# Renormalized perturbation theory for anisotropic spin systems: Crossover behavior of the susceptibility

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A cutoff-dependent formulation of renormalized perturbation theory is extended to study the crossover of the parallel susceptibility in systems with quadratic symmetry breaking near a bicritical point, to first order in $\epsilon = 4-d$. In terms of the true temperature variable $t = T - T_c(g)$, for anisotropy $g$, and crossover variable $x = g/t^\phi$, the effective susceptibility exponent $\gamma_{\text{eff}}(t,x)$ is calculated for two $\phi^4$-field models, one with a sharp momentum cutoff $\Lambda$ and the other with a smooth term $\Lambda^{-2}(\nabla^2\phi)^2$. Corrections to the first model due to insertions of composite operators of canonical dimension six, inside the scaling regime, are explicitly calculated and shown to yield small but not negligible contributions to crossover behavior. The role of new, finite renormalizations is discussed and it is shown that either a fairly monotonic $\gamma_{\text{eff}}(t,x)$ is obtained or a displaced maximum may appear depending on the outcome of a higher-order renormalization-group study, which is not done here.

## I. INTRODUCTION

Crossover phenomena from high- to low-symmetry critical behavior in anistropic spin systems[1-4] have already been extensively studied by means of high-temperature series expansions[4,5] and renormalization-group (RG) procedures.[3,6-14] Besides the intrinsic interest in such systems, they are recognized to be relevant to spin-flop bicritical points that appear in materials like $GdAlO_3$ and $MnF_2$.[15,16] The appropriate RG procedures that have been employed for anisotropic spin systems have also been found useful for the study of crossover phenomena in dipolar ferromagnets.[17]

According to phenomenological crossover scaling theory,[1,4] the parallel susceptibility near a bicritical point, $(t_0,g)=(0,0)$, under crossover to a critical line with exponent $\dot{\gamma}$, is assumed to have the form

$$\chi(t_0,g) \approx t_0^{-\gamma} X(g/t_0^\phi) \approx t_0^{-\gamma}(1-y)^{-\dot{\gamma}} P(y) \ , \tag{1.1}$$

where $g$ is the anisotropy parameter,

$$t_0 = [T - T_c(0)]/T_c(0) \tag{1.2}$$

is the reduced critical temperature, in which $T_c(0) = T_c(g)$ at $g=0$, $\gamma$ and $\phi$ (crossover) being the exponents associated with the bicritical point and $y \equiv x/\dot{x}$ is a reduced crossover variable to the singularity in the scaling function $X(x)$ at $x=\dot{x}$. The universality of the scaling functions $X(x)$ and $P(y)$, except for nonuniversal constant amplitudes, has been verified to some extent, and so has that for the effective susceptibility exponent[18]

$$\gamma_{\text{eff}}(t) = \frac{d\ln\chi^{-1}}{d\ln t} \ , \tag{1.3}$$

which is of direct experimental interest, in terms of

$$\dot{t} = [T - T_c(g)]/T_c(0) \ . \tag{1.4}$$

Except for nonuniversal scale factors, one expects the shape of the curves for $\gamma_{\text{eff}}(\dot{t})$ to be universal, and it is of considerable importance to establish their precise shape. So far, there is only semiquantitative agreement, theoretically, that this shape is rather monotonic.

This has been established by Nelson and Domany,[10] using a RG trajectory-integral procedure,[19] combined with the matching of an effective-temperature-like parameter, to first order in $\epsilon = 4-d$, and also by Horner[12] and by Amit and Goldschmidt,[14] in renormalized perturbation theory (RPT), keeping selected terms beyond the first order in $\epsilon$. Calculations by Bruce and Wallace,[13] in a Feynman-diagram expansion matched onto RG results, to second order in $\epsilon$ and, independently, high-temperature series expansions, yield to a similar conclusion.[5]

With the aim of establishing more definite results, one may look for further contributions to crossover scaling functions and effective critical exponents due to "irrelevant" variables or operators of higher canonical dimension,[20-22] as well as corrections to the scaling regime itself due to a finite-lattice spacing. The precise role of the former has, to our knowledge, not been determined so far, although the operator

 

$\Lambda^{-2}[\nabla^2\phi(x)]^2$, of canonical dimension six in $\phi^4$-field theory with a momentum cutoff $\Lambda$, has been used before in works on the Gaussian-to-Heisenberg crossover,[13,23] where it yields contributions to crossover scaling functions *within* the scaling regime. Finite-momentum-cutoff corrections to the scaling regime itself have been obtained by Lawrie,[24] for the same type of crossover, and further contributions inside the scaling regime, due to the above operator, seem to be implicit in there.

With a finite-momentum cutoff, the bare dimensional quartic couplings in RPT remain finite and can have fixed points which are prevented from going to infinity. Deviations of the actual flowing quartic coupling from its stable fixed-point value give a measure of the extent to which the crossover takes place and, at the same time, provides the corrections to asymptotic critical behavior, within the scaling regime. The finite momentum cutoff can either be absorbed in *non*linear scaling fields[24] or else, in an essentially equivalent way, one can do without it by working directly with *dimensionless* quartic couplings and mass variables.[13,23]

The main purpose of the present work is to study the role of operators of canonical dimension six for the crossover in anisotropic spin systems, *within* the scaling regime. We will not be concerned here with the corrections to crossover scaling due to a finite-momentum cutoff, although that is certainly worthwhile studying. The work here is restricted to a very small anisotropy.[25]

Our second purpose is to extend a cutoff-dependent version of RPT, in terms of the "bare" parameters of the orginial Hamiltonian,[20,21,26] which has been employed before by Bruce and Wallace,[13] and by the author,[23] to study the Gaussian-to-Heisenberg crossover in isotropic spin systems. This is a RG procedure which, as that of Amit and Goldschmidt,[14] involves the flow of the actual temperature variable $t = T - T_c(g)$, and the quartic coupling $u$, which does not involve a matching of these parameters as in other works.[10] Moreover, it is simpler to work with than the renormalized theory with dimensional regularization and generalized minimal subtraction (GMS),[14] and, as will be seen, it yields directly contributions that amount to finite renormalizations in nonlogarithmic terms.

The work in this paper is restricted to one-loop order, that is, to first order in $\epsilon$. This is sufficient, as a first step, to treat some new features and to discuss a certain extent of arbitrariness that one meets in applying RPT to anisotropic spin systems.

The regularization in RPT with GMS consists in subtracting the leading singularity in the form of a logarithm, for infintiely large anisotropy, in addition to the subtraction of the usual dimensional pole.[14] This does not completely specify the argument of the logarithm, which amounts to a finite renormalization as discussed by Amit and Goldschmidt. There are further finite renormalizations behind the logarithms, with which GMS is not concerned, that can yield an additional change in the results as we point out in this work. Possible further finite renomalizations can be in the form of rational functions of polynomials in a scaled anisotropy variable which either vanish or remain finite for very large anisotropy. In order to have universality, the results should be independent of the regularization procedure and, it will be shown here, that this cannot be achieved to one-loop order. This is the third purpose of the paper.

Furthermore, in restricting ourselves to one-loop order, we point out a place where the calculations done so far in earlier work, may not be completely justified, and we work out the possible consequences. This is in solving the flow equation for the quartic coupling $u(\rho)$, $\rho$ being the flow parameter. The leading term is of $O(\epsilon)$ and the next-to-leading one of $O(\epsilon^2)$. It has beenfound in Ref. 14 that the full dependence on anisotropy enters only through $O(\epsilon^2)$. It has not been shown, however, that these terms cannot be modified by contributions from two-loop order. This is of concern since the solution for $u(\rho)$ is in terms of an exponentiated singularity of the scaling variable of the problem. We study here what happens if either only the leading terms in $u(\rho)$ or the complete form is kept, both in our version of RPT. The latter seves to point out the role of the finite renormalization, referred to above.

Among all the operators of canonical dimension

$$\delta_A = \frac{p}{2}(d-2) + q = p + q = 6 \quad \text{at } d = 4 \ , \tag{1.5}$$

where $p$ is the number of fields on which $q$ derivatives act, the operator $\Lambda^{-2}(\nabla^2\phi)^2$ can be used to implement a smooth cutoff when included in the propagator.[24,27] Its role in this sense is compared in this work, first, against the conventional sharp cutoff.[22] It will be seen that it can yield quite different results before the asymptotic critical region is reached. This is an interesting result because a smooth cutoff has been found convenient for higher-order calculations in $\epsilon$. Then, separately, we consider $\Lambda^{-2}(\nabla^2\phi)^2$ and the other operators with $\delta_A = 6$ as insertions to the model with a sharp cutoff, to first order in the insertion coupling. Part of the work has been reported briefly before.[28]

The outline of the paper is the following. In Sec. II we discuss the models and their relationship to those of other authors. The smooth cutoff term is included there. In Sec. III the RG equations for the bare, one-particle irreducible (1PI) vertex functions are derived and their solutions are discussed. The role of terms that amount to finite renormalizations is pointed out here. The explicit results for the effective susceptibility exponent are obtained in Sec. IV. Also, the comparison with the results of RPT with GMS is done there. The insertion of composite operators of canonical dimension six and their lowest-order contribution is discussed in Sec. V. We conclude in Sec. VI with a further discussion and a summary of our results.

## II. MODELS WITH QUADRATIC ANISOTROPY

The effective Ginzburg-Landau-Wilson Hamiltonian with quadratic spin anisotropy that we consider first is

$$\mathcal{H}_1 = \int d^dx \left\{ \tfrac{1}{2}(\nabla\phi)^2 + \tfrac{1}{2}\tilde{m}^2\phi_2^2 + \tfrac{1}{2}t\phi^2 + \frac{1}{4!}u\Lambda^{\epsilon}(\phi^2)^2 \right\} , \tag{2.1}$$

where $\phi$ is an $n$-component field, with $m$ "longitudinal" and $(n-m)$ "transverse" components, $\phi_1$ and $\phi_2$, such that $\phi^2 = \phi_1^2 + \phi_2^2$ with

$$\phi_1^2 = \sum_{i=1}^{m} \phi_i^2(x), \quad \phi_2^2 = \sum_{i=m+1}^{n} \phi_i^2(x) \ . \tag{2.2}$$

The *non*critical bare mass $\tilde{m}$, which is a measure of the anisotropy, to be made more explicit below, is defined as

$$\tilde{m}^2 \equiv r_T(r_L = 0) \ , \tag{2.3}$$

$r_L$ and $r_T$ being the true longitudinal and transverse inverse susceptibilities, with $r_L = 0$ at criticality for the longitudinal components. The temperature variable is

$$t = T - T_c(g) \tag{2.4}$$

and the dimensionless quartic coupling $u$ is assumed to be isotropic. This is justified for $n < n_x(d)$,[20] the crossover spin dimensionality to cubic anisotropy, and since $n_x(d) > 3$,[29] this will cover the crossover involving three-component Heisenberg fields. Momentum-space integrations in perturbation expansion for $\mathcal{H}_1$ are done with the sharp cutoff $\Lambda$ in

$$\int_{\vec{q}}^{\Lambda} \equiv (2\pi)^{-d}\int d^dq = K_d \int_0^{\Lambda} dq\, q^{d-1} \ , \tag{2.5}$$

$$K_d = 2^{1-d}\pi^{-d/2}[\Gamma(\tfrac{1}{2}d)]^{-1} \ . \tag{2.6}$$

This completes the description of the model which, except for the cutoff, is the counterpart in the bare theory of the renormalized Hamiltonian of Amit and Goldschmidt,[14] with $\phi$, $\tilde{m}^2$, $u$, and $t$ being replaced there by renormalized quantities, and the role of the cutoff in the dimensional $\phi^4$ coupling being taken by an arbitrary momentum scale parameter.

The free-field longitudinal and transverse propagators for this model are

$$G_1^L(q) = (t+q^2)^{-1} \quad \text{for } q \leq \Lambda \ , \tag{2.7}$$

$$G_1^T(q) = (t+\tilde{m}^2+q^2)^{-1} \quad \text{for } q \leq \Lambda \ , \tag{2.8}$$

and zero otherwise.

With Eq. (2.1) written

$$\mathcal{H}_1 = \int d^dx \left\{ \tfrac{1}{2}(\nabla\phi)^2 + \tfrac{1}{2}r_0\phi^2 - \frac{g}{2n}[(n-m)\phi_1^2 - m\phi_2^2] + \frac{1}{4!}u\Lambda^{\epsilon}(\phi^2)^2 + \mathcal{C} \right\} \tag{2.9}$$

(where $\mathcal{C}$ represents counter terms) the explicit part becomes the more familiar Hamiltonian of Nelson and Domany,[10] where $r_0$ is linear in the temperature $t_0$ of Eq. (1.2) and $g$ is the anisotropy parameter which favors criticality of the longitudinal components if $g > 0$, and that of the transverse components if $g < 0$. The counter terms are

$$\mathcal{C} = \tfrac{1}{2}\left[ \left( t + \frac{1}{n}g(n-m) - r_0 \right)\phi_1^2 + \left( \tilde{m}^2 + t - \frac{1}{n}gm - r_0 \right)\phi_2^2 \right] \ . \tag{2.10}$$

The second model we wish to consider makes use of the operator $\Lambda^{-2}(\nabla^2\phi)^2$ to provide for a smooth momentum cutoff. For that purpose, this is taken together with the quadratic part of the Hamiltonian $\mathcal{H}_1$, to define the model described by

$$\mathcal{H}_2 = \int d^dx \left\{ \tfrac{1}{2}(\nabla\phi)^2 + \tfrac{1}{2}\tilde{m}^2\phi_2^2 + \tfrac{1}{2}t\phi^2 + \tfrac{1}{2}\Lambda^{-2}(\nabla^2\phi)^2 + \frac{1}{4!}u\Lambda^{\epsilon}(\phi^2)^2 \right\} , \tag{2.11}$$

in which

$$(\nabla^k\phi)^2 = \sum_{i=1}^{n}\sum_{j=1}^{d} [\nabla_j^k\phi_i(x)]^2 \tag{2.12}$$

and with the sharp momentum cutoff removed in

$$\int_{\vec{q}} \equiv (2\pi)^{-d}\int d^dq = K_d \int_0^{\infty} dq\, q^{d-1} \ . \tag{2.13}$$

The smooth cutoff is now ensured by the new free-field propagators

$$G_2^L(q) = (t+q^2+\Lambda^{-2}q^4)^{-1} \ , \tag{2.14}$$

$$G_2^T(q) = (t+\tilde{m}^2+q^2+\Lambda^{-2}q^4)^{-1} \ , \tag{2.15}$$

for all $q$.

For the model based on $\mathcal{H}_1$, the longitudinal and transverse 1PI two-point vertex functions can be written formally as

$$\Gamma_L^{(2)}(q;r_L,r_T) = r_L + q^2 - [\Sigma_L(q;r_L,r_T) - \Sigma_L(0;r_L,r_T)] \ , \tag{2.16}$$

$$\Gamma_T^{(2)}(q;r_L,r_T) = r_T + q^2 - [\Sigma_T(q;r_L,r_T) - \Sigma_T(0;r_L,r_T)] \ , \tag{2.17}$$

in terms of the self-energy parts, and similarly for $\mathcal{H}_2$. For a fixed anisotropy $g > 0$, the inverse susceptibilities can be related to the actual critical temperature by means of

$$\begin{aligned} t &\equiv T - T_c(g) \\ &= r_L + [\Sigma_L(r_L, r_T; g) - \Sigma_L(0, \tilde{m}^2; g)] \\ &= r_T - \tilde{m}^2 + [\Sigma_T(r_L, r_T; g) - \Sigma_T(0, \tilde{m}^2; g)] \ , \end{aligned} \tag{2.18}$$

with the self-energy parts at $q = 0$. It also follows that the noncritical mass squared, $\tilde{m}^2$, is related to $g$ by means of

$$\tilde{m}^2 = g + \Sigma_L(0, \tilde{m}^2; g) - \Sigma_T(0, \tilde{m}^2; g) \ . \tag{2.19}$$

Thus, to leading order, $\tilde{m}^2 = g + O(u)$.

## III. RENORMALIZATION-GROUP EQUATIONS

First we discuss the nature of the RG equations and the way in which spin anisotropy enters, as a necessary background for the calculations that follow.

The RG equations for the bare 1PI vertex functions are derived from the requirement that, at each finite order in perturbation theory, the *renormalized* vertex functions be asymptotically independent of the cutoff,[20,21,26]

$$\Lambda \frac{d}{d\Lambda}\bigg|_R \Gamma_{\text{ren}}^{(N)}(q_i; u_R, t_R, m_R, \mu) \simeq 0 \ , \tag{3.1}$$

with the renormalized parameters $u_R, m_R$, and the arbitrary mass parameter $\mu$ being held fixed. This is an approximate statement from which we find, in standard way,[20,23] that

$$\left\{ \Lambda \frac{\partial}{\partial \Lambda} + \beta\left(u, \frac{\tilde{m}}{\Lambda}\right) \frac{\partial}{\partial u} - \left[ \gamma_4\left(u, \frac{\tilde{m}}{\Lambda}\right) - \gamma_3\left(u, \frac{\tilde{m}}{\Lambda}\right) \right] t \frac{\partial}{\partial t} - \tfrac{1}{2} N \gamma_3\left(u, \frac{\tilde{m}}{\Lambda}\right) \right\} \Gamma^{(N)}(q_i; u, t, \tilde{m}, \Lambda) = 0 \ , \tag{3.2}$$

for the *bare* vertex function $\Gamma^{(N)}(q_i; u, t, \tilde{m}, \Lambda)$, and the approximation consists in neglecting terms that will be smaller by an order $t/\Lambda^2$ and $q^2/\Lambda^2$, up to powers of $\ln(t/\Lambda^2)$ and $\ln(q/\Lambda)$, in an expansion in powers of $u$ and $\epsilon$, for fixed but arbitrary $\tilde{m}/\Lambda$. Although corrections to scaling due to a finite-momentum cutoff are excluded thereby, this is in accordance with the usual approximation that defines the critical region, and it is all we need for the purpose of our work. The explicit dependence on $\tilde{m}/\Lambda$ in the coefficient functions $\beta(u, \tilde{m}/\Lambda)$, $\gamma_3(u, \tilde{m}/\Lambda)$, and $\gamma_4(u, \tilde{m}/\Lambda)$ is necessary to ensure the crossover in the parameters $u$ and $t$ that satisfy the characteristic equations

$$\Lambda \frac{du}{d\Lambda}\bigg|_R = \beta(u, \tilde{m}/\Lambda) \ , \tag{3.3}$$

$$\frac{\Lambda}{t} \frac{dt}{d\Lambda}\bigg|_R = -[\gamma_4(u, \tilde{m}/\Lambda) - \gamma_3(u, \tilde{m}/\Lambda)] \ . \tag{3.4}$$

The independence of the coefficient functions on $t/\Lambda^2$ is due to the fact that the renormalization is done for the critical theory and that the combination of $\gamma_4(u, \tilde{m}/\Lambda) - \gamma_3(u, \tilde{m}/\Lambda)$ is the coefficient of $L$ in the RG equation for $\Gamma^{(N,L)}(q_i, p_j; u, t = 0, \tilde{m}, \Lambda)$, the bare vertex function with $N$ external lines and $L$ $\phi^2$ insertions.[23] The explicit forms of the coefficient functions, which are model dependent, follow from perturbation expansions for $\Gamma^{(N)}$. Within the calculations in this paper, to one-loop order, there is no contribution to $\gamma_3$ and it is sufficient to consider the simpler RG equations

$$\left\{ \Lambda \frac{\partial}{\partial \Lambda} + \beta(u, \tilde{m}/\Lambda) \frac{\partial}{\partial u} - \gamma_4(u, \tilde{m}/\Lambda) t \frac{\partial}{\partial t} \right\} \Gamma^{(N)}(q, u, t, \tilde{m}, \Lambda) = 0 \tag{3.5}$$

for the two- and four-point vertex functions at $q = 0$.

The integration of Eqs. (3.5), from which the physically interesting quantities are obtained, is done introducing first a scale parameter $\lambda = \ln(\Lambda/\Lambda_0)$ that accounts for a change in the cutoff, from an initial $\Lambda_0$, with $\lambda < 0$.[13,23] Then $u$ and $t$ become $u(\lambda)$ and $t(\lambda)$, where $u = u(0)$ and $t = t(0)$, in terms of which

$$\frac{du(\lambda)}{d\lambda} = \beta(u(\lambda), \tilde{m}/\Lambda_0 e^{\lambda}) \ , \tag{3.6}$$

$$\frac{1}{t(\lambda)} \frac{dt(\lambda)}{d\lambda} = -\gamma_4(u(\lambda), \tilde{m}/\Lambda_0 e^{\lambda}) \ , \tag{3.7}$$

to one-loop order. Equation (3.5) becomes then

$$\frac{d}{d\lambda} \Gamma^{(N)}(0; u(\lambda), t(\lambda), \tilde{m}(\lambda), \Lambda_0 e^{\lambda}) = 0 \ , \tag{3.8}$$

where

$$\tilde{m}(\lambda) \equiv \tilde{m}/\Lambda_0 e^{\lambda} \tag{3.9}$$

and this yields, of course,

$$\Gamma^{(N)}(0; u, t, \tilde{m}, \Lambda_0) = \Gamma^{(N)}(0; u(\lambda), t(\lambda), \tilde{m}(\lambda), \Lambda_0 e^{\lambda}) \ . \tag{3.10}$$

The vertex function in the critical region for arbitrary $u$ is then obtained, as usual, calculating the right-hand side, with an appropriately chosen $\lambda$ out of the critical region so that a perturbation expansion can be used.

In the subsections that follow and in Sec. IV a single-vertex function will be used in two different ranges of the parameters. One is to determine the coefficient functions within the validity of the approximate RG equations, discussed in connection with Eqs. (3.1) and (3.2). The other is to provide an expression for the right-hand side of Eq. (3.10).

### A. RG equations for model $\mathcal{H}_1$

The first line of Eq. (2.18) yields, with the usual subtraction of the self-energy parts at criticality, and noting that $r_T=\tilde{m}^2+t$, to leading order,

$$\Gamma_L^{(2)}(0;u,t,\tilde{m},\Lambda)=r_L=t\left[1-\frac{m+2}{6}u\Lambda^\epsilon\int_q^\Lambda\frac{1}{q^2(q^2+t)}-\frac{n-m)}{6}u\Lambda^\epsilon\int_q^\Lambda\frac{1}{(q^2+\tilde{m}^2)(q^2+\tilde{m}^2+t)}\right]+O(u^2)\ . \tag{3.11}$$

Also, to one-loop order,

$$\Gamma_L^{(4)}(0;u,t,\tilde{m},\Lambda)=u\Lambda^\epsilon\left[1-\frac{m+8}{6}u\Lambda^\epsilon I(t)-\frac{n-m}{6}u\Lambda^\epsilon I(\tilde{m}^2+t)\right]+O(u^2)\ , \tag{3.12}$$

where

$$I(m^2)\equiv\int_q^\Lambda\frac{1}{(q^2+m^2)^2}\ . \tag{3.13}$$

Explicit calculation then gives

$$\Gamma_L^{(2)}(0;u,t,\tilde{m},\Lambda)\simeq t\{1+\tfrac{1}{6}u[(m+2)\ln\tau+(n-m)f_1(\mu^2,\tau)]\}+O(u^2,u\epsilon)\ , \tag{3.14}$$

$$\Gamma_L^{(4)}(0;u,t,\tilde{m},\Lambda)\simeq u\Lambda^\epsilon\{1+\tfrac{1}{6}u[(m+8)(\ln\tau+1)+(n-m)f_2(\mu^2,\tau)]\}+O(u^3,u^2\epsilon)\ , \tag{3.15}$$

$$\mu^2\equiv\tilde{m}^2/\Lambda^2\ , \tag{3.16}$$

the integrations being done at $d=4$ with a factor of $\frac{1}{2}K_4$, defined in Eq. (2.6), adsorbed in $u$, while

$$\tau\equiv t/\Lambda^2\ll 1\ . \tag{3.17}$$

Contributions that vanish as $t\to 0$ are neglected in the first terms in square brackets behind $\ln\tau$, in consistency with the validity of Eq. (3.2), as in previous works with $O(n)$ symmetry.[30] The functions

$$f_1(\mu^2,\tau)=-\frac{1}{\tau}[(\mu^2+\tau)\ln(1+\mu^2)-\mu^2\ln(1+\mu^2)+\mu^2\ln\mu^2-(\mu^2+\tau)\ln(\mu^2+\tau)]\ , \tag{3.18}$$

$$f_2(\mu^2,\tau)=\ln\frac{\mu^2+\tau}{1+\mu^2}+\frac{1}{1+\mu^2} \tag{3.19}$$

are finite functions of $\tau$, for *nonzero* $\mu^2$, with limiting forms

$$f_1(\mu^2,0)=\ln\frac{\mu^2}{1+\mu^2}\ ,\qquad f_2(\mu^2,0)=\ln\frac{\mu^2}{1+\mu^2}+\frac{1}{1+\mu^2}\ . \tag{3.20}$$

On the other hand, for vanishing $\mu^2$,

$$\lim_{\mu^2\to 0}f_1(\mu^2,\tau)\simeq\ln\tau\ , \tag{3.21}$$

$$\lim_{\mu^2\to 0}f_2(\mu^2,\tau)\simeq\ln\tau+1\ . \tag{3.22}$$

In this case one recovers the vertex functions for an isotropic system with $O(n)$ symmetry. As $\mu^2\to\infty$, from Eqs. (3.18) and (3.19) it can be seen that $f_1(\mu^2,\tau)$ and $f_2(\mu^2,\tau)\to 0$, and one is left with the vertex functions for the isotropic case of $O(m)$ symmetry. The full $\tau$-dependent $f_1(\mu^2,\tau)$ and $f_2(\mu^2,\tau)$ are needed for the right-hand side of Eq. (3.10), which is to be calculated with a $\tau$ of $O(1)$. Indeed, $\lambda$ will be chosen so that[13,23]

$$\tau(\lambda)\equiv t(\lambda)/\Lambda_0^2e^{2\lambda}=1\ , \tag{3.23}$$

and this eliminates the terms in $\ln\tau$. In calculating the coefficient functions, however, it turns out that only the limiting forms Eq. (3.20) contribute, providing a check on their independence of $\tau$. Indeed, the RG Eqs. (3.5) yield

$$\beta(u,\mu)=-\epsilon u+\tfrac{1}{3}\left[m+8+\frac{n-m}{(1+\mu^2)^2}\right]u^2+O(u^3,u^2\epsilon)\ , \tag{3.24}$$

$$\gamma_4(u,\mu)=-\tfrac{1}{3}\left[m+2+\frac{n-m}{(1+\mu^2)}\right]u+O(u^2,u\epsilon)\ . \tag{3.25}$$

The first one follows from $\Gamma_L^{(4)}$ in Eq. (3.15) and the second from $\Gamma_L^{(2)}$ in Eq. (3.14). Note that, in the limits $\mu^2\to 0$ and $\infty$ one recovers the known results, to one-loop order, for a system with $O(n)$ and $O(m)$ symmetry, respectively.

At this point, some comments are in order. First, note that Eqs. (3.14) and (3.18) yield a longitudinal susceptibility, defined in Eq. (3.11), which is formally the same as the form derived by Amit and Goldschmidt for the renormalized theory with GMS, in terms of the renormalization constant $Z_{\phi^2}$ that makes finite the two-point function $\Gamma_L^{(2,1)}$, with one $\phi^2$ insertion.[31] Our $\gamma_4$ in Eq. (3.25) is, accordingly, the same as their $\gamma^{(1)}_{\phi^2}$. On the other hand, we cannot compare our four-point function with the form derived in Ref. 14, since the latter involves only $\Gamma_L^{(4)}$ *at* criticality. We expect, however, that if calculated for *non*zero $\tau$, $\Gamma_L^{(4)}$ in RPT with the GMS defined in that work will yield a result different from Eqs. (3.15) and (3.19). Indeed, our $\beta(u,\mu)$ differs from theirs in the quadratic dependence on $(1+\mu^2)^{-1}$. It can easily be seen that *if* the constant terms behind the logarithms in Eqs. (3.15) and (3.19) would not be there, $\beta(u,\mu)$ would turn out to depend *linearly* on $(1+\mu^2)^{-1}$, in agreement with GMS. In our form of RPT, however, we have no reason to neglect such terms. The approximation that is made in the RG statement of Eq. (3.1) is only the neglect of terms that are smaller by a *power* of $\tau$—or a power times a logarithm—and the presence of the constant terms is needed to derive the appropriate coefficent functions. This can be verified within the work of Bruce and Wallace[13] for the Gaussian-to-Heisenberg crossover.

According to Eq. (3.3), $\beta(u,\mu)$ serves to determine the flow equation for $u(\rho)$ which may, therefore, be expected to be different from that in the renormalized theory with GMS, for any finite $\mu^2$. Everything else being the same in the two-point vertex $\Gamma_L^{(2)}$, the two theories could apparently yield different results for $\gamma_{\text{eff}}(t)$. Note, however, that the difference between a linear and a quadratic dependence on $(1+\mu^2)^{-1}$, in the renormalized theory with GMS, can be accounted for by a finite renormalization in the terms of $O(\epsilon^2)$ in the expansion of the bare quartic coupling $u_0$ in terms of the renormalized $u$. This has not been discussed by Amit and Goldschmidt and it is clearly a limitation of GMS. The only requirement on a finite renormalization is that it vanishes in the limit of the scaled variable $\mu^2=m^2/k^2\to\infty$. To verify the independence of the results of RPT on such a finite renormalization, in order to confirm universality, one may have to carry out a systematic calculation to $O(\epsilon^2)$.

As a summary to these comments, we conclude that the bare theory of the present work corresponds to a renormalized theory in which the two-point function is made finite by GMS while the four-point function has an added finite renormalization. There is nothing in the renormalized theory that excludes such an unsymmetric renormalization.

The zeros of the $\beta$ function,

$$\beta(u^*,\mu)=0 \tag{3.26}$$

are $u^*=0$ and the nontrivial

$$u^*=3\epsilon\bigg/\left[m+8+\frac{n-m}{(1+\mu^2)^2}\right] . \tag{3.27}$$

The flow of $u(\lambda)$ under crossover, however, is described by Eq. (3.6) with $\mu$ being replaced by

$$\mu(\lambda)\equiv\tilde{m}(\lambda)=\tilde{m}/\Lambda_0 e^{\lambda} \tag{3.28}$$

and this becomes very large for large negative $\lambda$, even with a small initial anisotropy $\mu=\tilde{m}/\Lambda_0$, so that ultimately the true fixed point of the $O(m)$ isotropic system,

$$u_m^*=3\epsilon/(m+8) \tag{3.29}$$

is reached. Of course, if $\mu^2$ is strictly zero, the true nontrivial fixed point is just the

$$u_n^*=3\epsilon/(n+8) \tag{3.30}$$

for an isotropic system with $O(n)$ symmetry.

To obtain the flow of the coupling $u(\lambda)$ that goes into Eq. (3.10) we integrate Eq. (3.6) in standard way[14,21] and find

$$u^{-1}(\rho)=\rho^{\epsilon}u^{-1}+\frac{m+8}{3\epsilon}(1-\rho^{\epsilon}) -\frac{n-m}{6}\rho^{\epsilon}\int_1^{\rho^2}dx\,\frac{x^{-\epsilon/2+1}}{(x+\tilde{m}^2/\Lambda^2)^2} , \tag{3.31}$$

where

$$\rho\equiv e^{\lambda} . \tag{3.32}$$

According to the discussion following Eq. (3.24), this differs essentially from the $u^{-1}(\rho)$ in Ref. 14 in the form of the last term. For small, but finite $\epsilon$, this yields

$$u^{-1}(\rho)=\frac{n+8}{3\epsilon}-\frac{n-m}{6}\left[\frac{\mu^2/\rho^2}{1+\mu^2/\rho^2}+\ln(1+\mu^2/\rho^2)\right]+\rho^{\epsilon}\left[u^{-1}-\frac{n+8}{3\epsilon}+\frac{n-m}{6}\left[\frac{\mu^2}{1+\mu^2}+\ln(1+\mu^2)\right]\right] , \tag{3.33}$$

for $\mu\le\rho\le 1$, $\mu^2$ being given by $\mu^2=\tilde{m}^2/\Lambda^2$, while

$$u^{-1}(\rho)=\frac{m+8}{3\epsilon}-\frac{n-m}{6}\left[-\frac{\rho^2/\mu^2}{1+\rho^2/\mu^2}+\ln(1+\rho^2/\mu^2)\right]+\rho^{\epsilon}\left[u^{-1}-\frac{n+8}{3\epsilon}+\frac{n-m}{6}\left[\frac{\mu^2}{1+\mu^2}+\ln(1+\mu^2)\right]\right] +\frac{n-m}{3\epsilon}\left[\frac{\rho^2}{\mu^2}\right]^{\epsilon/2}-\frac{n-m}{6}\left[\frac{\rho^2}{\mu^2}\right]^{\epsilon/2} \tag{3.34}$$

for $\rho \leq \mu$.

The function $u^{-1}(\rho)$ is continuous and the two forms match at $\mu=\rho$ into a single expression. The next-to-leading terms in $u^{-1}(\rho)$ are of $O(1)$, which means of $O(\epsilon^2)$ in $u(\rho)$. It will be seen further down that when these terms are included a fairly monotonic $\gamma_{\text{eff}}$ is obtained, as was the case in Ref. 14. Note, however, that in distinction to that work, we have additional *non*logarithmic terms which one does not have in GMS.

In the limit of small $\epsilon$, we obtain a single expression for all nonzero $\rho$,

$$u^{-1}(\rho)=\rho^{\epsilon}u^{-1}+\frac{m+8}{3\epsilon}(1-\rho^{\epsilon}) - \frac{n-m}{6}\left[\frac{\mu^2/\rho^2}{1+\mu^2/\rho^2}+\ln(\mu^2+\rho^2)\right] + \frac{n-m}{6}\rho^{\epsilon}\left[\frac{\mu^2}{1+\mu^2}+\ln(1+\mu^2)\right] . \tag{3.35}$$

This is a result in expansion in $\epsilon$, with a leading term

$$u^{-1}(\rho)=\rho^{\epsilon}u^{-1}+\frac{m+8}{3\epsilon}(1-\rho^{\epsilon}) , \tag{3.36}$$

and Eqs. (3.33) and (3.34) should be viewed rather as appropriate forms for extrapolation to $\epsilon=1$. Unless a two-loop-order calculation shows that the next-to-leading terms in Eq. (3.35) are not modified, Eq. (3.36) will give the only genuine contribution to one-loop order. Clearly, this does not account fully for the anisotropy, and the way one has to drive the crossover is by choosing $u=u_n^*$, the fixed-point value of the initial $n$-component system and let $\rho$ vary in the interval $(1,0)$. The consequences of using Eq. (3.36) against (3.33) and (3.34) will be worked out further down.

Finally, Eq. (3.7) may be integrated to yield

$$\ln[t(\rho)/t]=\tfrac{1}{3}\int_1^{\rho}\frac{dx}{x}\left[(m+2)+\frac{n-m}{1+\mu^2(x)}\right]u(x) , \tag{3.37}$$

with

$$\mu^2(x)\equiv\tilde{m}^2/x^2 \tag{3.38}$$

and $\Lambda_0=1$ (this will be done in all that follows). One does not need $t(\rho)/t$ to be given more explicitly than in Eq. (3.37) to get $\gamma_{\text{eff}}(t)$.

### B. RG equations for model $\mathcal{H}_2$

With the free-field propagators being now given by Eqs. (2.14) and (2.15), and the integrations done at $d=4$ in a calculation to one-loop order, we find

$$\Gamma_L^{(2)}(0;u,t,\tilde{m},\Lambda)\simeq t\{1+\tfrac{1}{6}u[(m+2)(\ln\tau+1) + (n-m)g_1(\mu^2,\tau)]\} + O(u^2,u\epsilon) , \tag{3.39}$$

$$\Gamma_L^{(4)}(0;u,t,\tilde{m},\Lambda)\simeq u\Lambda^{\epsilon}\{1+\tfrac{1}{6}u[(m+8)(\ln\tau+2) + (n-m)g_2(\mu^2,\tau)]\} + O(u^3,u^2\epsilon) , \tag{3.40}$$

with $\mu$ and $\tau$ defined as in Eqs. (3.16) and (3.17), and again neglecting contributions that vanish as $\tau\to 0$ in the first terms in square brackets. Now

$$g_1(\mu^2,\tau)=-\frac{1}{4\tau}\left[\ln\frac{\mu^2+\tau}{\mu^2}+L[4(\mu^2+\tau)] - L(4\mu^2)\right] , \tag{3.41}$$

$$g_2(\mu^2,\tau)=\frac{1}{1-4(\mu^2+\tau^2)}\{1-\tfrac{1}{2}L[4(\mu^2+\tau)]\} , \tag{3.42}$$

are finite functions of $\tau$ for nonzero $\mu^2$, in which

$$L(x)=(1-x)^{-1/2}\ln\{[1+(1-x)^{1/2}] \times[1-(1-x)^{1/2}]^{-1}\},\quad x<1$$
$$=2(x-1)^{-1/2}\tan^{-1}[(x-1)^{1/2}],\quad x>1 . \tag{3.43}$$

In the calculation of the coefficient functions only the limiting form

$$g(\mu^2)=g_1(\mu^2,0)=g_2(\mu^2,0) =\frac{1}{1-4\mu^2}[1-\tfrac{1}{2}L(4\mu^2)] \tag{3.44}$$

contributes but, as before, the general expressions will be kept for the integrated vertex functions. For this purpose, both forms of Eq. (3.43) are relevant, even if one starts with a weak initial anisotropy, $\mu^2 \ll 1$. Finally, it can be verified that, for vanishing $\mu^2$, $g_1(0,\tau)$ and $g_2(0,\tau)$ have the appropriate forms which lead to the vertex functions for the isotropic system with $O(n)$ symmetry.

The coefficient functions that are obtained by using Eqs. (3.39) and (3.40) in the RG Eqs. (3.5) are now

$$\beta(u,\mu)=-\epsilon u+\tfrac{1}{3}[m+8+(n-m)\phi(\mu^2)]u^2 + O(u^3,u^2\epsilon) , \tag{3.45}$$

$$\gamma_4(u,\mu)=-\tfrac{1}{3}[m+2+(n-m)\phi(\mu^2)]u+O(u^2,u\epsilon) \tag{3.46}$$

where

$$\phi(\mu^2)=(1-4\mu^2)^{-2}[8\mu^2+1-6\mu^2L(4\mu^2)] \quad . \tag{3.47}$$

It can easily be verified that the correct limiting forms are obtained for both $\mu^2\to 0$ and $\infty$.

Unfortunately, $\phi(\mu^2)$ is too complicated to make feasible the detailed calculation of $u(\rho)$ in the way we could do for model $\mathcal{H}_1$. However, we may replace $\phi(\mu^2)$ by an effective form $(1+\mu^2)^{-2}$ that leads to a $u^{-1}(\rho)$ which does not differ appreciably from $u^{-1}(\rho)$ obtained from the limiting forms of Eq. (3.47) for small and large $\mu^2$. The full anisotropy dependence of this effective form is that of the next-to-leading terms in Eqs. (3.33) and (3.34). It will be shown further down that this yields an approximately, rather monotonic $\gamma_{\rm eff}(t)$ for model $\mathcal{H}_2$. In contrast, we also work out the results keeping only the leading terms in the form given by Eq. (3.36). These are, clearly, also the leading terms for model $\mathcal{H}_2$, which may again be the only ones that should be kept to one-loop order.

Integration of Eq. (3.7) yields for model $\mathcal{H}_2$

$$\ln[t(\rho)/t]=\frac{1}{3}\int_1^\rho \frac{dx}{x}[(m+2)+(n-m)\phi(\mu^2(x))]u(x) \quad . \tag{3.48}$$

Both here and in Eq. (3.37), $\rho$ can be replaced to leading order by $t^{1/2}$.

The flow equations derived in this section will now be used in the next one to calculate the effective critical exponent $\gamma_{\rm eff}(t)$.

## IV. EFFECTIVE CRITICAL EXPONENT

The inverse longitudinal susceptibility,

$$\chi_L^{-1}(t,g)=\Gamma^{(2)}(0;u,t,\tilde{m}(g),1) \tag{4.1}$$

can now be written, by means of Eq. (3.10) as

$$\chi_L^{-1}(t,g)\simeq t(\rho)\{1+\tfrac{1}{6}u(\rho)[(m+2)+(n-m)f_1(\mu^2,\tau)|_{x-c}]\}+O(u^2,u\epsilon) \quad \text{for model } \mathcal{H}_1 \ , \tag{4.2}$$

$$\chi_L^{-1}(t,g)\simeq t(\rho)\{1+\tfrac{1}{6}u(\rho)[(m+2)+(n-m)g_1(\mu^2,\tau)|_{x-c}]\}+O(u^2,u\epsilon) \quad \text{for model } \mathcal{H}_2 \ , \tag{4.3}$$

with $t(\rho)$ given by Eqs. (3.37) and (3.48), $u(\rho)$ by Eqs. (3.33) and (3.34) or Eq. (3.36), while

$$f_1(\mu^2,\tau)|_{x-c}\equiv f_1(\mu^2,\tau) \ , \tag{4.4}$$

$$\mu^2\to\mu^2(\lambda)=\frac{\tilde{m}^2}{\rho^2}\simeq\frac{\tilde{m}^2}{t}$$

$$g_1(\mu^2,\tau)|_{x-c}\equiv g_1(\mu^2,\tau) \tag{4.5}$$

are the functions $f_1(\mu^2,\tau)$ and $g_1(\mu^2,\tau)$, given by Eqs. (3.18) and (3.41) out of the critical region, with $\tau\to\tau(\lambda)=1$ as in Eq. (3.23).

The effective susceptibililty exponent,

$$\gamma_{\rm eff}(t,x)=\frac{d\ln\chi_L^{-1}(t,g)}{d\ln t} \quad , \tag{4.6}$$

will depend on the crossover variable $x=g/t^\phi\simeq\tilde{m}^2/t=\mu^2/\rho^2$, and is given, to first order in $\epsilon$, by

$$\gamma_{\rm eff}(t,x)\simeq 1+\tfrac{1}{6}u(t^{1/2})\left[m+2+\frac{n-m}{(1+x)}+(n-m)\frac{d}{d\ln t}[f_1(\mu^2,\tau)|_{x-c}]\right] \quad \text{for model } \mathcal{H}_1 \ , \tag{4.7}$$

noting that Eq. (3.37) yields

$$\frac{d\ln t(\rho)}{d\ln t}\simeq 1+\tfrac{1}{6}t^{1/2}\frac{d}{dt^{1/2}}\int_1^{t^{1/2}}\frac{dx}{x}\left[m+2+\frac{n-m}{1+\mu^2(x)}\right]u(x) \tag{4.8}$$

and that $du(t^{1/2})/d\ln t=O(\epsilon^2)$.

Also, $d[f_1(\mu^2,\tau)|_{x-c}]/d\ln t$ turns out to depend only on the scaling variable $x$. Then we find, for model $\mathcal{H}_1$,

$$\gamma_{\rm eff}(t,x)=1+\tfrac{1}{6}u(t^{1/2})\left\{(m+2)+(n-m)\left[\frac{1}{1+x}+x\left[\ln\frac{x}{1+x}+\frac{1}{1+x}\right]\right]\right\} \quad , \tag{4.9}$$

where $u(t^{1/2})$ is $u(\rho)$ for $\rho=t^{1/2}$, to leading order.

To start with the right primary critical behavior when Eq. (3.36) is employed, but actually not necessary if instead we take Eqs. (3.33) and (3.34), for the full $u(\rho)$, the choice $u=u_n^*$ is made. It can easily be seen then that

$$\begin{aligned}\gamma_{\rm eff}(t,x)|_{t=1}&=\gamma_{\rm eff}(1,x)\simeq\gamma(n) \quad \text{as } x\simeq 0\\ &=1+(n+2)\epsilon/(n+8)+O(\epsilon^2) \ ,\end{aligned} \tag{4.10}$$

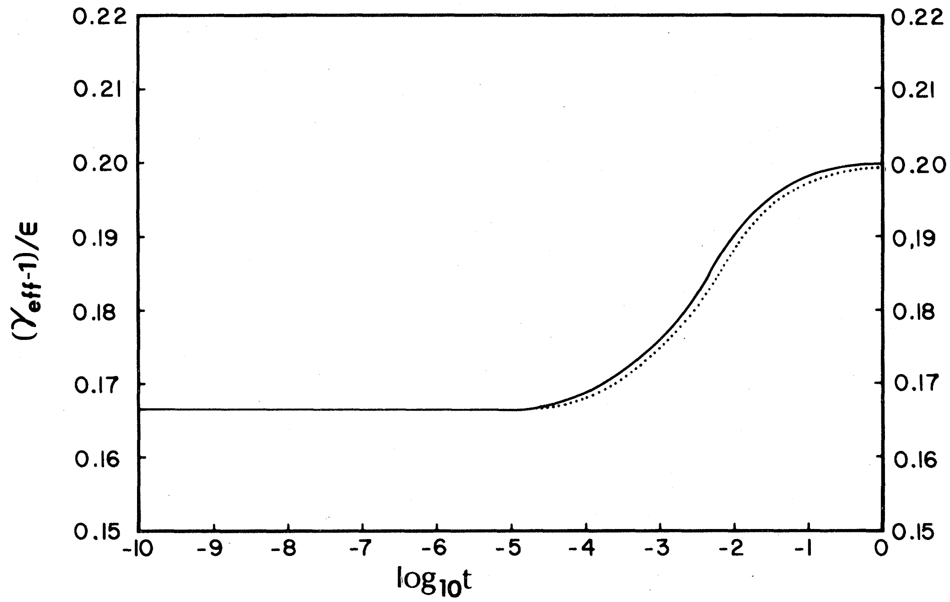


FIG. 1. Effective susceptibility exponent for model $\mathcal{H}_1$ (full curve) and approximate exponent—see text for model $\mathcal{H}_2$ (dotted line) when the flowing quartic coupling with next-to-leading terms is used for $n=2$, $m=1$, and anisotropy $\tilde{m}^2=10^{-3}$, when $\epsilon=1$.

while for small but fixed $\tilde{m}^2$ and vanishingly small $t$,

$$\gamma_{\mathrm{eff}}(t,x)\simeq\gamma_{\mathrm{eff}}(0,\infty)=\gamma(m)$$
$$=1+(m+2)\epsilon/(m+8)+O(\epsilon^2)\ , \qquad (4.11)$$

the susceptibility exponent for the secondary critical behavior.

The behavior of $\gamma_{\mathrm{eff}}(t,x)$ throughout the crossover region, for model $\mathcal{H}_1$, is shown in Figs. 1 and 2, the first making use of the full $u(\rho)$ given by Eqs. (3.33) and (3.34), while the latter uses the leading form in Eq. (3.36). Note that if $\epsilon$ is not too large

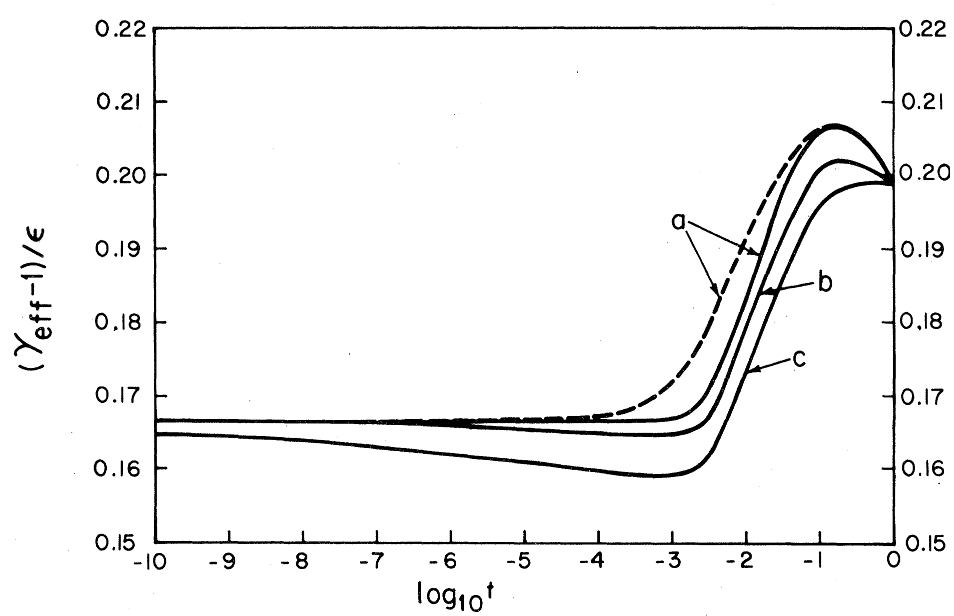


FIG. 2. Effective susceptibility exponent for model $\mathcal{H}_1$ (full curves) when only the leading term in $\epsilon$ for the flowing quartic coupling is kept for $n=2$, $m=1$, anisotropy $\tilde{m}^2=10^{-3}$, and $\epsilon=1$ (a), 0.5 (b), and 0.2 (c). The result for model $\mathcal{H}_2$ taken from Fig. 3, is shown for comparison (dashed line) when $\epsilon=1$.

($\epsilon < 0.5$), the latter reflects rather closely the monotonic crossover displayed in Fig. 1, but otherwise there could be a marked difference which results in a displaced maximum near the primary critical behavior. Without the results of a two-loop-order calculation at hand to ensure that the next-to-leading-order terms in Eqs. (3.33) and (3.34) remain unchanged, we cannot rule out the possibility that the maximum in $\gamma_{\rm eff}$ is really there, at least in a calculation to one-loop order.

On the other hand, when the curve of Fig. 1 is compared with the result shown in Ref. 14, one notes that they do not quite agree with each other. This is not surprising, in view of the role of nonlogarithmic terms, discussed in Sec. III.

The effective susceptibility exponent for model $\mathcal{H}_2$ that follows from Eq. (4.3) is given, to first order in $\epsilon$, by

$$\gamma_{\rm eff}(t,x) \simeq 1+\tfrac{1}{6}u(t^{1/2})\left[m+2+(n-m)\phi(x)+(n-m)\frac{d}{d\ln t}[g_1(\mu^2,\tau)|_{x-c}]\right] \quad (4.12)$$

in which $\phi(x)$ with the argument $\mu^2$ replaced by the crossover variable $x \simeq \tilde{m}^2/t$ is given by Eq. (3.47), while

$$\frac{d}{d\ln t}[g_1(\mu^2,\tau)|_{x-c}] = -\tfrac{1}{6}\left[\Sigma(y)-\frac{1}{1-y}[1-\tfrac{1}{2}yL(y)]\right] , \quad (4.13)$$

where $L(y)$ is given by Eq. (3.43) with $y=4x$, and

$$\Sigma(y) \equiv \frac{1}{y+3}[3+y(3+y)^{-1/2}\tan^{-1}(3+y)^{1/2}] . \quad (4.14)$$

Setting $u=u_n^*$ and using either Eq. (3.36) or the pair (3.33) and (3.34), if $\phi(x)$ is replaced by an effective $(1+x)^{-2}$, one can easily verify the limiting forms in Eqs. (4.10) and (4.11).

The behavior of $\gamma_{\rm eff}(t)$ for model $\mathcal{H}_2$ is shown in Figs. 1 and 3, the former with terms beyond the leading part in $u(\rho)$ and the latter with the use of Eq. (3.36). Figure 1 shows a rather close agreement between the results for a sharp and a smooth cutoff, for the case where the one-loop calculation does not become modified by a higher-order contribution. The two cutoff results are also compared in Fig. 2 for a leading $u(\rho)$, and they are rather similar. However, this is not always the case as illustrated in Fig. 4, where a rather pronounced dip is shown to appear with a *decrease* in anisotropy. Nothing similar occurs in the case of a sharp cutoff, as shown in the same figure, and this may be an indication that the operator $\Lambda^{-2}(\nabla^2\phi)^2$ when included in the usual propagator becomes a "dangerous irrelevant variable"[32] for very weak anisotropy. It will be seen in Sec. V that the

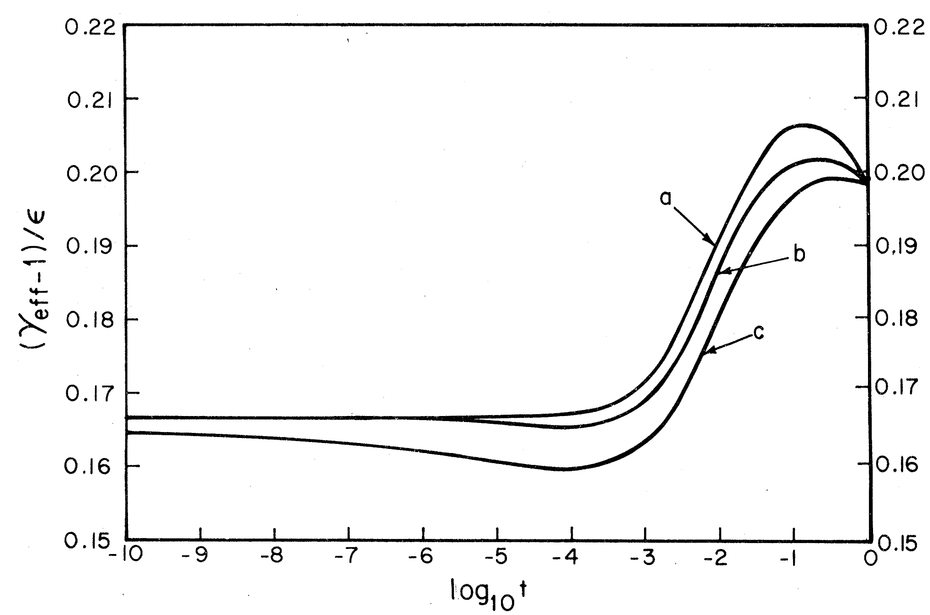


FIG. 3. Effective susceptibility exponent for model $\mathcal{H}_2$ when only the leading term in $\epsilon$ for the flowing quartic coupling is kept for $n=2$, $m=1$, anisotropy $\tilde{m}^2=10^{-3}$, and $\epsilon=1$ (a), 0.5 (b), and 0.2 (c).

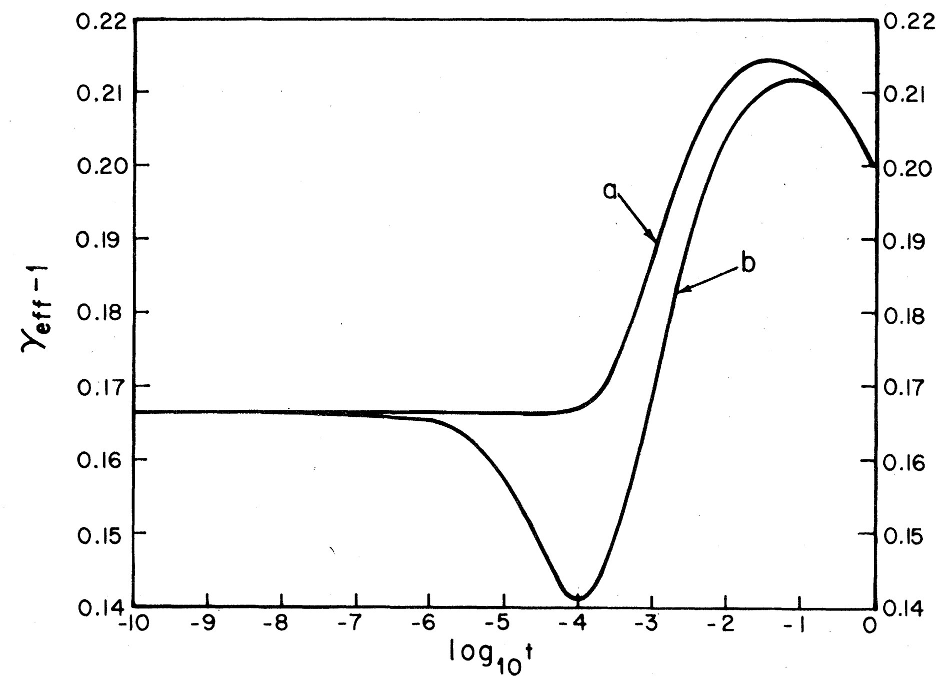


FIG. 4. Result of decreasing the anisotropy to $\bar{m}^2 = 10^{-4}$ in model $\mathcal{H}_2$ (b), as compared to model $\mathcal{H}_1$ (a), when the leading term in the flowing quartic coupling is used, for $n = 2$, $m = 1$, and $\epsilon = 1$.

situation is different as long as that operator is viewed as an insertion of higher canonical dimension into the effective Hamiltonian of model $\mathcal{H}_1$.

It is not the first time that a maximum in $\gamma_{eff}(t)$ is found. Indeed, it has already been reported by Amit and Goldschmit[14] that the use of a single expression for $u^{-1}(\rho)$, in the limit of small $\epsilon$, i.e., the analog of our Eq. (3.35) in the renormalized theory, is responsible for a rather pronounced maximum in $\gamma_{eff}(t)$ when the results are extrapolated to $\epsilon = 1$. However, there again next-to-leading terms are included and it would be interesting to see what comes out if instead

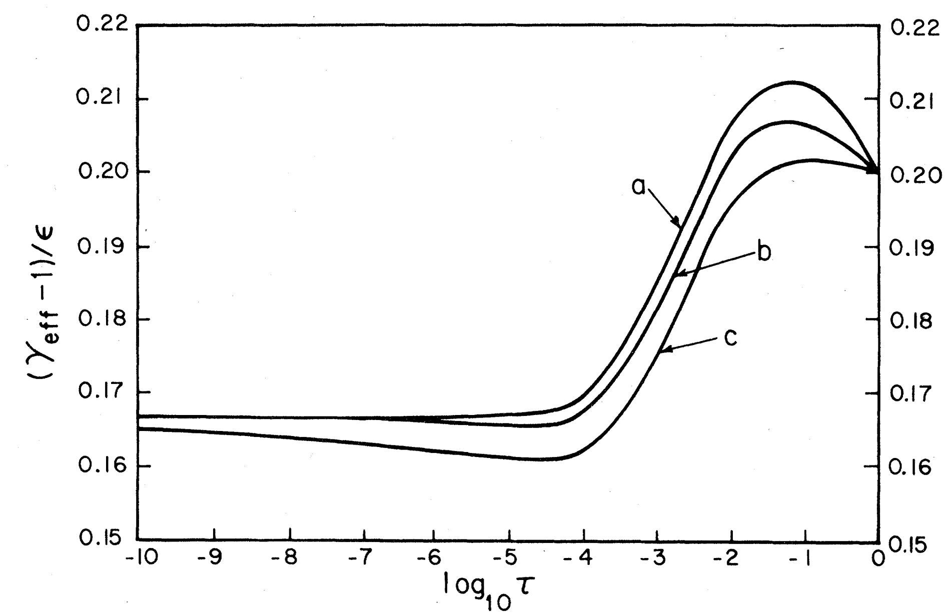


FIG. 5. Result of using the leading part in $\epsilon$ for the flowing quartic coupling in the renormalized theory with Eq. (4.15), for $n = 2$, $m = 1$, renormalized anisotropy $m_R^2 = 10^{-3}$, and $\epsilon = 1$ (a), 0.5 (b), and 0.2 (c).

only the form in Eq. (3.36) is kept. When this is used in their

$$\gamma_{\rm eff}(t,z) \simeq 1 + \tfrac{1}{6} u(\tau) \times \{ m + 2 + (n-m)[1 - z \ln(1+1/z)] \} , \quad (4.15)$$

in which

$$z \equiv m_R^2/\tau \quad (4.16)$$

is the crossover variable in the normalized theory, to leading order in $\epsilon$, $m_R^2$ is the square of the noncritical mass, and $\tau$ is the normalized reduced temperature, the curves shown in Fig. 5 are obtained for $m_R^2 = 10^{-3}$. Although the precise relationship between bare and renormalized noncritical mass is not known, the curves are rather close to our previous results and, moreover, in all of them the height of the maximum is much smaller than in Ref. 14, for $\epsilon = 1$.

Both here, and in the other results discussed above, for the sharp cutoff, there is rather close qualitative agreement with the renormalized theory, for small $\epsilon$. It should be noted that the results of both theories have been obtained only, so far, as expansions in $\epsilon$. Therefore, one should be cautious with expressions like Eq. (3.34), particularly when extended to $\epsilon = 1$—a similar one appears in the renormalized theory—containing an exponentiated form of the scaling variable $x = \mu^2/\rho^2$ in the last two terms. It is here where a further RG analysis is needed, or at least a calculation to two-loop order, together with a check that the other anisotropy-dependent terms remain unchanged.

## V. INSERTION OF COMPOSITE OPERATORS OF CANONICAL DIMENSION SIX

The operator $\Lambda^{-2}(\nabla^2\phi)^2$ is included in the unperturbed part of the Hamiltonian $\mathcal{H}_2$ only for the purpose of simulating a smooth cutoff. That is not what is usually referred to as an insertion of a composite operator of higher canonical dimension. Indeed, if $G_2^L(q)$ of Eq. (2.14) is expanded in powers of $\Lambda^{-2}q^4$, so that $\Lambda^{-2}(\nabla^2\phi)^2$ can be viewed as a perturbation to model $\mathcal{H}_1$, the first-order term is $-\Lambda^{-2}q^4/(t+q^2)^2$ and this already has the wrong sign, as will be seen next. Therefore, the propagator $G_2^L(q)$ cannot be taken as a resummation of the insertions of $\Lambda^{-2}(\nabla^2\phi)^2$.

Given a Hamiltonian $\mathcal{H}_0(\phi)$ in terms of the fields and their gradients and the set of all operators $\mathcal{O}_A$ of canonical dimension $\delta_A$, the new Hamiltonian to first order in the insertions is[20,21]

$$\mathcal{H}(\phi) = \mathcal{H}_0(\phi) + \Lambda^{-\delta} g \Sigma_A \mathcal{O}_A , \quad (5.1)$$

$g$ being the dimensionless coupling of the insertions, $\Lambda^{-\delta} g \ll 1$, and where

$$\delta \equiv \delta_A - d \quad (5.2)$$

is the (positive) degree of the composite operators $\mathcal{O}_A$. It will be assumed in what follows that $g = 1$, since no attempt will be made of including insertions to higher order. From Eq. (1.5) we have $\delta = 2$ at $d = 4$. The bare 1PI vertex functions for the Hamiltonian $\mathcal{H}$ can then be written, to first order in the insertions,[20,21]

$$\Gamma_{\mathcal{H}}^{(N)}(\vec{p}_1, \ldots, \vec{p}_N; \vec{q}; \mathcal{O}_A) \simeq \Gamma_{\mathcal{H}_0}^{(N)}(\vec{p}_1, \ldots, \vec{p}_N) + \Lambda^{-2}\Sigma_A \Gamma_A^{(N)}(\vec{p}_1, \ldots, \vec{p}_N; \vec{q}) , \quad (5.3)$$

$\Gamma_{\mathcal{H}_0}^{(N)}$ being the vertex functions for $\mathcal{H}_0$ and $\Gamma_A^{(N)}$ the vertex functions with insertions of operators $\mathcal{O}_A$, calculated for $\mathcal{H} = \mathcal{H}_0$. The momentum of the insertion is $\vec{q}$.

The set of all operators of canonical dimension six, denoted by $\mathcal{O}_{[\phi^6]}$, is

$$\mathcal{O}_{[\phi^6]}: \ \phi^6 ; \quad (\nabla^2\phi)\phi^3, \ \nabla^2\phi^4 ; \quad (\nabla^2\phi)^2, \ \phi\nabla^4\phi, \ \nabla^2(\phi\nabla^2\phi), \ \nabla^4\phi^2 . \quad (5.4)$$

As far as the operator $(\nabla^2\phi)^2$ is concerned, this yields a contribution of $\Lambda^{-2}q^4/(t+q^2)^2$ to the two-point vertex, in distinction to the first term in the expansion of the propagator $G_2^L(q)$, referred to above.

In what follows we take $\mathcal{H}_0(\phi)$ to be the Hamiltonian of model $\mathcal{H}_1$ and we are primarily interested in the two-point vertex function $\Gamma_{\mathcal{H}}^{(2)}$ under crossover. The RG equation for $\Gamma_A^{(N)}$, which has been used in the literature to determine the anomalous dimensions of composite operators of canonical dimension six, will not be needed here.[20,21] To lowest order in the insertion, $\Gamma_A^{(2)}$ can be calculated with the couplings of model $\mathcal{H}_1$, the flow of which have already been determined in Sec. III. It is only to second order in the insertions that the couplings have to be recalculated to first order and that a RG equation for $\Gamma_A^{(N)}$, $N = 2$ and 4, is needed.

Not all the operators in Eq. (5.4) contribute to one-loop order to

$$\Gamma_{\mathcal{H}}^{(2)}(0;\vec{q};\mathcal{O}_A)|_{\vec{q}=0} = \Gamma_{\mathcal{H}_1}^{(2)}(0) + \Lambda^{-2}\Sigma_A \Gamma_A^{(2)}(0;\vec{q})|_{\vec{q}=0} , \quad (5.5)$$

the quantity we are interested in, when all the external momenta, including the insertion, are set to zero. First, $\phi^6$ does not contribute to one-loop order.[22(a)] Next, the operators in the second line of Eq. (5.4) contribute only to $\Gamma_{\mathcal{H}}^{(4)}$, which is necessary to find the first-order (in $\Lambda^{-2}$) change in $u(\rho)$. This contributes to $\Gamma_{\mathcal{H}}^{(2)}$ only to second order in the insertion, and need not be considered here. The remaining four operators can contribute to $\Gamma_{\mathcal{H}}^{(2)}$, to first order. However, they produce additional factors $q_1^2 q_2^2$, $q_1^4 + q_2^4$, $q^2(q_1^2 + q_2^2)$, and $q^4$ to a pair of propagators or to a propagator-insertion pair, as shown in Fig. 6. In here, $q_i$ are the momenta of the propagators and $q$ is the momentum of the insertion. None of these graphs will contribute, with the restriction of zero momenta. The only nonzero contribution to one-loop order comes from the insertions to internal propagators, as shown in Fig. 7. Since the external momenta have to be zero, there is no distinction in this case between the last four operators in Eq. (5.4). Finally, the point of insertion carries a factor of $\Lambda^{-2}$.

The additional contributions to the longitudinal self-energy parts, to one-loop order, with insertions of internal longitudinal and transverse propagators, is proportional to the integrals

$$I_A(t) = \frac{1}{\Lambda^2} \int_{\vec{q}}^{\Lambda} \frac{q^4}{(q^2 + t)^2} \;, \tag{5.6a}$$

$$I_A(\tilde{m}^2 + t) = \frac{1}{\Lambda^2} \int_{\vec{q}}^{\Lambda} \frac{q^4}{(q^2 + \tilde{m}^2 + t)^2} \;. \tag{5.6b}$$

In the usual subtraction at $t=0$, the approximation

$$\frac{1}{\Lambda^2} \int_{\vec{q}}^{\Lambda} q^4 \left[ \frac{1}{(q^2+t)^2} - \frac{1}{q^4} \right] \simeq -\frac{2}{\Lambda^2} t \int_{\vec{q}}^{\Lambda} \frac{q^4}{q^2(q^2+t)^2} \tag{5.7}$$

is made in (5.6a), and similarly, in (5.6b). The additional contributions to $\Gamma_L^{(2)}(0;u,t,\tilde{m},\Lambda)$, Eq. (3.11),

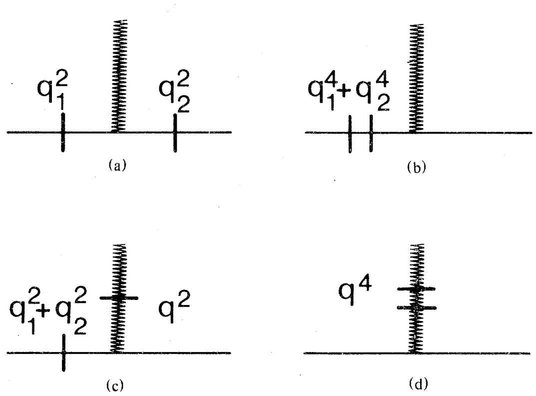


FIG. 6. Insertions of the last four operators of canonical dimension six in Eq. (5.4) to the leading part of $\Gamma_{\mathcal{H}_1}^{(2)}$. The wiggly line indicates the insertion point. Factors of $q_1^2 q_2^2$, indicated by dashes, go with (a), $q_1^4 + q_2^4$ with (b), $q^2(q_1^2 + q_2^2)$ with (c), and $q^4$ with (d), when $q_i$ are the momenta of the propagators and $q$ is the momentum of the insertion. These graphs do not contribute when all momenta are zero.

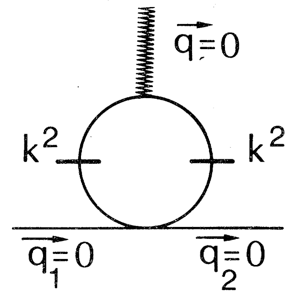


FIG. 7. One-loop contribution to the longitudinal susceptibility with one insertion of canonical dimension six at zero momentum. The only nonzero contribution is due to insertions at the momentum of the internal propagators, which can be either longitudinal or transverse.

are thus

$$\Lambda^{-2}\Sigma_A \Gamma_A^{(2)}(0;\vec{q})|_{\vec{q}=0} = -\frac{u\Lambda^{\epsilon}}{6} t[(m+4)J(0,\tau) + (n-m)J(\mu^2,\tau)] \;, \tag{5.8}$$

where

$$J(\mu^2,\tau) \equiv 2K_4 \int_0^1 dq\; q^3 \frac{q^4}{(q^2+\mu^2)(q^2+\mu^2+\tau)^2} \;, \tag{5.9}$$

in which the factor $\Lambda^{-2}$, associated with the insertion, has been absorbed, while $\mu^2$ and $\tau$ are defined as in Eqs. (3.16) and (3.17). Explicitly,

$$J(0,\tau) \simeq K_4 \left[ 1 + \frac{\tau}{1+\tau} - 2\tau \ln\frac{1+\tau}{\tau} \right] \simeq K_4 \;, \tag{5.10}$$

plus terms that vanish as $\tau \to 0$, whereas for finite $\mu^2$ the full form

$$J(\mu^2,\tau) = K_4 \left[ 1 + \frac{1}{\tau^2} \left[ (\mu^2+\tau)^3 \ln\frac{1+\mu^2+\tau}{\mu^2+\tau} - \mu^6 \ln\frac{1+\mu^2}{\mu^2} \right] - \frac{1}{\tau}(\mu^2+\tau)^2 \left[ 3\ln\frac{1+\mu^2+\tau}{\mu^2+\tau} - \frac{1}{1+\mu^2+\tau} \right] \right] \tag{5.11}$$

must be kept to avoid terms that diverge in the limit of small $\tau$ when $1+\mu^2+\tau \approx 1+\mu^2$. The rest of the calculation proceeds as in Sec. III. This requires first the form in Eq. (5.8), out of the critical region, which yields the additional contribution to $\gamma_{\text{eff}}(t,x)$.

Noting that $du(t^{1/2})/d\ln t = O(\epsilon^2)$, we find

$$\gamma_{\rm eff}(t,x) = \gamma_{\rm eff}(t,x)|_{\mathcal{H}_1} + \delta\gamma_{\rm eff}(t,x) \ , \qquad (5.12)$$

the first term on the right-hand side being given by Eq. (4.7), while

$$\delta\gamma_{\rm eff}(t,x) = \frac{1}{6}(n-m)u(t^{1/2})x \times \left[3(1-x^2)\ln\frac{2+x}{1+x} - (7+x-x^2)\frac{1+x}{(2+x)^2} + x^2\left[3\ln\frac{1+x}{x} - \frac{1}{1+x}\right]\right] \ , \qquad (5.13)$$

$x$ being the scaling variable $\mu^2/\rho^2 \approx m^2/t$. The result has been calculated with either the complete forms for $u^{-1}(\rho)$ in Eqs. (3.33) and (3.34) or the leading expression in Eq. (3.36) and $\delta\gamma_{\rm eff}(t,x)$ is given in Table I, for $n=2$, $m=1$, and $\tilde{m}^2 = 10^{-3}$ when $\epsilon = 1$. Clearly, it is rather small in the crossover region and smaller by several orders of magnitude near the fixed points. The sharp decrease near the latter is at least in qualitative accordance with the fact that the anomalous dimensions of the inserted operators is two integers larger than the correction-to-scaling exponent $\omega$ for $u(\rho) \neq u_m^*$, or $u_n^*$. It should be noted that the anomalous dimensions of composite operators are, however, defined and calculated *at* the fixed points and that gives no idea of the amount by which an effective critical exponent can be modified in the crossover region. As we show here, the effect is clearly small, perhaps too small to be of practical importance for the time being, but not completely negligible.

Note that that smallness of $\delta\gamma_{\rm eff}$ is not due to the $\Lambda^{-2}$ at the insertion point, as can be seen from Eqs. (5.7) and (5.9), but rather to the larger number of propagators that enter into the one-loop contribution, as compared to the main part $\gamma_{\rm eff}(t,x)|_{\mathcal{H}_1}$. This leads us to expect smaller contributions from further insertions of composite operators of canonical dimension six, or of first-order insertions of higher-dimensional operators.

TABLE I. $\delta\gamma_{\rm eff}(t,x)$, defined in Eq. (5.12) due to the insertion of composite operators of canonical dimension six into model $\mathcal{H}_1$: (I) when the complete $u(\rho)$ is used and (II) with the leading terms only.

| $\log_{10}t$ | I | II |
|---|---|---|
| −0.6990 | 0.000 00 | 0.000 07 |
| −1.0000 | 0.000 15 | 0.000 17 |
| −1.6990 | 0.000 65 | 0.000 68 |
| −2.0000 | 0.001 00 | 0.000 88 |
| −2.6990 | 0.001 70 | 0.001 92 |
| −3.0000 | 0.001 20 | 0.003 10 |
| −3.6990 | 0.000 00 | 0.000 00 |

## VI. DISCUSSION AND SUMMARY OF RESULTS

We have shown here how the cutoff-dependent version of RPT that has been used before to study the Gaussian-to-Heisenberg crossover in isotropic systems can be extended to a problem with quadratic spin anisotropy. This, as that of Amit and Goldschmidt, is a proper RG procedure in terms of the actual temperature variable $t = T - T_c(g)$ and the flowing coupling $u(\rho)$, which does not involve a matching of those parameters, as done in some previous works,[10] a point that does not seem to have been investigated in detail. Our main results are the following.

First, we have shown that there is an important difference, already at the one-loop level, between a cutoff-dependent version of RPT and the renormalized theory with GMS, in that the former yields other than logarithmic terms in the 1PI vertex functions, which corresponds to finite renormalizations in the renormalized theory. The relevance of this can best be seen in the case of a sharp cutoff. We found there that the added nonlogarithmic terms in the four-point vertex function yield a coefficient function $\beta(u,\mu)$ that is formally different from that in the renormalized theory, while $\gamma_4(u,\mu)$ is formally the same. The former implies a flowing $u(\rho)$ with other than logarithmic terms in the scaling variable $\mu^2/\rho^2 \approx m^2/t$, even when the full forms in Eqs. (3.33) and (3.34) are kept. This is the only formal difference with GMS, for the sharp-cutoff model, and a different result from that of Amit and Goldschmidt[14] is obtained. Although the difference is not very large, clearly the detailed *shape* of $\gamma_{\rm eff}(t,x)$ is not the same and this is relevant to the problem of universality. It is worth noting that the reason why our *two*-point function $\Gamma_L^{(2)}$, for model $\mathcal{H}_1$, is formally the same as that of the renormalized theory with GMS, is that the coefficients of the nonlogarithmic terms are accidentally zero.

The further significance of finite renormalizations, in order to understand our resutls, can be seen as follows. Given a specific regularization in a bare theory one can construct RG equations that are equivalent to those of a wide class of renormalized theories. In this sense, RPT with GMS is just one special renormalized theory, with purely logarithmic terms, of the class that corresponds to a bare theory with a sharp cutoff. Although the finite renormalizations that are equivalent to the latter are not expected to make a difference at fixed-point values, we see no reason for neglecting them under crossover.

So far, this refers to the results of our work compared to those of the renormalized theory. In comparing our results between models $\mathcal{H}_1$ and $\mathcal{H}_2$, it should be noted that further finite-renormalization terms can follow from a change in regularization, and that is precisely what happens in our case. The difference in the results between the two models is large enough to give them further thought. A calculation to two-loop order may give some further clue about the relevance of a change of regularization.

Second, in view of the exponentiated singularity of the scaling variable that enters in our full form of $u^{-1}(\rho)$, or in that of previous work,[14] and in the absence of a two-loop-order calculation that demonstrates that these terms, and the other nonleading contributions to $u^{-1}(\rho)$, are not modified, we calculated the effect of the leading terms in expansion in $\epsilon$, both for a sharp and a smooth cutoff. We find a displaced maximum for $\gamma_{\rm eff}(t,x)$ when $0.5 \lesssim \epsilon \lesssim 1$. A similar result has been obtained before using, however, nonleading terms in $\epsilon$.[14] Furthermore, we show that, in the case of a smooth cutoff, a rather pronounced dip may appear in the crossover region for sufficiently small anisotropy. This is due to the particular nonlogarithmic terms in the vertex functions for this type of cutoff. We do not find the same behavior to occur with a sharp cutoff, and the result may just indicate that the large momentum cutoff, $\Lambda^{-2}q^4$, becomes a dangerous irrelevant variable under crossover. This part of our work suggests that a systematic second-order calculation in $\epsilon$ becomes necessary in order to see which of the various situations discussed here hold through.

The third main result of this work concerns the correction to the leading effective singular behavior due to composite operators of canonical dimension six, to one-loop order and to first order in the insertions. Although the effect is quite small, it is not completely negligible away from primary and secondary critical behavior. A calculation of this effect has not been done before, and the extension to include operators of canonical dimension higher than six can be carried out along the same lines. More urgent, however, may be an extension to two-loop order and second order in the insertions, for operators of canonical dimension six, since only then the complete set of operators plays a role, as discussed in this paper. This, and other extensions referred to above, will be considered in future work. The asymptotic behavior of a crossover scaling function has been reported before,[28] to one-loop order, and a more complete form would be desirable. However, since the same complications as for $\gamma_{\rm eff}(t,x)$ discussed here also arise there, further progress will depend on the results of work to two-loop order.

As far as the universality of $\gamma_{\rm eff}(t,x)$ is concerned, we hope to have shown that this is not a settled (or almost so) problem, to one-loop order. We have shown that eventually a fairly monotonic $\gamma_{\rm eff}(t,x)$ can be obtained even to that order. In this case we cannot ensure, however, that partial contributions to higher order have not been included in the calculations. This is not an uncommon problem in RG studies of crossover phenomena.[33] For anisotropic spin systems, already, Horner[12] noted that higher-order contributions in $\epsilon$ entered in an unsystematic fashion into his calculations to lowest order. In this case, however, if certain leading terms in expansion in $\epsilon$ turn out to be the only genuine terms to first order, a rather nonmonotonic $\gamma_{\rm eff}(t,x)$ should be expected, as we have shown here, for $\epsilon \simeq 1$.

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*Address since August 16, 1980.
[1]E. K. Riedel and F. J. Wegner, Z. Phys. 225, 195 (1969).
[2]M. E. Fisher and P. Pfeuty, Phys. Rev. B 6, 1889 (1972).
[3]F. Wegner, Phys. Rev. B 6, 1891 (1972).
[4]P. Pfeuty, D. Jasnow, and M. E. Fisher, Phys. Rev. B 10, 2088 (1974).
[5]S. Singh and D. Jasnow, Phys. Rev. B 11, 3445 (1975); 12, 493 (1975).
[6]A. D. Bruce, Phys. Lett. A 48, 317 (1974).
[7]A. D. Bruce and A. Aharony, Phys. Rev. B 10, 2078 (1974).
[8]A. D. Bruce, J. Phys. C 8, 2992 (1975).
[9]A. Aharony, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and M. S. Green (Academic, New York, 1976), Vol. 6.
[10]D. R. Nelson and E. Domany, Phys. Rev. B 13, 236 (1976); E. Domany, D. R. Nelson, and M. E. Fisher, *ibid.* 15, 3493 (1977).
[11]J. M. Kosterlitz, J. Phys. C 9, 497 (1976).
[12]H. Horner, Z. Phys. B 23, 183 (1976).
[13]A. D. Bruce and D. J. Wallace, J. Phys. C 9, 1117 (1976).
[14]D. J. Amit and Y. Y. Goldschmidt, Ann. Phys. (N.Y.) 114, 356 (1978).
[15]M. E. Fisher and D. R. Nelson, Phys. Rev. Lett. 32, 1350 (1974).

[16]A. Aharony and A. D. Bruce, Phys. Rev. Lett. 33, 427 (1974).
[17]A. D. Bruce, J. M. Kosterlitz, and D. R. Nelson, J. Phys. C 9, 825 (1976); T. Nattermann and S. Trimper, *ibid.* 9, 2589 (1976); A. D. Bruce *ibid.* 10, 419 (1977); D. Bailin and A. Love, *ibid.* 10, 3633 (1977).
[18]E. K. Riedel and F. J. Wegner, Phys. Rev. B 9, 294 (1974).
[19]D. R. Nelson, Phys. Rev. B 11, 3504 (1975).
[20]E. Brézin, J. LeGuillou, and J. Zinn-Justin, in *Phase Transitions and Critical Phenomena,* edited by C. Domb and M. S. Green (Academic, New York, 1976), Vol. 6.
[21]D. J. Amit, *Field Theory, the Renormalization Group and Critical Phenomena* (McGraw-Hill, New York, 1978).
[22](a) K. G. Wilson and J. Kogut, Phys. Rep. C 12, 77 (1974); (b) M. E. Fisher, Rev. Mod. Phys. 46, 597 (1974).
[23]W. K. Theumann, Phys. Rev. B 21, 1930 (1980).
[24]I. D. Lawrie, J. Phys. A 10, 1557 (1977).
[25]It has been shown in Ref. 24 that, if the initial value of the parameter that drives the crossover is large enough, the corrections to crossover scaling due to a finite-momentum cutoff can become quite appreciable over a non-negligible temperature range.
[26]J. Zinn-Justin, in *1973-Cargése Lectures in Field Theory and Critical Phenomena,* edited by E. Brézin and J. M. Charap (Gordon and Breach, New York, 1975).
[27]D. J. Wallace, in *Phase Transitions and Critical Phenomena,* edited by C. Domb and M. S. Green (Academic, New York, 1976), Vol. 6.
[28]W. K. Theumann, Bull. Am. Phys. Soc. 25, 248 (1980). Also, 42nd Statistical Mechanics Meeting, Rutgers University, December 13–14 (1979) (unpublished).
[29]E. Brézin, J. C. LeGuillou, and J. Zinn-Justin, Phys. Rev. B 10, 892 (1974).
[30]Note, however, that because of the sharp momentum cutoff, the constant behind $\ln\tau$ is not the same as in Refs. 13 and 23. See also, Eqs. (3.39) and (3.40). A pure $\ln\tau$ term also follows in dimensional regularization for the renormalized $\Gamma_L^{(2)}$.
[31](a) The precise correspondence is with their Eq. (11.9), corrected for a misprint in their last term. (b) Our $u$ is one-half theirs, since it absorbs a factor $\frac{1}{2}K_4$ against the $K_4$.
[32]M. E. Fisher, in *Renormalization Group in Critical Phenomena and Quantum Field Theory,* edited by J. Gunton and M. S. Green (Temple University Press, Philadelphia, 1974).
[33]W. K. Theumann, Phys. Rev. B 18, 6372 (1978).