Transition in magnetoresistance behavior from classical to quantum regime

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A theory developed earlier by Arora and Peterson is analyzed both in the classical as well as in the quantum limit to explain the observed changes in the power law in going from the classical to the quantum regime. An expression for the critical value of the magnetic field at which this transition takes place is obtained as a function of temperature. The results so obtained are consistent with actual experimental observations.

Electrical transport in a magnetic field has been widely investigated, both theoretically and experimentally, as such studies provide useful information about the scattering mechanisms and band structure in solids. For low magnetic fields, the semiclassical Boltzmann transport equation is adequate for the description of electronic transport, where the magnetic field is treated as a perturbation under the condition $\hbar\omega_c \lesssim k_B T$ (classical limit), and $\omega_c = eB/m^*c$ is the cyclotron frequency of the electron of effective mass m^* in a magnetic field B. These galvanomagnetic effects in the classical limit have been extensively reviewed by Beer.[1]

In strong magnetic fields, when $\hbar\omega_c \gtrsim k_B T$ (quantum limit), the quantization of the energy of an electron needs to be included in a theoretical framework. Although the theories[2,3] for the longitudinal case which take quantization into account have been reasonably successful, quantum theories for strong-field transverse magnetoresistance continue to remain a puzzle[4,5] in spite of the fact that several theorists have worked on the problem.

Recent experiments[6-8] indicate a change in the power law of the magnetoresistance as the magnetic field is increased from the classical to the quantum regime. A quadratic increase of magnetoresistance with magnetic field is observed at low magnetic fields, while a linear increase is observed at higher magnetic fields. This change in power law has been interpreted[6,9] as arising due to Wigner condensation or to the charge-density-wave transition of the system of conduction electrons. The first observation of the change in power law of the transverse magnetoresistance in n-InSb was reported in the early careful experiments of Bate, Willardson, and Beer.[10] The results obtained have been successfully interpreted in the classical limit[10] by including the mixed scattering of electrons via acoustic phonons and ionized impurities. At high magnetic fields, skepticism was indicated regarding the approach to the quantum limit and a correct theory for the mixed scattering in the quantum limit. Pearson and Suhl[11] also found a quadratic behavior at low magnetic fields consistent with the classical theory of magnetotransport.[1] These studies are supplemented by the high-field experiments of Gallagher and Love,[12] who observed an approximately linear increase in the transverse magnetoresistance with the magnetic field. This linear rise could not be interpreted[13] by a quantum theory developed by Argyres,[14] which takes into account the interference effect of the electric field during collisions.

A quantum theory of magnetotransport valid for arbitrary values of the magnetic field, within the effective-mass approximation, was developed by Arora and Peterson[15] for isotropic scattering and was later extended by us[16] to include anisotropic impurity scattering. It was shown[16] that at high magnetic fields, the acoutic-phonon scattering predominates and essentially controls the magnetoresistance behavior. In the quantum limit this theory gives, for the transverse magnetoresistance, an expression[17]

$$\frac{\Delta\rho_{xx}^{Q}}{\rho_0} = \frac{2}{3\pi} a \frac{b_a}{b_0} \left[\ln\left[\frac{3\pi^{1/2}}{2b_a} \right]^2 - 0.577 \right], \quad (1)$$

with

$$a = \hbar\omega_c / k_B T , \quad (2)$$

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$$b_0 = \hbar\tau_0^{-1}/k_B T \,, \tag{3}$$

$$b_a = \hbar\tau_a^{-1}/k_B T \,, \tag{4}$$

$$\rho_0 = m^*/n_e e^2 \tau_0 \,, \tag{5}$$

where τ_0 is the zero-field relaxation time for the predominant scattering mechanism, τ_a is the zero-field relaxation time for elastic acoustic-phonon scattering, and n_e is the electronic concentration per unit volume. It may be noted that Eq. (1) differs from the corresponding Eq. (20) of Ref. 17 by a factor $b_a/b_0 = \tau_0/\tau_a$. This is due to the fact that at high magnetic fields, the acoustic-phonon scattering is the dominant mechanism of scattering,[16] whereas at low magnetic fields including the value zero, mixed scattering may be present. Thus $\Delta\rho_{xx} \propto \tau_a^{-1}$, whereas $\rho_0 \propto \tau_0^{-1}$. In a recent study[18] of cyclotron resonance at low temperature, we have specifically indicated that, in a sample of n-InSb, impurity scattering may be predominant at low magnetic fields even in the quantum limit, but may cease to be operative at sufficiently high magnetic fields. $\Delta\rho_{xx}/\rho_0 \approx \rho_{xx}/\rho_0$ in the quantum limit as $\Delta\rho_{xx} >> \rho_0$.

In the classical limit, our theoretical results for the transverse conductivity components reduce to those obtained from the Boltzmann transport equation[19]:

$$\sigma_1^C = \frac{n_e e^2}{m^*} \frac{4}{3\sqrt{\pi}} \int_0^\infty \frac{\tau(x)}{1+\omega_c^2\tau^2(x)} e^{-x} x^{3/2} dx \,, \tag{6}$$

$$\sigma_2^C = \frac{n_e e^2}{m^*} \frac{4}{3\sqrt{\pi}} \int_0^\infty \frac{\omega_c\tau^2(x)}{1+\omega_c^2\tau^2(x)} e^{-x} x^{3/2} dx \,, \tag{7}$$

with

$$x = \epsilon/k_B T \,. \tag{8}$$

The above expressions, for mixed scattering via acoustic phonons and ionized impurities, have been analyzed in terms of integrals, which are tabulated by Beer, Armstrong, and Greenberg.[20] The transverse magnetoresistane, $\Delta\rho_{xx}^C/\rho_0$, so obtained is proportional to B^2 at low fields ($\omega_c\tau << 1$) and becomes field independent at high fields (classical saturation magnetoresistance), but is dependent on the mixing parameter $\beta = 6\tau_a/\tau_i$ at all magnetic fields. The results so obtained can be represented by an equation of the form

$$\frac{\Delta\rho_{xx}^C}{\rho_0} = A_0(\beta) a^2 b_0^{-2}, \quad \Delta\rho_{xx}^C << \rho_0 \,. \tag{9}$$

$A_0 = 0.38$ for pure acoustic-phonon scattering [$\tau(x)\alpha x^{-1/2}$], and $A_0 = 2.15$ for pure ionized-impurity scattering [$\tau(x)\alpha x^{+3/2}$].

If plotted on a log-log scale, Eqs. (1) and (9) will transform to

$$\ln\frac{\Delta\rho_{xx}^C}{\rho_0} = \ln\left\{\frac{2}{3\pi}\frac{b_a}{b_0}\left[\ln\left[\frac{3\pi^{1/2}}{2b_a}\right]^2 - 0.577\right]\right\} + \ln a \,, \tag{10}$$

$$\ln\frac{\Delta\rho_{xx}^C}{\rho_0} = \ln(A_0 b_0^{-2}) + 2\ln a \,. \tag{11}$$

A straight line obtained from Eq. (10) when plotted as $\ln \Delta\rho_{xx}^Q/\rho_0$ vs $\ln a$ gives a slope of unity, while that obtained from Eq. (11) has a slope of 2. The intersection of these two straight lines gives the critical value a_c of the magnetic field parameter a:

$$a_c = \frac{2}{3\pi A_0(\beta)} b_0 b_a \left[\ln\left[\frac{3\pi^{1/2}}{2b_a}\right]^2 - 0.577\right] = S_Q/S_C \,, \tag{12}$$

where

$$S_Q \equiv \frac{\Delta\rho_{xx}^Q/\rho_0}{a} = \frac{2}{3\pi}\frac{b_a}{b_0}\left[\ln\left[\frac{3\pi^{1/2}}{2b_a}\right]^2 - 0.577\right] \,, \tag{13}$$

$$S_C \equiv \frac{\Delta\rho_{xx}^C/\rho_0}{a^2} = A_0(\beta) b_0^{-2} \,. \tag{14}$$

a_c describes the transition in power law from the classical to the quantum regime and is a function of temperature and scattering parameters. When acoustic-phonon scattering is the predominant scattering mechanism at low fields ($b_0 = b_a$), $a_c \sim T$, or the critical magnetic field $B_c \sim T^2$. When ionized impurity scattering is the predominant mechanism of scattering ($b_0 = b_i$), $a_c \sim T^{-2}$, or $B_c \sim T^{-1}$. We thus see that the critical value of the magnetic field, at which the classical to quantum transition takes place, may increase or decrease with temperature depending upon which scattering mechanism is dominant at low fields.

To make a comparison with the experimental data, we need the value of $b_a/b_0 = \tau_0/\tau_a$ for the

scattering parameter β, which is estimated to be unity.[10] It may be noted that the τ's given in the above expressions are energy averages[19] of energy-dependent $\tau(\epsilon)$, where ϵ is the carrier energy. For mixed scattering by acoustic phonons and ionized impurities,[19] the energy-dependent inverse relaxation time $\tau_0^{-1}(\epsilon)$ is the sum of separate inverse relaxation times:

$$\tau_0^{-1}(\epsilon) = \tau_a^{-1}(\epsilon) + \tau_i^{-1}(\epsilon) = \tau_a^{-1}(\epsilon)(\epsilon/k_BT)^{-2}[\beta + (\epsilon/k_BT)^2] . \quad (15)$$

Equation (15) is to be contrasted to Eq. (16) below, which describes the summation of averaged relaxation rates:

$$\tau_0^{-1} = \tau_a^{-1} + \tau_i^{-1} , \quad (16)$$

where τ_a and τ_i are given by[18]

$$\tau_a^{-1} = 3(2m^*k_BT)^{3/2}E_1^2/8\pi^{1/2}\rho U^2\hbar^4 , \quad (17)$$

$$\tau_i^{-1} = [\pi^{3/2}Z^2e^4n_i/2^{7/2}\chi^2(k_BT)^{3/2}]C(T) , \quad (18)$$

$$C(T) = \ln[1 + (7\chi k_BT/2e^2n_i^{1/3})^2] . \quad (19)$$

Here, E_1 is the deformation potential constant, ρ the crystal density, U the sound velocity, Ze the impurity charge, n_i the impurity concentration, and χ the dielectric constant. If we use Eq. (16) to find $b_a/b_0 = \tau_0/\tau_a = (1 + \beta/6)^{-1}$, we get $\tau_0/\tau_a = 0.86$ for $\beta = 1$. If Eq. (15) is used for the mixed scattering, $\tau_0/\tau_a = \mu_0/\mu_a$ is correctly given by [Eq. (6f.10) in Ref. 19]:

$$\tau_0/\tau_a = \mu_0/\mu_a = [1 - \beta g(\sqrt{\beta})] , \quad (20)$$

where

$$g(x) = -\mathrm{Ci}(x)\cos x - \mathrm{si}(x)\sin x , \quad (21)$$

$$-\mathrm{si}(x) = \int_x^\infty \frac{\sin t}{t}dt, \quad -\mathrm{Ci}(x) = \int_x^\infty \frac{\cos t}{t}dt . \quad (22)$$

For $\beta = 1$, $g(\sqrt{\beta}) = 0.34$ from tabulated values[21] of Ci(x) and si(x). Equation (20) then gives $\tau_0/\tau_a = 0.66$, which differs from the value $\tau_0/\tau_a = 0.86$ obtained from Eq. (16). Obviously, $\tau_0/\tau_a = 0.66$, obtained from the rigorous calculation, should be used below for correct interpretation of the experimental results.

For n-InSb at 77 K, the value of the critical magnetic field $B_c = 283$ G $= 0.0283$ T is observed by Bate *et al.*[10] in a sample for which $\beta = 1$. The value $S_C = 64$, obtained from the theoretical analysis, is found to be in agreement with the low-field data.[10] The value of S_Q calculated from Eq. (13) is equal to 2.4 for an acoustic-phonon deformation-potential constant $E_1 = 7.2$ eV, a value quoted by Rode[22] (see Ref. 17). This agrees well with the experimental value,[10] indicating the dominance of acoustic-phonon scattering at high magnetic fields. From these values, $a_c = S_Q/S_C = 0.04$, which agrees very well with its value of $a_c = 0.038$ at $B_c = 283$ G, the experimental observed value of the critical field.[10]

Regarding the experimental results of Nimtz and Schlicht,[6] the above model can not be tested in detail as the zero-field mobility results are not given in their work, but some of the experimental features can be explained. First, it should be noted that the system behaves, for the most part, classically if $\frac{1}{2}\hbar\omega_c << \zeta$, where ζ is the Fermi energy. Under this condition, an electron gas is essentially degenerate. A quasiparticle effect mentioned by Lodder and Fujita,[23] which gives the collision broadening for damped oscillations, should be included. This can be done by extending the formalism[15] to the so-called generalized Born approximation (GBA), which is the next approximation in Mori's hierarchy of approximations.[24] Apparently, this regime has not been studied by Nimtz and Schlicht.[6] Now, if the magnetic field is increased, a transition from degenerate to nondegenerate behavior is expected[25] at $\frac{1}{2}\hbar\omega_c \sim \zeta$. This prediction of a degenerate-nondegenerate transition is supported by other workers[9,26] and experimental observations appear in the published literature.[27] In the regime $\frac{1}{2}\hbar\omega_c - \zeta >> k_BT$, the quantum limit is expected to have been reached for the results of our theory to be valid. Obviously, the quantum limit will arrive at much higher fields. At low temperatures and low magnetic fields, normally the impurity scattering is the dominant mechanism of scattering and this dominance prevails sometimes even in the quantum limit.[18] If this is assumed to be the predominant mechanism of scattering, the critical value of the magnetic field will decrease with temperature and may fall in the oscillatory regime where it may be difficult to observe experimentally. For example, for $Hg_{0.784}Cd_{0.216}Te$, at $T = 1.43$ K, $B_c = 36$ kG is observed. When the temperature is increased to $T = 4.2$ K, $B_c = 12.3$ kG, which falls below the range (20–130 kG) investigated by Nimtz and Schlicht. A careful investigation is, therefore, deemed necessary to investigate the classical to

quantum transition.

Another possibility when a sample has a large number of point defects, such as in alloys,[26] should not be overlooked. The energy dependence of the relaxation time due to point defects is essentially the same as that for the acoustic-phonon scattering,[16] but the temperature dependence is not the same. If point defect scattering is predominant over the acoustic-phonon scattering, the high-field magnetoresistance will be controlled by the relaxation process appropriate to this scattering mechanism. The τ_a in Eq. (12) should be replaced by τ_{PD}, the relaxation time for point defect scattering.[16] In such cases, if $b_0 \sim b_{PD} \sim T^{-1/2}$, $a_c \sim T^{-1}$ or $B_c \sim T^0$, and if $b_0 \sim b_i$, $a_c \sim T^{-3}$ or $B_c \sim T^{-2}$.

To conclude, we have explicitly shown that the theory developed by Arora and Peterson extrapolates very well the classical as well as the quantum behavior of the observed magnetoresistance in semiconducting samples. In the light of our arguments presented above, the transition in the power law of magnetoresistance exists due to the transition from classical to quantum regimes as the magnetic field is increased.

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