Potential-scattering models for the quasiparticle interactions in liquid ^{3}He

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Using a Legendre function expansion, we obtain analytic results for the transport coefficients and superfluid Ginzburg-Landau parameters of liquid ^{3}He from any model quasiparticle scattering amplitude with the form of a matrix element of a local two-body potential. We perform a least-squares fit to the measured Landau parameters and transport coefficients and obtain nearly perfect agreement for models including Legendre components with $l \leq 3$.

Levin and Valls[1] have recently obtained good results for the transport coefficients and Landau parameters of normal liquid ^{3}He from model quasiparticle scattering amplitudes of the form (see Fig. 1)

$$T_{\alpha\beta;\gamma\rho}(\vec{p}_1\vec{p}_2;\vec{p}_3\vec{p}_4) = \delta_{\alpha\gamma}\delta_{\beta\rho}v(|\vec{p}_1-\vec{p}_3|) - \delta_{\alpha\rho}\delta_{\beta\gamma}v(|\vec{p}_1-\vec{p}_4|) + \vec{\sigma}_{\alpha\gamma}\cdot\vec{\sigma}_{\beta\rho}j(|\vec{p}_1-\vec{p}_3|) - \vec{\sigma}_{\alpha\rho}\cdot\vec{\sigma}_{\beta\gamma}j(|\vec{p}_1-\vec{p}_4|) \quad . \tag{1}$$

We call these potential scattering models because Eq. (1) has the same form as the matrix element of a local two-body potential. Levin and Valls studied several different functional forms for the potentials $v(q)$ and $j(q)$. To fix the parameters of their models, they first calculated the Landau parameters, which are given by one-dimensional integrals of $v(q)$ and $j(q)$, and fitted these as closely as possible to the measured Landau parameters for liquid ^{3}He. They used the scattering amplitudes determined in this way to calculate the transport coefficients and superfluid strong-coupling corrections, which are all given by two-dimensional integrals.

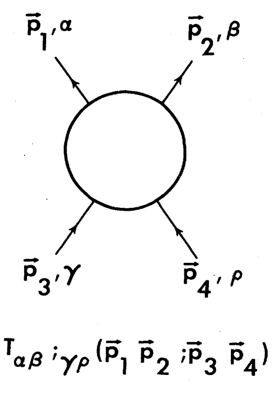

FIG. 1. Scattering amplitude conventions. For all four momenta on the Fermi surface T depends on only two independent angles. The particle-hole angles $x_2 = \hat{p}_3\cdot\hat{p}_1$ and $x_3 = \hat{p}_4\cdot\hat{p}_1$ are a convenient choice.

In this paper we point out that with a parametrization of the potential scattering models suggested by Wölfle[2] (and also used in a restricted form by Levin and Valls in their models *c,d,e*, and *g*), all the properties of interest can be calculated analytically. This then allows us to fit the model scattering amplitude to any or all the measured quantities with minimal computational effort. Furthermore, because our variational procedure in principle covers all potential scattering models, we can determine the extent to which the experimental results constrain the form of the scattering amplitude, within the class of potential scattering models.

For quasiparticles on the Fermi surface, the scattering amplitude depends on only two independent angular variables. We choose for these the two "particle-hole" angles, which are related to the more familiar Abrikosov-Khalatnikov angles θ and ϕ by

$$x_2 = \hat{p}_1\cdot\hat{p}_3 = \cos^2(\theta/2) + \sin^2(\theta/2)\cos\phi \quad ,$$
$$x_3 = \hat{p}_1\cdot\hat{p}_4 = \cos^2(\theta/2) - \sin^2(\theta/2)\cos\phi \quad . \tag{2}$$

The momentum transfers in the potential scattering models are

$$|\vec{p}_1 - \vec{p}_3| = 2k_F\sqrt{(1-x_2)/2} \quad ,$$
$$|\vec{p}_1 - \vec{p}_4| = 2k_F\sqrt{(1-x_3)/2} \quad , \tag{3}$$

and hence these momentum transfers may be replaced by x_2 and x_3 as the independent variables in Eq. (1). We find it most convenient to work with the singlet and triplet components of the scattering amplitude in the particle-particle channel. In potential scattering models these have the form

$$T_s(x_2,x_3) = W_s(x_2) + W_s(x_3) \ , \qquad T_t(x_2,x_3) = W_t(x_2) - W_t(x_3) \ , \tag{4}$$

which automatically satisfy the exchange antisymmetry conditions

$$T_s(x_2,x_3) = T_s(x_3,x_2) \ , \quad T_t(x_2,x_3) = -T_t(x_3,x_2) \ . \tag{5}$$

W_s and W_t are related to the potentials $v(q)$ and $j(q)$ in Eq. (1) by

$$W_s(x) = v(q) - 3j(q) \ , \quad W_t(x) = v(q) + j(q) \ . \tag{6}$$

We parametrize the potential scattering models by expanding W_s and W_t in the Legendre polynomials

$$W_s(x) = \sum_{l=0}^{\infty} W_l^s P_l(x) \ , \quad W_t(x) = \sum_{l=0}^{\infty} W_l^t P_l(x) \ . \tag{7}$$

Since the $l=0$ term disappears from $T_t(x_2,x_3)$, W_0^t does not enter any physical quantity. Consequently $v(q)$ and $j(q)$ cannot be uniquely determined by experiment; all physical quantities are unchanged if we replace $v(q)$ by $v(q)+3C$ and $j(q)$ by $j(q)+C$, where C is any constant.

To find the Landau parameters we use Landau's exact result for the forward scattering amplitude,

$$T^{(s)}(\theta,\phi=0) = \tfrac{1}{4}(3T_t + T_s) = \sum_{l=0}^{\infty} A_l^s P_l(\cos\theta) \ , \qquad T^{(a)}(\theta,\phi=0) = \tfrac{1}{4}(T_t - T_s) = \sum_{l=0}^{\infty} A_l^a P_l(\cos\theta) \ , \tag{8}$$

which together with Eqs. (4) and (7) gives[2]

$$A_l^s = \tfrac{1}{4}\{[3W_t(1) + W_s(1)]\delta_{l,0} - (3W_l^t - W_l^s)\} \ , \qquad A_l^a = \tfrac{1}{4}\{[W_t(1) - W_s(1)]\delta_{l,0} - (W_l^t + W_l^s)\} \ , \qquad W_{t,s}(1) = \sum_{l=0}^{\infty} W_l^{t,s} \ . \tag{9}$$

We note that the Landau parameters depend only on the potential scattering parameters with the same l, except for A_0^s and A_0^a which also depend on the full $q=0$ potentials $W_{t,s}(1)$. From Eq. (9) we immediately obtain the forward scattering sum rule

$$\sum_{l=0}^{\infty} (A_l^s + A_l^a) = 0 \ , \tag{10}$$

which is satisfied by all potential scattering models as a direct consequence of Eq. (4) for T_t. Inverting Eq. (9) yields

$$W_l^s = A_l^s - 3A_l^a, \quad l \geq 1 \ , \qquad W_l^t = -(A_l^s + A_l^a), \quad l \geq 1 \ , \qquad W_0^s = \tfrac{1}{2}\left[(A_0^s - 3A_0^a) - \sum_{l=1}^{\infty} (A_l^s - 3A_l^a) \right] \ . \tag{11}$$

In place of an equation for W_0^t we obtain

$$A_0^s + A_0^a = W_t(1) - W_0^t \ , \tag{12}$$

which combined with Eq. (11) simply reproduces the forward scattering sum rule. We note in passing that the *s-p* approximation by Dy and Pethick[3] is a potential scattering model if only Landau parameters with $l \leq 1$ are included and the forward scattering sum rule is satisfied.

The normal-state transport coefficients and the superfluid strong-coupling corrections through order T_c/T_F are determined by integrals quadratic in the quasiparticle scattering amplitude. The expressions for these quantities are summarized in the Appendix. For our parametrized potential scattering models, these integrals reduce to quadratic forms in $W_l^{t,s}$. For example, the quasiparticle lifetime $\tau(0)$ is given by

$$\frac{1}{\tau(0)} = \frac{\pi^3}{8} \left[\frac{T}{T_F} \right]^2 \frac{k_B T_F}{\hbar} \sum_{l \geq 0} \sum_{l' \geq 0} (A_{ll'}^{\tau} W_l^t W_{l'}^t + B_{ll'}^{\tau} W_l^s W_{l'}^s + C_{ll'}^{\tau} W_l^t W_{l'}^s) \ . \tag{13}$$

The coefficients in Eq. (13) and in the corresponding expressions for the other physical quantities are linear combinations of integrals of the following types:

$$j_{ll'}^1(m,n) = \left\langle P_l(x_2) P_{l'}(x_2) \left[\frac{1-x_2}{2}\right]^m \left[\frac{1-x_3}{2}\right]^n \right\rangle \ , \qquad j_{ll'}^2(m,n) = \left\langle P_l(x_2) P_{l'}(x_3) \left[\frac{1-x_2}{2}\right]^m \left[\frac{1-x_3}{2}\right]^n \right\rangle \ , \qquad j_{ll'}^3(m,n) = \left\langle P_l(x_3) P_{l'}(x_3) \left[\frac{1-x_2}{2}\right]^m \left[\frac{1-x_3}{2}\right]^n \right\rangle \ , \tag{14}$$

where

$$\langle G(x_2,x_3)\rangle=\frac{\sqrt{2}}{4\pi}\int_{-1}^{1}dx_2\int_{-x_2}^{1}dx_3\frac{G(x_2,x_3)}{[(x_2+x_3)(1-x_2)(1-x_3)]^{1/2}}\ . \tag{15}$$

Our analytic results for these integrals are

$$\begin{aligned} j_{ll'}^{\alpha}(m,n)&=\sum_{p=0}^{l}\sum_{q=0}^{l'}\frac{(-1)^{p+q}}{2(p+q+m+n)+1}\binom{l+p}{l}\binom{l}{p}\binom{l'+q}{l'}\binom{l'}{q}R^{\alpha}(m,n;p,q)\ ,\\ R^{1}(m,n;p,q)&=\binom{p+q+m+n}{n}\Big/\binom{2p+2q+2m+2n}{2n}\ ,\\ R^{2}(m,n;p,q)&=\binom{p+q+m+n}{p+m}\Big/\binom{2p+2q+2m+2n}{2m+2p}\ ,\\ R^{3}(m,n;p,q)&=\binom{p+q+m+n}{m}\Big/\binom{2p+2q+2m+2n}{2m}\ . \end{aligned} \tag{16}$$

In Ref. 4 we have tabulated the coefficients through $l=3$ in expansions analogous to Eq. (13) for the transport quantities and the superfluid Ginzburg-Landau parameters.

To determine potentials which fit the Landau parameters, transport coefficients, and superfluid strong-coupling coefficients, we adjust the scattering parameters $W_l^{t,s}$ to minimize the sum of squared deviations of the calculated physical quantities from their corresponding experimental values. Table I summarizes the results obtained by fitting to the melting pressure values of A_0^s, A_0^a, A_1^s, $\tau(0)T^2$, λ_K, λ_η and λ_D.[5,6] [See the Appendix for definitions of the λ's and of the functions $S_{E,0}(\lambda)$ discussed

TABLE I. Melting pressure calculations. The scattering amplitudes were obtained using the transport coefficients and Landau parameters as fitting parameters.

	F_0^s	F_0^a	F_1^s	F_1^a	F_2^s	F_2^a	A_0^s	A_0^a	A_1^s	A_1^a	A_2^s	A_2^a
Exp.	94.13	−0.738	15.66	· · ·	~1.0	· · ·	0.9895	−2.822	2.518	· · ·	~0.8	· · ·
$l\leq 2$	67.49	−0.724	14.21	−0.753	0.390	−0.191	0.9854	−2.622	2.477	−1.005	0.362	−0.199
$l\leq 3$	93.34	−0.737	14.51	−0.766	0.497	0.289	0.9894	−2.799	2.486	−1.029	0.452	0.273

	$\tau(0)T^2(\mu\text{sec mK}^2)$	λ_κ	λ_η	λ_D	κT(erg/cm sec)	$\eta T^2(P\ \text{mK}^2)$	$DT^2(\text{cm}^2\,\text{mK}^2/\text{sec})$
Exp.	0.26	1.31	0.70	0.01	10.7	0.88	0.17
$l\leq 2$	0.26	1.30	0.71	0.03	10.5	0.89	0.17
$l\leq 3$	0.26	1.31	0.70	0.04	10.6	0.88	0.17

	W_0^s	W_1^t	W_1^s	W_2^t	W_2^s	W_3^t	W_3^s
$l\leq 2$	1.1997	−1.4729	5.4912	−0.1633	0.9594	0	0
$l\leq 3$	1.0120	−1.4576	5.5732	−0.7252	−0.3666	0.3732	2.1556

	$\Delta\tilde{C}_A$	$\Delta\tilde{C}_B$	$\Delta\tilde{C}_{A1}$	$x=\Delta\tilde{\beta}_{245}$	$y=\Delta\tilde{\beta}_{12}+\frac{1}{3}\Delta\tilde{\beta}_{345}$	$\Delta\tilde{\beta}_{24}$	$(y-x)$
Expt.	2.00	1.90	0.74	−0.72	−0.33	−0.47	0.39
$l\leq 2$	2.21	2.13	0.70	−0.86	−0.48	−0.40	0.38
$l\leq 3$	2.24	2.12	0.70	−0.87	−0.47	−0.40	0.40

	$\Delta\tilde{\beta}_1$	$\Delta\tilde{\beta}_2$	$\Delta\tilde{\beta}_3$	$\Delta\tilde{\beta}_4$	$\Delta\tilde{\beta}_5$	$\Delta\tilde{\beta}_1^{\text{wc}}$
$l\leq 2$	−0.066	−0.130	−0.124	−0.266	−0.460	0.004
$l\leq 3$	−0.074	−0.103	−0.123	−0.298	−0.469	0.003

below.] With only $l \leq 3$ scattering parameters included, all the known Landau parameters and transport coefficients are fit to within 1% of experiment.

In our fits of the scattering potential parameters to the experimental quantities we use $\tau(0)T^2$, λ_K, λ_η, and λ_D as constraints rather than the transport coefficients because the latter are primarily determined by the quasiparticle lifetime $\tau(0)$. In fact, $\tau(0)$, K, and D constrain the quasiparticle scattering amplitude less than would three independent angular averages of the quasiparticle scattering rate, because (1) the λ_α are less sensitive to changes in the scattering amplitudes than is $\tau(0)$ and (2) $S_{E,0}(\lambda)$ is a slowly varying function of λ in the regions of physical interest for K and D (but not for η).

The extent to which the Landau parameters and transport coefficients determine the form of $v(q)$ and $j(q)$ is indicated in Fig. 2.[7] Potentials (a) were determined by optimizing a scattering amplitude with $W_l^{t,s} \neq 0$ only for $l \leq 2$ (see also Table I). Although this scattering amplitude fits the transport coefficients accurately, the fit to the Landau parameters is not as good as can be obtained by including $l=3$ terms in the potentials. The optimum potentials for $l \leq 3$ are shown in Fig. 2(b). The shape of the scattering potentials in Fig. 2(b) and the accuracy of the fit to the experimental quantities are essentially unchanged by increasing $l_{\max}$ to five.

Levin and Valls call the potentials in Fig. 2 "spin-fluctuation-like" because $-j(q)$ has a maximum at $q=0$. In all our calculations, including fits with $\Delta\tilde{\beta}_{245}$ and $\Delta\tilde{\beta}_{12}+\frac{1}{3}\Delta\tilde{\beta}_{345}$ as constraints and calculations at all pressures with as many as 13 scattering parameters, we find a maximum in $-j(q)$ at $q=0$; in this sense we also find that the scattering amplitude is "spin-fluctuation-like." However, we find the best agreement with the Landau parameters and transport coefficients from a scattering potential $j(q)$ which is much less sharply peaked than those obtained by Levin and Valls. This is consistent with the transport coefficients being most sensitive to the low-order moments of the scattering potentials because the weight factors determining $\tau(0)$, λ_K, λ_η, and λ_D depend linearly or bilinearly on the particle-hole angles.

To check the rate of convergence of the Legendre expansions we have calculated $\tau(0)T^2$ using Eq. (13) with $W^{t,s}$ calculated from the spin-fluctuation model with $\bar{I}=0.96$. The expansion for $\tau(0)T^2$ [Eq. (13)] converges to within 8% of the exact value if we retain only terms with $l \leq 3$ and to within 4% if we add the $l=4$ terms.

In Table I we also list the strong-coupling corrections to the Ginzburg-Landau parameters calculated with the same scattering amplitude which fits the transport coefficients, etc. In this calculation the linear combinations of $\Delta\tilde{\beta}$'s which can be extracted from experiment were not used to determine the scattering amplitude. Our calculations of the $\Delta\tilde{\beta}$'s include, in addition to the scattering amplitude in Table I, a smooth cutoff in the frequency sums S_α [see Eqs. (A10), (A11), (A12), and Ref. 8]. This cutoff comes from the frequency dependence of the pairing interaction, quasiparticle lifetime, etc. Since the details of this cutoff are not known, we have evaluated the frequency sums using a frequency-dependent order parameter of the form

$$\Delta(\epsilon_n;T)=\Delta(T)/[1+(\epsilon_n/\epsilon_c)^2] \ . \tag{17}$$

Figure 3 shows the S_α calculated as a function of ϵ_c. Our calculated $\Delta\tilde{\beta}$'s are in best agreement with ex-

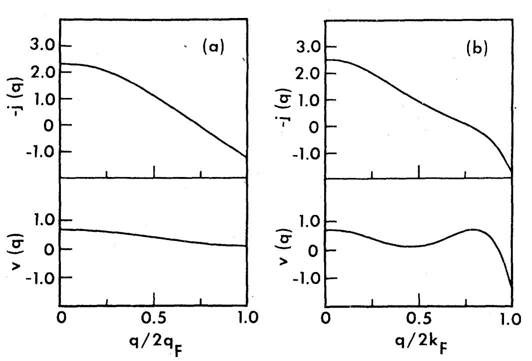

FIG. 2. Scattering potentials for ^{3}He at melting pressure. Potentials (a) were optimized with $l_{\max}=2$. Potentials (b) were optimized with $l_{\max}=3$.

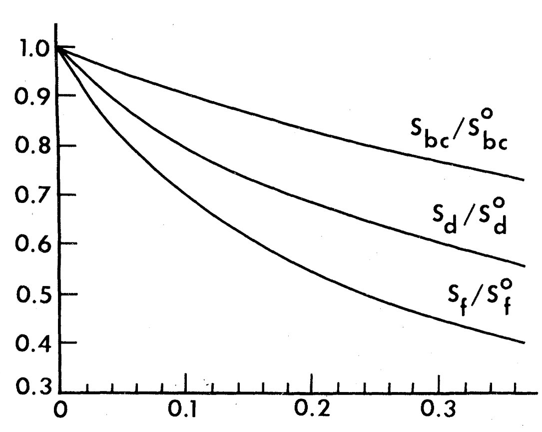

FIG. 3. Cutoff dependence of the frequency sums. The curves are normalized to the cutoff independent values: $S_{bc}^0=6.8$, $S_d^0=10.1$, and $S_f^0=30.4$ With $\epsilon_c=0.068k_BT_F$, x_c decreases from 0.25 at 34 bars to 0.15 at 12 bars and 0.06 at 0 bar.

TABLE II. Calculated corrections to the Ginzburg-Landau parameters.

P(bars)	$\Delta\tilde{\beta}_1$	$\Delta\tilde{\beta}_2$	$\Delta\tilde{\beta}_3$	$\Delta\tilde{\beta}_4$	$\Delta\tilde{\beta}_5$
12	−0.034	−0.080	−0.117	−0.199	−0.195
16	−0.041	−0.088	−0.128	−0.229	−0.235
20	−0.048	−0.095	−0.136	−0.254	−0.277
24	−0.056	−0.101	−0.140	−0.273	−0.321
28	−0.062	−0.105	−0.139	−0.286	−0.369
34.4	−0.074	−0.103	−0.123	−0.298	−0.469

periment if $\epsilon_c = 0.068k_B T_F$. The calculated specific-heat discontinuities are 11% larger than the melting pressure results in Ref. 9; equivalently the calculated values $\Delta\tilde{\beta}_{245} = -0.87$ and $\Delta\tilde{\beta}_{12} + \frac{1}{3}\Delta\tilde{\beta}_{345} = -0.47$ are moderately large compared with experimentally determined values of −0.72 and −0.33, respectively.[10] However, the difference in these two quantities, which is the most sensitive indicator of A-phase stability, is 0.40 compared to the experimental value of 0.39. To emphasize the tenuous stability of ^{3}He-A we note that the smallest this difference can be for a stable A phase is 0.33. Thus, a precise theoretical determination of the pressure at the polycritical point (PCP) is difficult. We have calculated the $\Delta\tilde{\beta}_i$ at lower pressures (see Table II) using scattering amplitudes determined by the procedure described above, and taking $\epsilon_c(p) = 0.068k_B T_F(p)$. From these $\Delta\tilde{\beta}_i$'s we obtain 27 bars for the PCP, compared to the experimental value of 22 bars.

To summarize, we have obtained analytic expressions for the normal-state transport coefficients and superfluid strong-coupling corrections for ^{3}He assuming that the scattering amplitude has the form of a matrix element of a local two-body operator. We have optimized our scattering potentials to obtain accurate results for all the known Landau parameters and normal-state transport coefficients. Our calculated strong-coupling free energy coefficients are in reasonable agreement with experiment.

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APPENDIX

In this Appendix we list the formulas for the normal-state transport properties and superfluid strong-coupling corrections in terms of the normal-state quasiparticle scattering amplitude.[11]

(i) Quasiparticle lifetime:

$$\tau(0) = \frac{8\pi^2\hbar^6}{(m^*)^3 k_B^2 T^2} \frac{1}{\langle W(\theta,\phi)\rangle} . \quad \text{(A1)}$$

(ii) Thermal conductivity:

$$\kappa = (\pi^2/2) n k_B (T/T_F) v_F^2 \tau(0) S_E(\lambda_\kappa) ,$$
$$\lambda_\kappa = \langle W(\theta,\phi)(1+2\cos\theta)\rangle / \langle W(\theta,\phi)\rangle . \quad \text{(A2)}$$

(iii) Viscosity:

$$\eta = \tfrac{1}{5} n v_F p_F \tau(0) S_0(\lambda_\eta) ,$$
$$\lambda_\eta = \langle W(\theta,\phi)[1 - 3\sin^4(\theta/2)\sin^2\phi]\rangle / \langle W(\theta,\phi)\rangle . \quad \text{(A3)}$$

(iv) Spin diffusion:

$$D = \tfrac{1}{3} v_F^2 (1 + F_0^a) \tau(0) S_0(\lambda_D) ,$$
$$(1 - \lambda_D) = \langle W_{\uparrow\downarrow}(\theta,\phi)\sin^2(\theta/2)(1-\cos\phi)\rangle / \langle W(\theta,\phi)\rangle . \quad \text{(A4)}$$

The scattering rates $W(\theta,\phi)$ and $W_{\uparrow\downarrow}(\theta,\phi)$ are given in terms of $T_t(\theta,\phi)$ and $T_s(\theta,\phi)$ by

$$W(\theta,\phi) = \frac{2\pi}{\hbar} \nu(0)^{-2} [\tfrac{3}{8} T_t(\theta,\phi)^2 + \tfrac{1}{8} T_s(\theta,\phi)^2 + \tfrac{1}{4} T_t(\theta,\phi) T_s(\theta,\phi)] ,$$
$$W_{\uparrow\downarrow}(\theta,\phi) = \frac{2\pi}{\hbar} \nu(0)^{-2} [\tfrac{1}{4} T_t(\theta,\phi)^2 + \tfrac{1}{4} T_s(\theta,\phi)^2 + \tfrac{1}{2} T_t(\theta,\phi) T_s(\theta,\phi)] , \quad \text{(A5)}$$

where $\nu(0)=m^* k_F/\pi^2\hbar^2$. The function $S_E(\lambda)$ $(S_0(\lambda))$ is

$$S_{E(0)}(\lambda)=\sum_{\substack{\nu=\text{even}\\(\nu=\text{odd})}}^{\infty}\frac{2n+1}{\nu(\nu+1)[\nu(\nu+1)-2\lambda]} \quad . \tag{A6}$$

Our angular averages are defined by[12]

$$\langle G(\theta,\phi)\rangle=\int_0^{2\pi}\frac{d\phi}{2\pi}\int_0^1 d\left[\cos\frac{\theta}{2}\right]G(\theta,\phi) \quad . \tag{A7}$$

The definition of the strong-coupling corrections is partly a matter of choice. From a theoretical point of view it is most convenient to define all corrections to weak-coupling BCS theory as strong-coupling terms. In the Ginzburg-Landau region these terms naturally divide into three types:

$$\alpha'=N(0)(1+\delta\tilde{\alpha}_{sc}) \quad ,$$
$$\beta_i=\beta_i^{BCS}+\Delta\beta_i \quad , \tag{A8}$$
$$\Delta\beta_i=\Delta\beta_i^{wc}+\Delta\beta_i^{f}+\Delta\beta_i^{bc}+\Delta\beta_i^{d} \quad .$$

(1) $\delta\tilde{\alpha}_{sc}$ is the strong-coupling correction to the quadratic free energy coming from the finite quasiparticle lifetime and is related to $\tau(0)$ by[10]

$$\delta\tilde{\alpha}_{sc}=\frac{\pi}{4}\left[\frac{k_B T_c}{v_F p_F}\right]\frac{\hbar T_F}{k_B\tau(0)T^2} \quad . \tag{A9}$$

(2) The $\Delta\beta_i^{wc}$ are strong-coupling corrections from the frequency dependence of the normal-state pairing interaction. These terms enter only through the weak-coupling diagram [Fig. 3(a) of Ref. 8]; consequently the $\Delta\beta_i^{wc}$ have the same ratios as the BCS β's. From Serene and Rainer,[13]

$$\Delta\beta_1^{wc}=(\tilde{\eta}/16)S_{wc}\langle[5T_t(\theta,\phi)T_t(\theta',\phi')+T_s(\theta,\phi)T_s(\theta',\phi')+T_s(\theta,\phi)T_t(\theta',\phi')+T_t(\theta,\phi)T_s(\theta',\phi')]x_2\rangle \quad . \tag{A10}$$

For all the scattering amplitudes that we have studied, $\Delta\beta_1^{wc}$ is negligible.

(3) The remaining $\Delta\beta_i^{\alpha}$ are not related by the BCS ratios. These terms are discussed extensively by Rainer and Serene[8] and are given by

$$\Delta\beta_i^{\alpha}=-(\tilde{\eta}/16)S_\alpha\langle X_i^\alpha(\theta,\phi)T_t(\theta,\phi)^2+Y_i^\alpha(\theta,\phi)T_s(\theta,\phi)^2+Z_i^\alpha(\theta,\phi)T_t(\theta,\phi)T_s(\theta,\phi)\rangle \quad , \tag{A11}$$

for $\alpha=$f, bc, and

$$\Delta\beta_i^{d}=-(\tilde{\eta}/4)S_d\langle X_i^{d}(\theta,\phi)T_t(\theta,\phi)T_t(\theta',\phi') + Y_i^{d}(\theta,\phi)[T_s(\theta,\phi)T_s(\theta',\phi')+T_t(\theta,\phi)T_s(\theta',\phi')+T_s(\theta,\phi)T_t(\theta',\phi')]\rangle \quad , \tag{A12}$$

where $\tilde{\eta}=N(0)/(30k_B T_c v_F p_F)$. The weight factors $X(\theta,\phi)$, $Y(\theta,\phi)$, and $Z(\theta,\phi)$ are given in Ref. 4. The $S_\alpha(\alpha=$wc, f, bc, d) are frequency sums over products of quasiparticle propagators; these sums are listed in Refs. 4 and 8.

The specific-heat jumps for the A, B, and $A1$ phases are

$$\left[\frac{\Delta C_A}{C_N}\right]=1.188\frac{2(1+\delta\tilde{\alpha}_{sc})^2}{(2+\Delta\tilde{\beta}_{245})} \quad , \tag{A13}$$

$$\left[\frac{\Delta C_B}{C_N}\right]=1.426\frac{\frac{5}{3}(1+\delta\tilde{\alpha}_{sc})^2}{[\frac{5}{3}+(\Delta\tilde{\beta}_{12}+\frac{1}{3}\Delta\tilde{\beta}_{345})]} \quad , \tag{A14}$$

$$\left[\frac{\Delta C_{A1}}{C_N}\right]=0.594\frac{4(1+\delta\tilde{\alpha}_{sc})^2}{(4+\Delta\tilde{\beta}_{24})} \quad , \tag{A15}$$

where $\Delta\tilde{\beta}_i=\Delta\beta_i/|\beta_1^{BCS}|$, and C_N is the low-temperature limit of the specific heat (evaluated at T_c), $C_N=(\frac{2}{3}\pi^2)k_B^2 N(0)T_c$.

The condition for stability of the A phase relative to the B phase is

$$(\Delta\tilde{\beta}_{12}+\tfrac{1}{3}\Delta\tilde{\beta}_{345})-\Delta\tilde{\beta}_{245}\geq\tfrac{1}{3} \quad . \tag{A16}$$

Patton and Zaringhalam[14] have obtained a result for the transition temperature in terms of the quasiparticle scattering amplitude and a cutoff ϵ_0 which approximately accounts for the frequency dependence of the pairing interaction. For potential scattering models this relation is

$$k_B T_c^{(l)}=1.13\epsilon_0 e^{1/\lambda_l} \quad ,$$
$$\lambda_l=\frac{1}{2(2l+1)}\begin{cases}W_l^s, & l \text{ even}\\ W_l^t, & l \text{ odd}\end{cases} \tag{A17}$$

for $\lambda_l<0$.

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