# Soliton damping and topological order in quasi-one-dimensional systems

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The effects of the soliton damping due to the scattering from impurities or from other thermal solitons on the dynamical correlation functions are considered within a phenomenological model. It is shown that the spin-correlation functions of quasi-one-dimensional magnetic systems like $(CH_3)_4NMnCl_3$ (TMMC) and $CsCoCl_3$ depend strongly on the soliton damping.

## I. INTRODUCTION

As is well known, the one-dimensional system with a finite-range interaction never undergoes a phase transition at finite temperatures. On the other hand a number of systems exhibit a crossover behavior from the high-temperature disordered phase to the low-temperature quasi-ordered phase. Even at low temperatures the topological order of the system is partially broken due to the presence of thermally excited solitons or kinks.

In a seminal paper Krumhansl and Schrieffer[1] have studied the $\phi^4$ model and identified that the central peak observed in the quasi-one-dimensional system with displacive phase transition is due to kinks. It is known that the correlation functions in a number of systems can be written in a similar form as the $\phi$-$\phi$ correlation function in the $\phi^4$ model, if the soliton behaves like ideal gas. These examples include the spin correlation of the planar antiferromagnet like TMMC $[(CH_3)_4NMnCl_3]$ in a magnetic field,[2–4] the Ising-like antiferromagnetic chain[5] like $CsCoCl_3$, and the dimerization field correlation function in polyacetylene.[6,7] In all of these examples the correlation length and the correlation time are inversely proportional to the soliton density of the system.

In all of the above analyses it was assumed that the soliton behaves like a free particle. However in the real experimental situations the soliton may have a finite lifetime due to scattering with impurities or with other solitons. The object of the present paper is to study the effect of the soliton damping on the corresponding dynamical correlation functions. We do not imply here by the soliton damping that the soliton will decay into other excitations but that the soliton changes its velocity from time to time due to scattering.[8] Such effects may be studied experimentally by introducing impurities in the quasi-one-dimensional magnetic systems described already.[9] However, the impurities should not disturb directly the topological order of the system but only introduce the localized potential which scatters the soliton in the system. We shall treat the effect of the soliton damping phenomenologically: we introduce a characteristic frequency $\tau^{-1}$, with which the soliton velocity will change. In the limit of the large damping our phenomenological model reproduces recent results[10,11] for $\langle[\phi(x,t)-\phi(0,0)]^2\rangle$ in the sine-Gordon (sG) system.

Although the effects of the soliton damping are not apparent in the static correlation functions, the soliton damping modifies strongly the dynamic correlation of the systems. In general the central peak associated with the soliton becomes sharper due to the soliton damping, which should be readily accessible by the neutron scattering experiments in the quasi-one-dimensional magnetic systems.

## II. CORRELATION FUNCTIONS IN THE IDEAL-GAS LIMIT

Here we shall summarize properties of the correlation functions in the limit that the soliton behaves like ideal gas. For model Hamiltonians given by[12,13]

$$H_{\phi^4}=\frac{1}{2}\int dz\left[\pi(z)^2+\left(\frac{\partial\phi}{\partial z}\right)^2-\frac{1}{2}(m^*)^2\phi^2+\frac{1}{4}\lambda\phi^4\right] \quad (1)$$

and

$$H_{sG}=\frac{1}{2}\int dz\left[\pi(z)^2+\left(\frac{\partial\phi}{\partial z}\right)^2-\frac{2m^{*2}}{g^2}\cos(g\phi)\right] , \quad (2)$$

where $\pi(z)=\partial\phi/\partial t$, $m^*$ is the bare phonon (or magnon) mass and $C$ the phonon velocity is taken to be unity, the correlation function $\langle\phi(z,t)\phi(0,0)\rangle$ in the $\phi^4$ system and $\langle\cos[\frac{1}{2}g\phi(z,t)]\cos[\frac{1}{2}g\phi(0,0)]\rangle$ in the sine-Gordon system are given by[1,3,4]

$$\langle\phi(z,t)\phi(0,0)\rangle\simeq\phi_0^2\exp\left(-2\int\frac{dp}{2\pi}n_s(v)|z-vt|\right) \quad (3)$$

 

and

$$\langle \cos[\tfrac{1}{2}g\phi(z,t)]\cos[\tfrac{1}{2}g\phi(0,0)]\rangle \simeq \frac{m(T)}{m^*}\exp\left[-4\int \frac{dp}{2\pi}n_s(v)|z-vt|\right] , \tag{4}$$

where $n_s(v)$ is the density of the soliton with velocity $v$ and $p$ is the momentum of the soliton.

The soliton density $n_s(v)$ is given as[12]

$$n_s(v)=\begin{cases}\exp(-\beta E_s\gamma), & \text{for } T<m\\ \frac{\beta m}{\sqrt{3}}(1+\gamma)(2+\gamma^{-1})\exp(-\beta E_s\gamma), & \text{for } T>>m,\end{cases} \tag{5}$$

and

$$n_s(v)=\begin{cases}\exp(-\beta E_s\gamma), & \text{for } T<m\\ \beta m(1+\gamma)\exp(-\beta E_s\gamma), & \text{for } T>>m\end{cases} \tag{6}$$

for the $\phi^4$ system and the sG system, respectively, where $E_s$ is the (temperature-independent) soliton energy, $m$ is the (temperature-dependent) renormalized mass of phonon,[12] $\beta=T^{-1}$ and $\gamma=(1-v^2)^{-1/2}$. Hereafter we take $k_B=\hbar=1$ for simplicity.

In Eqs. (3) and (4) we have neglected the pure phonon terms which are much smaller than the soliton term at least for $T\lesssim\frac{1}{3}E_s$.

Both Eqs. (3) and (4) are well approximated by[4,13]

$$\langle\phi(z,t)\phi(0,0)\rangle \simeq \phi_0^2\exp[-2\bar{n}_s(z^2+v_0^2t^2)^{1/2}] \tag{7}$$

and

$$\langle\cos[\tfrac{1}{2}g\phi(z,t)]\cos[\tfrac{1}{2}g\phi(0,0)]\rangle \simeq \frac{m}{m^*}\exp[-4\bar{n}_s(z^2+v_0^2t^2)^{1/2}] , \tag{8}$$

where $\bar{n}_s$ is the total soliton density

$$\bar{n}_s=\int\frac{dp}{2\pi}n_s(v) \tag{9}$$

and $v_0$ is the thermal soliton velocity

$$v_0=\left[\frac{1}{\pi}\int_0^\infty dp\, v\, n_s(v)\right]\bar{n}_s^{-1}\simeq\left[\frac{2}{\pi\beta E_s}\right]^{1/2} . \tag{10}$$

The Fourier transforms of Eqs. (7) and (8) are then given by

$$\langle[\phi,\phi]\rangle(\omega,q)=\phi_0^2\frac{4\pi\bar{n}_s}{v_0}\left[q^2+\left(\frac{\omega}{v_0}\right)^2+(2\bar{n}_s)^2\right]^{-3/2} \tag{11}$$

and

$$\langle[\cos(\tfrac{1}{2}g\phi),\cos(\tfrac{1}{2}g\phi)]\rangle(\omega,q) =\frac{m}{m^*}\frac{8\pi\bar{n}_s}{v_0}\left[q^2+\left(\frac{\omega}{v_0}\right)^2+(4\bar{n}_s)^2\right]^{-3/2} , \tag{12}$$

which are directly accessible to the neutron scattering and/or the x-ray scattering experiment.

In the low-temperature region ($T\lesssim 5$ K), the planar spin antiferromagnetic systems like TMMC in a magnetic field is mapped to the sine-Gordon system.[3,4] Therefore the transverse spin-correlation function in the planar spin antiferromagnet in a magnetic field is described by Eqs. (8) and (12). In the case of TMMC, both $q$ and $\omega$ dependence of the spin-correlation functions have been studied recently by neutron scattering experiments.[14–16] It appears that Eq. (12) describes both $q$ and $\omega$ dependences of the spin-correlation function quite well.[16]

A quite similar expression applies also for the longitudinal spin-correlation function of the Ising-like antiferromagnetic chain described by the Hamiltonian,[5,17]

$$H=2J\sum_n[\sigma_n^z\sigma_{n+1}^z+\epsilon(\sigma_n^x\sigma_{n+1}^x+\sigma_n^y\sigma_{n+1}^y)] , \tag{13}$$

where $\vec{\sigma}_n$ is the Pauli spin operator attached at the site $n$. In this system the soliton is a domain wall between two distinct antiferromagnetic ground states.[17] For small $\epsilon$ the soliton energy is given by[17]

$$E_s(p)=J[1+2\epsilon\cos(2ap)] , \tag{14}$$

where $p$ is the soliton momentum and $a$ is the lattice constant. The longitudinal spin-correlation function of this system is again given by[5]

$$\langle\sigma_n^z(t)\sigma_{n+m}^z(0)\rangle \simeq\exp\left[i\frac{\pi}{a}x-\int_{-\pi/a}^{\pi/a}\frac{dp}{2\pi}e^{-\beta E_s(p)}|x-vt|\right] , \tag{15}$$

where $x=ma$, $E_s(p)$ is given by Eq. (14), and

$$v=\frac{\partial}{\partial p}E_s(p)=-4\epsilon aJ\sin(2ap) \tag{16}$$

is the soliton velocity. Equation (15) is approximated again by

$$\langle\sigma^z(x,t)\sigma^z(0,0)\rangle \simeq\exp\left[i\frac{\pi}{a}x-2\bar{n}_s(x^2+v_0^2t^2)^{1/2}\right] \tag{17}$$

with

$$\bar{n}_s = \frac{1}{2\pi} \int_{-\pi/a}^{\pi/a} dp\, e^{-\beta E_s(p)} = (2a)^{-1} e^{-\beta J} I_0(2\beta\epsilon J) \quad (18)$$

and

$$v_0 = \frac{4a}{\pi\beta} I_0^{-1}(2\beta\epsilon J)\sinh(2\beta\epsilon J) \ , \quad (19)$$

where $I_0(z)$ is the modified Bessel function. The Fourier transform of Eq. (17) is again

$$S_{zz}(\omega,q) = \frac{4\pi\bar{n}_s}{v_0}\left[\left(q-\frac{\pi}{a}\right)^2 + \left(\frac{\omega}{v_0}\right)^2 + (2\bar{n}_s)^2\right]^{-3/2} \ , \quad (20)$$

which should be valid for $|q-\pi/a| << \pi/a$.

Finally the correlation function for the displacement fields in the Su, Schrieffer, Heeger (SSH) model[6] for the trans-polyacetylene is given by[7]

$$\langle u(z,t)u(0,0)\rangle = u_0^2 \exp\left[i\frac{\pi}{a}z - 2\bar{n}_s(z^2+v_0^2t^2)^{1/2}\right] \ , \quad (21)$$

where the $\bar{n}_s$ is the total soliton density and $v_0$ is the average soliton velocity. However, in this last example the soliton energy $E_s(\sim 0.5 \text{ eV})$ is so large compared with the room temperature that, except in the extremely high-temperature region, the solitons are not thermally excited but rather introduced by doping for example. In this case $v_0$ is then no longer the thermal velocity of the soliton but rather like the Fermi velocity of the soliton gas.

## III. EFFECTS OF SOLITON DAMPING

We have described so far the dynamical correlation functions sensitive to the topological order in the limit when the soliton behaves like a free particle. In this limit the dynamical correlation function takes the simple general form given by Eqs. (11) or (12).

In actual physical systems the soliton can scatter each other or can be scattered by impurities or defects on the linear chain. We shall consider here these effects phenomenologically. For this purpose we shall consider first the displacement of the $\phi$ field in the sine-Gordon system [Eq. (2)]. In the limit of the ideal gas approximation, we obtain[13]:

$$\langle[\phi(z,t)-\phi(0,0)]^2\rangle = 2\bar{n}_s\left(\frac{2\pi}{g}\right)^2 (z^2+v_0^2t^2)^{1/2} \ , \quad (22)$$

where $\bar{n}_s$ and $v_0$ have been defined in Eqs. (9) and (10). The squared displacement $\phi$ is proportional to the soliton number passing through the line connecting two points $(z,t)$ and $(0,0)$ in the space-time plane. This agrees with the result by Gunther and Imry[10] in the same limit. In the presence of the soliton damping (i.e., a soliton can be scattered by impurities or other solitons) the soliton velocity is no longer constant of motion. If we assume that the velocity correlation is lost after a time $\tau$, due to scattering, we obtain

$$\left\langle\left(\int v(t)\,dt\right)^2\right\rangle = v^2\int_0^t dt_1\int_0^t dt_2 e^{-|t_1-t_2|/\tau} = v^2F(t,\tau) = 2v^2\tau[|t|-\tau(1-e^{-|t|/\tau})] \ . \quad (23)$$

Then Eq. (23) implies in the presence of the soliton damping Eq. (22) is replaced by

$$\langle[\phi(z,t)-\phi(0,0)]^2\rangle = 2\bar{n}_s\left(\frac{2\pi}{g}\right)^2 [z^2+v_0^2F(t,\tau)]^{1/2} \ . \quad (24)$$

In the limit $|t| << \tau$ Eq. (24) reduces to Eq. (22). On the other hand in the limit $|t| >> \tau$, Eq. (25) simplifies as

$$\langle[\phi(z,t)-\phi(0,0)]^2\rangle = 2\bar{n}_s\left(\frac{2\pi}{g}\right)^2 (z^2+2D|t|)^{1/2} \ , \quad (25)$$

with $D=v_0^2\tau$ the soliton diffusion constant, which agrees again with Gunther and Imry[10] and Büttiker and Landauer[11] in that the soliton motion becomes diffusive in this limit. Therefore Eq. (23) interpolates two known limits nicely.

Corresponding expressions for $\langle\phi(z,t)\phi(0,0)\rangle$ and $\langle\cos[\frac{1}{2}g\phi(z,t)]\cos[\frac{1}{2}g\phi(0,0)]\rangle$ in the presence of the soliton damping are given by

$$\langle\phi(z,t)\phi(0,0)\rangle = \phi_0^2\exp\{-2\bar{n}_s[z^2+v_0^2F(t,\tau)]^{1/2}\} \quad (26)$$

and

$$\langle\cos[\tfrac{1}{2}g\phi(z,t)]\cos[\tfrac{1}{2}g\phi(0,0)]\rangle = \frac{m}{m^*}\exp\{-4\bar{n}_s[z^2+v_0^2F(t,\tau)]^{1/2}\} \ , \quad (27)$$

respectively. Equations (26) and (27) are our fundamental results.

As is easily seen the equal-time correlation function is unaffected by the soliton damping. In the following we shall study the Fourier transform of Eq. (26).

## IV. DYNAMICAL CORRELATION FUNCTIONS

In the presence of the soliton damping, the dynamical correlation functions are given as Fourier transform of Eq. (26) or Eq. (27). Since both functions have the same $z$ and $t$ dependence we shall limit our consideration to Eq. (26) in the following:

$$S(\omega,q)=\int\int_{-\infty}^{\infty} dtdz\, e^{i(qz-\omega t)} \times \exp\{-K[z^2+v_0^2F(t,\tau)]^{1/2}\}\ , \tag{28}$$

where $K=2\bar{n}_s$ (or $4\bar{n}_s$) and

$$F(t,\tau)=2\tau[|t|-\tau(1-e^{-|t|/\tau})]\ . \tag{29}$$

Since the general expression for $S(\omega,q)$ is rather complicated, we shall confine ourselves in two limiting cases; $Kl \gg 1$ and $Kl \ll 1$ where $l=v_0\tau$ the soliton mean free path.

### A. $Kl \gg 1$

In this limit we can expand Eq. (28) as

$$S(\omega,q)=S_0(\omega,q)+S_1(\omega,q)+\cdots\ , \tag{30}$$

where

$$S_0(\omega,q)=\int_{-\infty}^{\infty}\int dtdz\, e^{i(qz-\omega t)}\exp[-K(z^2+v_0^2t^2)^{1/2}]=\frac{2\pi K}{v_0}\left[K^2+q^2+\left(\frac{\omega}{v_0}\right)^2\right]^{-3/2} \tag{31}$$

and

$$\begin{aligned}S_1(\omega,q)&=\tfrac{1}{6}Kl^{-1}\int_{-\infty}^{\infty}\int dtdz\,(v_0|t|)^3(z^2+v_0^2t^2)^{-1/2}e^{i(qz-\omega t)}\exp[-K(z^2+v_0^2t^2)^{1/2}]\\&=2(v_0l)^{-1}K\int_{-\infty}^{\infty}d\theta\,\mathrm{Re}\left[\left((K^2+q^2)^{1/2}\cosh\theta-\frac{i\omega}{v_0}\right)^{-4}\right]\\&=\tfrac{8}{3}(v_0l)^{-1}K\left[K^2+q^2+\left(\frac{\omega}{v_0}\right)^2\right]^{-3}\left[K^2+q^2-\frac{11}{4}\left(\frac{\omega}{v_0}\right)^2-\frac{3\omega}{4v_0}\left[3(K^2+q^2)-2\left(\frac{\omega}{v_0}\right)^2\right]\left[K^2+q^2+\left(\frac{\omega}{v_0}\right)^2\right]^{-1/2}\right.\\&\qquad\left.\times\tanh^{-1}\left\{\frac{\omega}{v_0}\left[K^2+q^2+\left(\frac{\omega}{v_0}\right)^2\right]^{-1/2}\right\}\right]\ .\end{aligned} \tag{32}$$

Equation (32) gives the lowest-order correction to the ideal gas limit in $l^{-1}$. For small $\omega$, the correction term may be approximated by

$$S_1(\omega,q)\simeq\tfrac{8}{3}(v_0l)^{-1}K\left[K^2+q^2+\left(\frac{\omega}{v_0}\right)^2\right]^{-2}+O\left(\frac{\omega}{v_0}\right)^2 \tag{33}$$

within the same approximation $S(\omega,q)$ is given by

$$S(\omega,q)\simeq\frac{2\pi K}{v_0}\left\{\left[K^2+q^2+\left(\frac{\omega}{v_0}\right)^2\right]^{1/2}-\frac{4}{9\pi}l^{-1}\right\}^{-3}\ , \tag{34}$$

which tells that in the limit $\omega=0$ (i.e., the elastic scattering) the correlation length is increased as

$$\xi=\left[K^2-\left(\frac{4}{9\pi}\right)^2l^{-2}\right]^{-1/2} \tag{35}$$

due to the soliton damping. The fact that the correlation length becomes longer in the presence of soliton damping may appear somewhat surprising. However, since the soliton is the agent which breaks the topological order of the system, it is natural that the presence of the soliton damping in general encourages the topological order.

However, it should be emphasized again that the correlation length associated with the equal-time correlation function is not affected by the soliton damping and given by $\xi_0=K^{-1}$. This prediction may be tested by introducing impurities in the quasi-one-dimensional magnetic systems for example.

### B. $Kl \ll 1$

In this limit the soliton motion is basically diffusive. We can now expand $S(\omega,q)$ as

$$S(\omega,q)=S_0(\omega,q)+S_0'(\omega,q)+\cdots\ , \tag{36}$$

$$S_D(\omega,q)=\int_{-\infty}^{\infty}\int dtdz\, e^{i(qz-\omega t)} \times\exp[-K(z^2+2D|t|)^{1/2}]\ , \tag{37}$$

and

$$S_D'(\omega,q) = KD\tau \int_{-\infty}^{\infty} \int dt dz (z^2 + 2D|t|)^{-1/2}(1 - e^{-|t|/\tau}) e^{i(qz - \omega t)} \exp[-K(z^2 + 2D|t|)^{1/2}] \; . \tag{38}$$

The $z$ integrals of Eqs. (37) and (38) are done easily and we obtain

$$S_D(\omega,q) = 2K(K^2 + q^2)^{-1/2} \int_{-\infty}^{\infty} dt (2D|t|)^{1/2} e^{-i\omega t} K_1(\alpha |t|^{1/2}) \tag{39}$$

and

$$S_D'(\omega,q) = 2KD\tau \int_{-\infty}^{\infty} dt (1 - e^{-|t|/\tau}) e^{-i\omega t} K_0(\alpha|t|^{1/2}) \; , \tag{40}$$

where

$$\alpha = (2D)^{1/2}(q^2 + K^2)^{1/2} \tag{41}$$

and $K_1(z)$ and $K_0(z)$ are the modified Bessel functions. Finally the $t$ integrals are done for $\omega >> \alpha^2$ and $\omega << \alpha^2$ as

$$S_D(\omega,q) = \begin{cases} \dfrac{8KD^{-1}}{(K^2+q^2)^2}\left[1 - 3\times 2^5 \left(\dfrac{\omega}{\alpha^2}\right)^2 + 3\times 5\times 2^{11}\left(\dfrac{\omega}{\alpha^2}\right)^4\right], & \text{for } \omega << \alpha^2, \quad (42) \\ \dfrac{2KD}{\omega^2}\left[\ln\left(\dfrac{4\omega}{\gamma\alpha^2}\right) + O\left(\dfrac{\alpha^2}{\omega}\right)^2\right], & \text{for } \omega >> \alpha^2, \quad (43) \end{cases}$$

and

$$S_D'(\omega,q) = \frac{4K\tau}{(K^2+q^2)}\left[1 - 2^5\left(\frac{\omega}{\alpha^2}\right) + 3\times 2^{11}\left(\frac{\omega}{\alpha^2}\right)^4, \ldots, \right] + K\tau^2(2D)[1 + (\tau\omega)^2]^{-1} \ln\left(\frac{\gamma'}{4}\frac{\tau\alpha^2}{1+(\tau\omega)^2}\right), \quad \text{for } \omega << \alpha^2 \; , \tag{44}$$

$$S_D'(\omega,q) \simeq -K\frac{D\tau}{2}\left(\frac{\alpha}{\omega}\right)^2\left[1 + \ln\left(\frac{\gamma'\alpha^2}{4\omega}\right)\right], \quad \text{for } \omega >> \alpha^2 \; , \tag{45}$$

where $\gamma' = 1,781, \ldots$ . We may interpolate Eqs. (42) and (43) by

$$S_D(\omega,q) \simeq 8KD^{-1}\left[(K^2+q^2)^2 + 24\left(\frac{\omega}{D}\right)^2\right]^{-1} \tag{46}$$

and similarly

$$S_D'(\omega,q) \simeq 4K(K^2+q^2)\tau\left[(K^2+q^2)^2 + 8\left(\frac{\omega}{D}\right)^2\right]^{-1} \; . \tag{47}$$

Equation (46) indicates that the dynamical correlation function in the diffusion limit is completely different from that in the ideal gas limit, although the equal-time correlation length is the same in these limits and independent of the soliton damping. In particular, Eq. (46) agrees with the phenomenological analysis by Imada[18] in the large damping limit.

Another quantity of interest is the local correlation function which is defined by

$$S(\omega) = \int \frac{dq}{2\pi} S(\omega,q) \; , \tag{48}$$

which can be also measured by the nuclear magnetic resonance technique.[15] Substituting Eq. (28) into Eq. (48), we obtain

$$S(\omega) = \int dt \exp\{-i\omega t - Kv_0[F(t,\tau)]^{1/2}\} \; . \tag{49}$$

Again the general expression is difficult to analyze. However, in two limiting cases $S(\omega)$ has rather simple expressions. For $Kl >> 1$, we obtain

$$S(\omega) = 2Kv_0^{-1}\left[K^2 + \left(\frac{\omega}{v_0}\right)^2\right]^{-1} \times \left\{1 + \frac{1}{3l}K\left[K^2 - 3\left(\frac{\omega}{v_0}\right)^2\right]\left[K^2 + \left(\frac{\omega}{v_0}\right)^2\right]^{-2}\right\} \; . \tag{50}$$

On the other hand in the diffusion limit $Kl << 1$ we have

$$S(\omega)=\begin{cases}\dfrac{2}{DK^2}(1-15y^2+945y^4)+2\tau(1-3y^2+105y^4), & \text{for } y(\equiv\omega/DK^2)<<1 \quad (51)\\ \omega^{-1}(\sqrt{\pi}y^{-1/2}-y^{-1}+\frac{1}{4}\sqrt{\pi}y^{-3/2})+\frac{1}{2}\tau\sqrt{\pi}y^{-1/2}, & \text{for } y>>1\ , \quad (52)\end{cases}$$

which may be interpolated as

$$S(\omega)\simeq 2KD^{-1}\left[K^4+20\left(\frac{\omega}{D}\right)^2\right]^{-3/4}+2\tau K\left[K^4+12\left(\frac{\omega}{D}\right)^2\right]^{-1/4}\ . \tag{53}$$

## V. CONCLUDING REMARKS

We have studied possible effects of the soliton damping on the dynamical correlation functions within a phenomenological model. We have shown that the static correlation length is unaffected by the soliton damping. On the other hand, the structure of the dynamical correlation functions will be modified strongly, which should be accessible to the neutron scattering experiments.

*Note added in proof.* Recently Büttiker[11] has shown that in the diffusion limit Eq. (25) is given exactly by

$$\langle[\phi(z,t)-\phi(0,0)]^2\rangle=2\bar{n}_s\left(\frac{2\pi}{g}\right)^2\left[\left(\frac{4D_0t}{\pi}\right)^{1/2}\exp\left(\frac{-z^2}{4D_0t}\right)+z\,\mathrm{erf}\left(\frac{z}{(4D_0t)^{1/2}}\right)\right]\quad \text{for } t>0$$

with $D_0=(\pi/2)D=\tau(\beta E_s)^{-1}$. Therefore, Eq. (25) gives the correct asymptotic behaviors for $z^2>>4D_0t$ and for $z^2<<4D_0t$. Furthermore, our results [Eqs. (24) and (25)] appear to differ from Gunther and Imry[10] in that we find diffusive behavior for $t>>\tau$ and free-particle-like behavior for $t<<\tau$. These agree completely with Büttiker and Landauer.[11] I thank Dr. Markus Büttiker for helpful correspondence.

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