Correction of experimental values of the critical exponent β of high-temperature ferromagnets to constant volume

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It is shown that for a high-temperature ferromagnet the critical exponent derived from magnetization or hyperfine-field measurements at constant pressure (β_P) is related to the value at constant volume by the expression: $\beta_P = \beta_V[1 + B_T\gamma_T(\partial T_c/\partial P)]$, where B_T and γ_T are the average values of the isothermal bulk modulus and coefficient of thermal expansion in the critical region, respectively, T_c is the Curie point, and P the pressure. For nickel $\beta_V = 0.966\beta_P$ and the correction is of the same order as the estimated random error in the experimental value of β_P.

Shaham *et al.*[1] (afterwards referred to as I) have recently reported NMR measurements of the hyperfine field in pure iron, nickel, and cobalt in the critical region $T \simeq T_c$, where T_c is the Curie point. According to the theory of critical phenomena the magnetization and, it will be assumed here, the hyperfine field in the critical region, obey a law of the form

$$M_T/M_0 = \nu_T/\nu_0 = Bt^{\beta_V} \quad , \tag{1}$$

where M_T and ν_T are the magnetization and hyperfine field at temperature T and constant volume, $t = (T_c - T)/T_c$ and β_V is the critical exponent. The subscript on β is usually omitted but is introduced here to emphasize that in a theoretical discussion the system is always considered to be at constant volume. In I a fit to Eq. (1) was obtained using hyperfine-field measurements made at constant pressure leading to an exponent we shall call β_P. A thermodynamic calculation was then given which claimed to show that $(\beta_V - \beta_P) \simeq 1 \times 10^{-3}$, an order of magnitude less than the random error in β_P. In the present note we show that this conclusion is incorrect, because it neglects the volume dependence of T_c, and that in particular $(\beta_V - \beta_P) = -0.012$ for nickel which is close to the random error given for β_P(0.354 $\pm$0.014) in I and some six times greater than the random error claimed for the most accurate values of β_P (see Table I of I).

The thermodynamic equation

$$\left(\frac{\partial \ln \nu_T}{\partial T}\right)_V = \left(\frac{\partial \ln \nu_T}{\partial T}\right)_P + B_T\gamma_T\left(\frac{\partial \ln \nu}{\partial P}\right)_T \quad , \tag{2}$$

where B_T is the isothermal bulk modulus and γ_T the coefficient of thermal expansion at temperature T, was integrated in I, to correct the hyperfine-field measurements to constant volume, by ignoring the temperature dependence of B_T and $(\partial \ln \nu/\partial P)_T$ and using the values found from experiments at room temperature. It is true that B_T is only a weak function of temperature, even in the critical region,[2] but it will be shown that it is not permissible to ignore the temperature dependence of $(\partial \ln \nu/\partial P)_T$.

Provided that at constant volume the temperature dependence of the hyperfine field can be written in the form, $\nu_T/\nu_0 = f(T/T_c)$ it is simple to show that

$$\left(\frac{\partial \ln \nu_T}{\partial P}\right)_T = \left(\frac{\partial \ln \nu_0}{\partial P}\right)_T - \left(\frac{\partial \ln T_c}{\partial P}\right)\left(\frac{\partial \ln \nu_T}{\partial \ln T}\right)_V \quad , \tag{3}$$

and, on replacing ν_T by M_T throughout, a similar equation must hold for the magnetization.[3] Equation (3) has in fact been shown[4] to describe the dependence of the magnetization of Ni Cu alloys on pressure in the temperature range $0-0.8T/T_c$. We assume that Eq. (3) will continue to hold in the critical region and hence

$$\left(\frac{\partial \ln \nu_T}{\partial P}\right)_T = \left(\frac{\partial \ln \nu_0}{\partial P}\right)_T - \beta_V\left(\frac{\partial \ln T_c}{\partial P}\right)\left(\frac{\partial \ln t}{\partial \ln T}\right)_V \quad , \tag{4}$$

where $\beta_V = (\partial \ln \nu_T/\partial \ln t)_V$, in agreement with Eq. (1).

Equation (4) is now substituted into Eq. (2). The constant volume of the system is defined to be that at the experimental value of T_c under a pressure of 1 atm and the equation is integrated over the range of temperature T_1 to T_2 for which the measurements were taken to be in the critical region at constant pressure. Since at constant volume $(T_1 - T_2) = T_c(t_2 - t_1)$ the equation becomes

$$\ln\left(\frac{\nu_2}{\nu_1}\right)_V = \ln\left(\frac{\nu_2}{\nu_1}\right)_P - B_T\gamma_T T_c(t_2 - t_1)\left[\left(\frac{\partial \ln \nu_0}{\partial P}\right) - \beta_V\left(\frac{\partial \ln T_c}{\partial P}\right)\right] - \beta_V B_T \gamma_T \left(\frac{\partial T_c}{\partial P}\right)\ln\left(\frac{t_2}{t_1}\right) \quad (5)$$

(for simplicity we ignore the weak-temperature dependence[2,5] of both B_T and γ_T for a ferromagnet of high T_c). On dividing throughout Eq. (5) by $\ln(t_2/t_1)$ the left-hand side is simply β_V and the first term on the right-hand side is equal to the experimental value of β_P found in I by ignoring the pressure dependence of T_c. Taking the range $2\times10^{-3} \leq t \leq 10^{-1}$ used in I for nickel the second term on the right-hand side is found to be $\simeq 10^{-4}$ in agreement with the conclusion in I that the direct effect of thermal expansion is negligible[6] but the third term gives

$$\beta_P = \beta_V\left[1 + B_T\gamma_T\left(\frac{\partial T_c}{\partial P}\right)\right] . \quad (6)$$

The experimental values for B_T and γ_T of nickel in the critical region are[2,5] 1.75×10^3 kbar and 5.5×10^{-5} K^{-1}, respectively, and the value of $\partial T_c/\partial P$ is[7] (0.35 ± 0.02) K kbar^{-1} so $\beta_V = 0.966\beta_P$.

The correction from β_P to β_V for nickel is therefore not $\sim10^{-3}$ as suggested in I but $\sim3\times10^{-2}$ which is of the same order as the estimated random error for β_P given in I. The correction for iron, however, is very small because $(\partial T_c/\partial P) \simeq 0 \pm 0.1$ K kbar^{-1} and is not known for cobalt since Patrick was only able to estimate a value of 0 ± 1 K kbar^{-1} for $(\partial T_c/\partial P)$.

It is clear that Eq. (6) can also be derived for the magnetization and hence the values of β_P and β_V found from magnetization and hyperfine-field measurements should be identical provided that bulk values of B_T and γ_T are appropriate to the hyperfine-field correction. This point is certainly open to question when the hyperfine field of an impurity in a ferromagnet is measured to find β_P although experiments[8] seem to support the view that the value found is independent of the impurity. One interesting example of an impurity for which β_P might be quite different from that of the host is Au in iron for which the hyperfine field has a most unusual temperature dependence at low temperature[9] that may be due to a local lattice mode at the Au site.

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