refer, strictly speaking, to an infinitesimal field.

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