PHYSICAL REVIEW B VOLUME 24, NUMBER 1 1 JULY 1981

Pairbreaking in superconductors near and below antiferromagnetic transitions

T. V. Ramakrishnan
Princeton University, Princeton, New Jersey 08544

C. M. Varma
Bell Laboratories, Murray Hill, New Jersey 07974

(Received 17 November 1980)

The Abrikosov-Gorkov theory for pairbreaking in superconductors due to a dilute concentration of magnetic impurities is generalized to the case when the magnetic ions are present in a large concentration or on a regular lattice. The case in which the magnetic ions undergo a transition to an antiferromagnetic phase is considered in detail. The physical considerations for increased or decreased pairbreaking over the Abrikosov-Gorkov value due to the increased range of correlations near the antiferromagnetic transition and due to inelastic scattering of electrons off spin waves in the antiferromagnetic phase are discussed in detail. Recent experimental results are considered in terms of the new theory.

I. INTRODUCTION

Recently many superconducting compounds that contain a lattice of magnetic rare-earth ions have been discovered.[1] Some of these become ferromagnetic and are then no longer superconducting.[2] Others have a transition to antiferromagnetic order which coexists with superconductivity.[3] The upper critical field H_{c2} near and below the Neél temperature T_N is unusual. Fischer *et al.*[3] have carefully analyzed the data for H_{c2} in terms of known pairbreaking mechanisms for two antiferromagnetic superconductors and conclude that an additional new process must be operative near and in the antiferromagnetic phase. This process has a magnitude ~ 10–20% of the usual Abrikosov-Gorkov (AG) impurity scattering process,[4] seems to depend very little on temperature below $T/T_N \lesssim 0.8$, and decreases rapidly near T_N. In another antiferromagnetic superconductor however, pairbreaking seems reduced in the antiferromagnetic phase.[5]

We discuss in this paper the effect of magnetic fluctuations, both above and below T_N, on superconductivity for systems with $T_c > T_N$. The interaction between localized rare-earth spins $\vec{S}_i$ and the conduction electrons is described by an exchange Hamiltonian

$$H_{ex} = -J \sum_i \vec{S}_i \cdot \vec{s}_i \ ,$$

where $\vec{s}_i$ is the conduction-electron spin density at site i. At high temperatures $T >> T_N$, the spins can be considered independent. The basic pairbreaking mechanism then is the elastic scattering of conduction electrons by spins, as discussed first by Abrikosov and Gorkov.[4] For temperatures in the neighborhood of T_N strong spatial and temporal spin correlations develop. Since the latter have a time scale $\sim (k_B T/\hbar)^{-1}$, conduction-electron scattering is still quasielastic. Because of the equal-time–same-site sum rule $\langle \vec{S}_i \cdot \vec{S}_i \rangle = S(S+1)$ the change in pairbreaking with respect to the Abrikosov-Gorkov value is small,[6] despite increased spatial correlation of the spins. We discuss the difference, which is similar to the temperature dependence of electrical resistivity near T_N in metallic antiferromagnets.[7] The size and the sign depend on the wave-vector dependence of electronic susceptibility near the sublattice Bragg peak wave vectors $\vec{G}$, and the temperature dependence is of the form $|(T-T_N)_{T_N}|^{1-\alpha}$ where α is the specific-heat exponent (Sec. II). Below T_N, transverse spin fluctuations are oscillatory (spin waves) with characteristic energy $\hbar\omega_s$ ($>> k_B T$ for $T << T_N$) while longitudinal fluctuations are diffusive and thermal. The relative weight in these changes with temperature. Thus there is pronounced inelastic scattering in the antiferromagnetic phase. We discuss the effect of this transfer of spectral weight using a method developed by Bergman and Rainer[8] to evaluate the relative effect of phonons of various frequencies on superconducting T_c. We find decreased pairbreaking unless the normal phase electronic susceptibility peaks at the Bragg vectors $\vec{G}$. We present numerical calculations for the size of this effect using the Eliashberg equations. For realistic values of physical parameters such as Debye energy, spin-wave energy, etc., we obtain a 10–20% change in the pairbreaking parameter. The actual value of all these effects depends upon details of the $\vec{q}$ dependence of the electronic susceptibility and hence on the details of the electronic structure. As such they are not easy to calculate.

 ©1981 The American Physical Society

II. GAP EQUATION AND ELASTIC PAIRBREAKING

A. Gap equation

We are interested in the effect of spin correlations on the upper critical field H_{c2} near which the linearized gap equation is valid. This equation simplifies in the dirty limit (short mean free path due to elastic scattering by nonmagnetic disorder), where the space- and frequency-dependent gap function $\Delta(r,\omega_i)$ factors as $f(r)\Delta(\omega_i)$ and $f(r)$ satisfies a harmonic oscillator equation.[9,10] Here ω_j is the Matsubara frequency, i.e., $i\omega_j=(i\pi/T)(2j+1)$. One then finds that

$$\Delta(\omega_i)=\pi T\sum_j \lambda(\omega_i-\omega_j)\frac{1}{|\tilde{\omega}_j|+\rho_0}\Delta(\omega_j)\ , \tag{2.1}$$

where the kernel $\lambda(\omega_i-\omega_j)$ is the momentum-transfer-averaged interaction between Cooper pairs due to phonons and inelastic spin fluctuations ($\omega_i\neq\omega_j$). $\tilde{\omega}_j$ is the self-energy-shifted fermion frequency; i.e.,

$$\tilde{\omega}_j=\omega_j+\pi T\sum_l(\mathrm{sgn}\omega_l)\lambda(\omega_j-\omega_l) \tag{2.2}$$

and in ρ_0 are lumped together *all* elastic pairbreaking effects, e.g., due the magnetic field H, elastic magnetic scattering, and spin-orbit coupling. The explicit form[10] of ρ_0 is

$$\rho_0=\frac{DeH}{3}+\frac{1}{\tau_s}+\lambda^{\mathrm{eff}}\ , \tag{2.3}$$

where D is the electron diffusion constant, τ_s is the lifetime for scattering off spin fluctuations, and $\lambda_{so}^{\mathrm{eff}}$ is the effective spin-orbit-scattering parameter. $H_{c2}(T)$ is the magnetic field at which Eq. (2.1) is satisfied for a given temperature T. In the gap equation (2.1), elastic and inelastic processes have been separated. This is quite general and can be done exactly. All processes which do not change the fermion energy can be considered together, and the resulting Cooper pair propagator is $(\tilde{\omega}_i+\rho)^{-1}$ for the lowest harmonic-oscillator eigenstate which determines H_{c2}.

B. Coupling of conduction electrons and f-spin excitations

The basic spin-mediated coupling between electrons can be written as

$$\lambda_s(\omega_i-\omega_j)=\frac{J^2}{\rho(\epsilon_F)}\sum_{\vec{k},\vec{q}}\delta(\epsilon_{\vec{k}}-\mu)\delta(\epsilon_{\vec{k}}-\epsilon_{\vec{k}+\vec{q}})\times\chi_f(\vec{q},\omega_i-\omega_j)\ , \tag{2.4a}$$

$$=J^2\sum_q\phi(q)\chi_f(\vec{q},\omega_i-\omega_j)\ . \tag{2.4b}$$

The δ functions in Eq. (2.4a) describe the restriction of the initial and the final electronic states $\vec{k}$ and $\vec{k}+\vec{q}$ to the vicinity of the Fermi surface, and the joint density of electronic states $\phi(q)$ for coupling to fluctuations of wave vector q depends on the Fermi surface geometry. The wave vector and frequency-dependent susceptibility $\chi_f(\vec{q},\omega_i)$ of the f-spin system determines the dynamic nature of the coupling. In terms of the spin operators S_i^α, it is given by

$$\chi_f^{\alpha\beta}(\vec{q},\omega_l)=\frac{1}{N}\int_0^\beta\left\langle\sum_i S_i^\alpha(\nu)e^{i\vec{q}\cdot\vec{R}_i}\sum_j S_j^\beta(0)e^{-i\vec{q}\cdot\vec{R}_j}\right\rangle e^{-\nu\omega_l}d\nu\ . \tag{2.5}$$

Equations similar to Eqs. (2.1) and (2.4) can be obtained in the context of a Ginzburg-Landau free-energy functional $F(\psi)$ where ψ is the superconducting order parameter. For small ψ, it is well known that the equilibrium condition $\delta F/\delta\psi=A\psi=0$ is equivalent to the linearized gap equation (2.1). The magnetic contribution to the kernel $\lambda(\omega_i-\omega_j)$ is merely that part of the functional derivative $\delta F\{\psi\}/\delta\psi$ which depends on the f electron magnetization fluctuations. To lowest nonvanishing order, one can write this term schematically as

$$\sum_q\left[\frac{\delta^2}{\delta M_q\delta M_{-q}}\left[\frac{\delta F}{\delta\psi}\right]\right]\langle M_qM_{-q}\rangle\ .$$

The functional derivatives

$$(\delta^2/\delta M_q\delta M_{-q})(\delta F/\delta\psi)=(\delta/\delta\psi)\chi_q(\psi)$$

can be calculated using standard techniques (see, e.g., Hassing and Wilkins[11]), and corresponds to evaluating a four-point vertex[12] with two Cooper pair and two spin-fluctuation vertices. The result for $\lambda_s(\omega_i-\omega_j)$ is identical with Eq. (2.4). This derivation shows directly that the coupling to f spin fluctuations is given by $\frac{1}{2}\delta^2\chi_q(\psi)/\delta\psi^2$, i.e., by the change in magnetic susceptibility at the superconducting transition. Wave-vector-dependent susceptibilities in the normal and superconducting phases have been calculated for a nested Fermi surface by Machida and Matsubara.[13] They find that change in χ_q is very large near the nesting wave vector $\vec{G}$, i.e., that $\phi(\vec{q}=\vec{G})$ is large. The above is illustrated in Figs. 1(a) and 1(b), where the q dependence of the normal and the superconducting state susceptibilities for a spherical Fermi surface and a Fermi surface nested near $\vec{q}=\vec{G}$ are schematically drawn. The $\vec{q}$ dependence of the coupling constant, $\phi(q)$ in Eq. (2.4b) is given essentially by the difference in the two suscep-

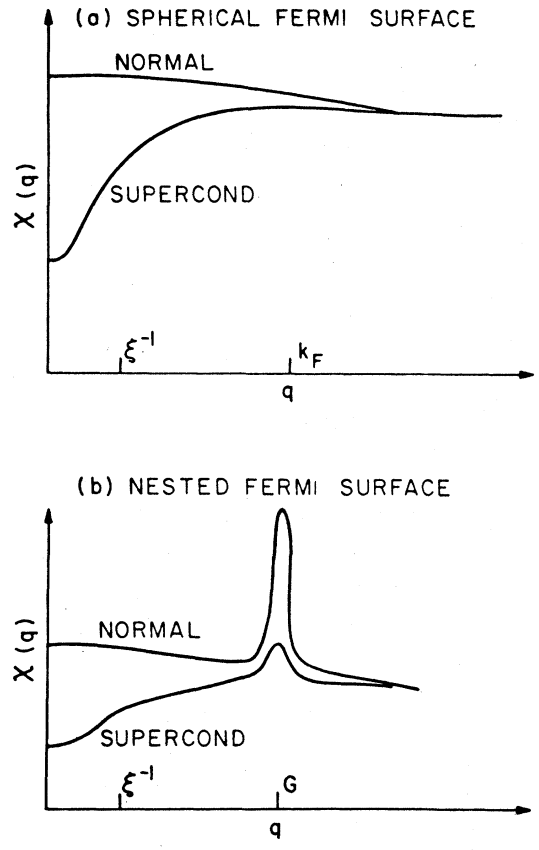

FIG. 1. (a) Schematic representation of the q dependence of the conduction-electron susceptibility for the normal state and the superconducting state for a metal with a spherical Fermi surface. The coupling constant $\phi(q)$ in the text is related to the difference in the two curves. (b) Same as (a) for a metal with a Fermi surface with nesting near $q = G$.

tibilities in each case. $\phi(q)$ is a maximum near $q = 0$ for a spherical Fermi surface, decreases initially as $(q\xi)^2$ where ξ is the coherence length and as $1/q$ for q near k_F. For nested Fermi surfaces $\phi(q)$ peaks near $\vec{q}=\vec{G}$ besides having a peak at $\vec{q}=0$.

C. Quasistatic-scattering limit

In the remainder of this section we discuss the quasistatic-scattering limit which obtains at high and intermediate temperatures $T \gtrsim T_N$. The strongly inelastic low-temperature situation is discussed in the next section. We note that since $\vec{S}_i \cdot \vec{S}_i = S(S+1)$, $\chi_f(\vec{q}, \omega_l)$ satisfies the important sum rule

$$\frac{1}{\beta} \sum_{\omega_l} \sum_{\vec{q},\alpha} \sum \chi_f^{\alpha\alpha}(\vec{q}, \omega_l) = S(S+1) \quad . \tag{2.6}$$

At temperatures much higher than the spin interaction energies ($T >> T_N$) the spins are nearly independent, and one has

$$\chi_f(\vec{q}, \omega_l) = \frac{S(S+1)}{k_B T} \delta_{\omega_l,0} \quad . \tag{2.7}$$

The scattering by spins is elastic and the interaction is of zero spatial range. This is the Abrikosov-Gorkov limit and is described by a spin-flip scattering time

$$\tau_s = [J^2 \rho(\epsilon_F) S(S+1)]^{-1}$$

or a dimensionless pairbreaking parameter

$$\tilde{\rho}_{\mathrm{AG}} = \frac{1}{\tau_s \pi T} = \frac{J^2 \rho(\epsilon_F) S(S+1)}{\pi T} \quad . \tag{2.8}$$

As temperature decreases towards T_N, the susceptibility peaks for wave vectors $\vec{q}=\vec{G}$ and frequencies $\omega_l = 0$. We have investigated the effect of diffusive nonthermal fluctuations by assuming a form $\chi_f(\vec{q}, \omega_l) = [\chi_f(\vec{q}, 0)^{-1} + c|\omega_l|]^{-1}$ for the dynamic susceptibility where c is a constant of order unity. Using this form and substituting in Eq. (2.4) and then Eq. (2.1) to estimate the effect on H_{c2} we find that the effect is small compared to that from the static term $\chi_f(\vec{q}, 0)$ and is very weakly (logarithmically) temperature dependent. Because of this and the sum rule (2.6), a quasistatic approximation where one assumes

$$\chi_f^{\alpha\beta}(\vec{q}, \omega_l) = \delta_{\alpha\beta} \chi(\vec{q}) \delta_{\omega_l,0} \tag{2.9a}$$

with

$$\sum_{\vec{q}} \chi(\vec{q}) = \frac{S(S+1)}{3k_B T} \tag{2.9b}$$

should be quite accurate. Under these conditions the pairbreaking parameter is simply

$$\tilde{\rho} = \frac{1}{\tau_s \pi T} = \frac{3J^2}{\pi \rho(\epsilon_F)} \sum_{\vec{k}} \delta(\epsilon_k - \mu) \sum_{\vec{q}} \delta(\epsilon_{\vec{k}} - \epsilon_{\vec{k}+\vec{q}}) \chi(\vec{q}) \quad . \tag{2.10}$$

If the joint density-of-states factor

$$\phi(q) = \frac{1}{\rho(\epsilon_F)} \sum_{k} \delta(\epsilon_k - \mu) \delta(\epsilon_k - \epsilon_{k+q}) \tag{2.11}$$

were independent of q, the sum rule (2.9b) ensures that $\tilde{\rho}_s = \tilde{\rho}_{\mathrm{AG}}$, i.e., the pairbreaking is unchanged. Thus deviation from the AG limit depends on the detailed functional forms of $\phi(q)$ and $\chi(q)$. The problem is similar to that encountered in analyzing the temperature dependence of electrical resistivity near T_N in metallic antiferromagnets. In that case $\phi(q)$ is proportional to the momentum transferred to the scattered electron by the spin fluctuation. This problem has been thoroughly discussed by Alexander, Helman, and Balberg.[6] They find that while

because of the sum rule the largest contribution shows no critical behavior, the leading correction goes as $|(T-T_N)/T_N|^{1-\alpha}$ where α is the specific-heat critical index. Its sign is positive if $\phi(\vec{q})$ peaks in q space where $\chi(q)$ does and is negative otherwise. Now $\chi(\vec{q})$ is maximum for $\vec{q}=\vec{G}$. In a free-electron gas, $\phi(q)\sim(1/q)$ (and zero if $q>2k_F$) so that for such a system one would expect, close to T_N, *decreased* pairbreaking with $\Delta\tilde{\rho}_s$ depending on temperature as $|(T-T_N)/T_N|^{1-\alpha}$. However, a metal undergoing an antiferromagnetic instability is most unlikely to have a spherical Fermi surface; it is often the peaking of the conduction-electron susceptibility $\chi_e(\vec{q})$ around $\vec{q}=\vec{G}$ and consequent spin-spin interaction that drives the f spins towards an antiferromagnetic instability. We might therefore assume $\chi_e(\vec{q})$ to peak around $\vec{q}=\vec{G}$, e.g., due to partial Fermi-surface nesting. This will be reflected in the joint density-of-states $\phi(\vec{q})$ [Eq. (2.11)] also peaking at $\vec{q}=\vec{G}$. In this case, pairbreaking increases around T_N, i.e.,

$$(\Delta\tilde{\rho}_s)_{\text{crit}}\sim\left(\frac{T-T_N}{T_N}\right)^{1-\alpha}. \tag{2.12}$$

For a Heisenberg antiferromagnet, $\alpha=\frac{1}{8}$.

The above analysis can be carried through to temperatures below T_N so long as spin-wave energies are not much greater than k_BT. The fluctuations are all thermal, and scattering is quasielastic. Well below T_N, longitudinal and transverse fluctuations have very different characteristics; their effect is discussed in the next section.

In earlier work,[6,14–17] the sum rule (2.9) has been either ignored[14–16] or its effect has been analyzed only for a spherical Fermi surface.[6] References 15–17 seem to ignore the $\vec{q}$ dependence of the coupling constant altogether. As a result, it is stated that near the antiferromagnetic transition point, pairbreaking necessarily increases[14–16] or always decreases.[6] Our analysis shows that pairbreaking can either increase or decrease, depending on whether the coupling constant $\phi(q)$ peaks at the sublattice Bragg wave vectors or not.

D. Effect of altered band structure

In discussing the elastic scattering in the antiferromagnetic phase, we have so far tacitly assumed for the conduction electrons the band structure of the paramagnetic phase. In fact the new periodic potential in the antiferromagnetic phase will mix the old states and one should strictly use the new conduction-electron eigenstates. This will in general lead to a weakening of superconductivity because the average density of conduction-electron states at the Fermi energy diminishes due to antiferromagnetism. The reduction in the average density of states at the Fermi energy is proportional to T_N, and this effect can approximately be taken into account by replacing the $\rho(\epsilon_F)$ factors occurring in the elastic pairbreaking by the reduced density of states. This will be quite a small effect, of $O(T_N\rho(\epsilon_F))$.

If one discusses the pairing in the antiferromagnetic phase in terms of the eigenstates of the paramagnetic phase, one may be inclined to say[18] that the pairing is between states k and $-k+G$, etc; where G are the antiferromagnetic Bragg vectors (because the new eigenstates are linear combinations of k and $k+nG$ of the old states). This is not very meaningful, however—the pairing is still between time-reversed states of the new band structure.

III. PAIRBREAKING IN THE ANTIFERROMAGNETIC PHASE

Well below the Néel temperature, the sublattice magnetization is large and attains the value S in an ideal antiferromagnet at $T=0$. Therefore longitudinal spin fluctuations are strongly suppressed. However, transverse spin fluctuations propagate as spin waves with characteristic energy $\hbar\omega_s\sim k_BT_N>>k_BT$. The longitudinal part of the sum rule (2.9) amounts to S^2; the remaining part S is the integrated strength of the transverse spin fluctuations. The longitudinal part is elastic, while the transverse part is not. Thus, relative to high temperatures ($T\gtrsim T_N$) there is some transfer of strength in $\chi(\vec{q},\omega_l)$ from the static to the dynamic part. This transfer is described quite generally by the susceptibility function

$$\chi_s(\vec{q},\omega_l)=\left[2\alpha\int\frac{\omega\tanh(\beta\omega/2)F_{\vec{q}}(\omega)}{\omega^2+\omega_l^2}d\omega+\frac{[S(S+1)-\alpha]}{T}\delta_{l,0}\right], \tag{3.1}$$

where $F_{\vec{q}}(\omega)$ is the normalized spectral density of spin-wave excitations, i.e.,

$$\int\sum_{\vec{q}}F_{\vec{q}}(\omega)d\omega=1 \tag{3.2}$$

and α represents the strength transferred from static ($l=0$) to dynamic part of χ. In an ideal antiferromagnet at $T=0$, $\alpha=S$. By construction the form (3.1) satisfies the sum rule (2.6). The transfer of weight from elastic to inelastic spin fluctuations obviously affects the form of the effective interaction $\lambda_s(\omega_i-\omega_j)$ [Eq. (2.4)]. The precise effect depends on the $\vec{q}$ dependence of the joint density of states (2.11). If this peaks at $\vec{q}=0$ or is flat in q space,

$$\lambda_s(\omega_i-\omega_j)=J^2\rho(\epsilon_F)\sum_{\vec{q}}\chi_s(\vec{q},\omega_i-\omega_j). \tag{3.3}$$

In particular, if we use a simple but reasonable model

for the spin-wave spectral density characterizing it by a single frequency ω_s, i.e.,

$$\sum_{\vec{q}} F_{\vec{q}}(\omega) = \delta(\omega - \omega_s) \tag{3.4}$$

the spin mediated interaction is

$$\lambda_s(\omega_i - \omega_j) = J^2 \rho(\epsilon_F) \left[2\alpha \frac{\omega_s \tanh(\beta\omega_s/2)}{\omega_s^2 + (\omega_i - \omega_j)^2} + \frac{[S(S+1) - \alpha]}{T} \delta_{ij} \right] . \tag{3.5}$$

The sum rule constraint and transfer of interaction strength from static to dynamic are especially transparent in Eq. (3.5). However, as mentioned earlier, it is quite probable that the joint density-of-states $\phi(q)$ peaks at the sublattice Bragg peak wave vectors $\vec{q} = \vec{G}$. The spin waves with small wave vectors $\vec{\eta} = \vec{q} - \vec{G}$ are the softest, and therefore $F_{\vec{q}}(\omega)$ peaks in q space where ϕ_q does. Thus the inelastic part of $\lambda(\omega_i - \omega_j)$ is enhanced with respect to the simple forms (3.3) or (3.5). One can still work with a form similar to Eq. (3.5), viz.,

$$\lambda_s(\omega_i - \omega_j) = J^2 \rho(\epsilon_F) \left[2\alpha' \frac{\omega_s \tanh(\beta\omega_s/2)}{\omega_s^2 + (\omega_i - \omega_j)^2} + \frac{[S(S+1) - \alpha]}{T} \delta_{ij} \right] \tag{3.6}$$

where quite generally, $\alpha' > \alpha$. It is easy to convince oneself, using plausible forms for $\phi(q)$ and the spin-wave spectrum (e.g., acoustic) that α' can be substantially larger than α, and that the characteristic spin-wave energy ω_s appearing in Eq. (3.6) is smaller than that appearing in $F_s(\vec{q}, \omega)$. The form (3.6) reflects the physical fact that the sum rule constraint is not absolute, i.e., effects which lead to antiferromagnetic instability peak in $\chi_e(\vec{q})$ or $\phi(\vec{q})$ at $\vec{q} = \vec{G}$ tend to enhance the dynamic interaction beyond naive sum rule expectations.

We now discuss the effect of the dynamic spin mediated interaction such as Eqs. (3.5) or (3.6) on H_{c2}. The method we use is due to Bergmann and Rainer.[7] We show analytically that if the transfer to inelastic scattering is constrained by the sum rule, as in Eq. (3.5), there is necessarily a reduction in pairbreaking, i.e., H_{c2} decreases. The size of the decrease is computed by numerically solving the Eliashberg equations, and shown for typical magnetic and superconducting parameters as a function of temperature. We also present calculations for the case (3.6) where inelastic scattering is enhanced, and show that pairbreaking and hence H_{c2} increase.

Bergmann and Rainer start with the linearized gap equation (2.1) (with $\rho_0 = 0$ in their paper, but this does not change the argument). Suppose we add a term ρ to ρ_0. The physically correct solution to the gap equation has $\rho = 0$; writing

$$\tilde{\Delta}(\omega_j) = \frac{\Delta(\omega_j)}{\tilde{\omega}_j + \rho_0 + \rho} \tag{3.7}$$

$\tilde{\Delta}(\omega_j)$ satisfies the eigenvalue equation

$$\rho \tilde{\Delta}(\omega_i) = \pi T \sum_j \left[\lambda(\omega_i - \omega_j) - \frac{(|\tilde{\omega}_i| + \rho_0)\delta_{ij}}{\pi T} \right] \tilde{\Delta}(\omega_j) \tag{3.8}$$

with zero eigenvalue ρ. This can be used in several ways. For a given ρ_0 and $\lambda(\omega_i - \omega_j)$ the eigenvalues are examined as T decreases. At a certain temperature a zero eigenvalue appears. One can thus find the critical pairbreaking parameter and from Eq. (2.3) H_{c2} as a function of T. Now suppose $\lambda(\omega_i - \omega_j)$ is altered by transfer of spectral weight as described by Eqs. (3.5) and (3.6). We now find anew the $\rho_0(T)$ for which a zero eigenvalue ρ of Eq. (3.8) appears. If ρ_0 increases, H_{c2} increases in direct proportion, and vice versa. The change in $\lambda(\omega_i - \omega_j)$ we consider is

$$\delta\lambda_s(\omega_i - \omega_j) = J^2 \rho(\epsilon_F) \left[\alpha' \frac{\omega_s \tanh(\beta\omega/2)}{\omega_s^2 + (\omega_s - \omega_j)^2} - \frac{\alpha}{T} \delta_{ij} \right] , \tag{3.9}$$

where $\alpha' = \alpha$ if the $\vec{q}$ dependence of the interaction is left unaltered. Detailed numerical calculations are described later below. In the case $\alpha = \alpha'$, we show analytically that pairbreaking decreases. Bergmann and Rainer[7] obtain the functional derivative $\delta\rho/\delta F(\omega)$, i.e., the change in ρ for a small change in the spectral density at a given frequency. Multiplying this by the actual change in $F(\omega)$ implied by the $\delta\lambda_s(\omega_i - \omega_j)$ of Eq. (3.5), and integrating over frequencies, one finds

$$\delta\rho = \frac{\pi T \alpha}{\sum_i \tilde{\Delta}_i^2} \left\{ \frac{2}{T} \sum_i \tilde{\Delta}_i^2 - \frac{2}{\omega_s} \tanh\left(\frac{\beta\omega_s}{2}\right) \sum_{i,j} \left[1 + \left(\frac{\omega_i - \omega_j}{\omega_s} \right)^2 \right]^{-1} (\tilde{\Delta}_i \tilde{\Delta}_j + \tilde{\Delta}_i^2 \operatorname{sgn}\omega_i \operatorname{sgn}\omega_j) \right\} . \tag{3.10}$$

Consider the limit $\beta\omega_s << 1$. Then $\delta\rho = 0$ as expected. In the low-temperature limit $\beta\omega_s >> 1$,

$$\delta\rho = \frac{4\pi T \alpha}{\omega_s \sum_i \tilde{\Delta}_i^2} \sum_{i,j} \left[1 + \left(\frac{\omega_i - \omega_j}{\omega_s} \right)^2 \right]^{-1} (2\tilde{\Delta}_i^2 - \tilde{\Delta}_i \tilde{\Delta}_j - \tilde{\Delta}_i^2 \operatorname{sgn}\omega_i \operatorname{sgn}\omega_j) . \tag{3.11}$$

This is clearly positive. Thus $\delta\rho$ increases. Since the eigenvalues increase as temperature decreases (other things being held fixed) an increase of $\delta\rho$ implies that one has to go to a higher temperature to find $\delta\rho = 0$, i.e., to get to the superconductor–normal-state transition. Thus there is decreased pairbreaking. The effect of Cooper-pair scattering by spin waves is to transfer strength originally in static spin mediated interaction to dynamic interaction. If this transfer of strength is constrained by sum rules and the momentum dependence of the interaction is left unaltered H_{c2} will necessarily increase. We now present results on the size of this increase as well as on the possible decrease of H_{c2} if (as is quite likely), the coupling to spin waves is stronger (i.e., $\alpha' > \alpha$) for reasons discussed earlier.

IV. NUMERICAL CALCULATIONS OF PAIRBREAKING

To estimate the magnitude of the change in pairbreaking due to inelastic effects, we have numerically solved the Eliashberg equations (2.1) and (2.2) including the effects of phonons, spin waves, and the effects due to elastic scattering off magnetic fluctuations. The phonons and spin waves are represented in the Einstein model, with frequencies ω_E and ω_S, respectively. We calculate the superconducting transition temperature for various values of an elastic pairbreaking parameter (proportional for instance to an external field plus elastic scattering off the magnetic fluctuations). The results are shown in Fig. 2. Curve (a) is for the case when no spectral weight is transferred to the spin waves, curve (b) for the case that 25% of the spectral weight is transferred to the spin-waves with the coupling constant to the spin waves and the elastic part left unchanged, and curve (c) is for the case with once again 25% of the spectral weight in the spin waves but with the coupling constant to the spin waves increased by 25%. As discussed earlier for nested Fermi surfaces, this is the direction in which the coupling constant will change.

Comparing curves (a) and (b), we find that higher pairbreaking is tolerated with spin waves. H_{c2} would thus tend to be higher in the antiferromagnetic phase than the AG value because below $T = T_N$, one will pass from curve (a) to (b). Curve (c) shows, however, that this effect can be overridden by a larger coupling constant to spin waves, so that a lower H_{c2} is obtained in the antiferromagnetic phase from that given by the AG value.

Turning to the experimental results, careful analysis of the data on antiferromagnetic superconductors $Gd_{1.2}Mo_6S_8$ and $Tb_{1.2}Mo_6S_8$ has revealed that pairbreaking increases above the AG value near the antiferromagnetic transition and saturates to about 10% above it at low temperatures. On the other hand, for $SmRh_4B_4$ the observed H_{c2} vs T data show

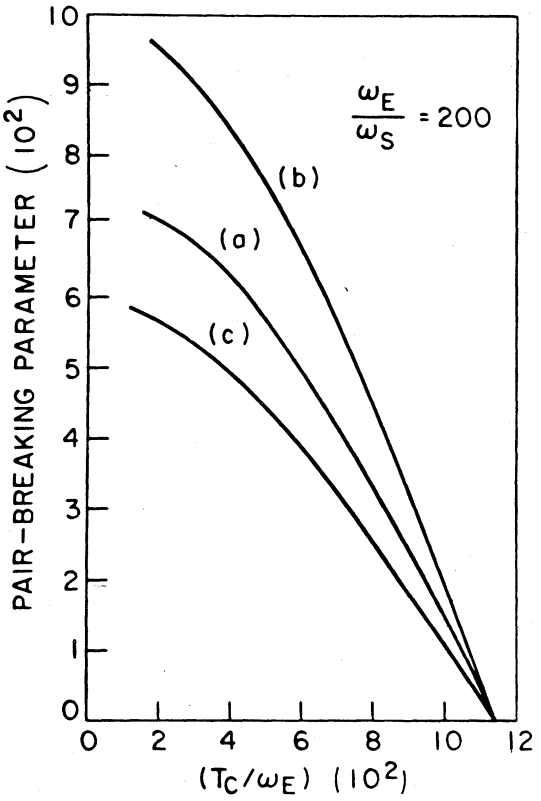

FIG. 2. Numerical results for (T_c/ω_E) from a solution of the Eliashberg equations for various values of the pairbreaking parameter. Curve (a) is for elastic scattering alone, curve (b) for the case when 25% of the spectral weight is transferred to spin waves with $\omega_s/\omega_E = 5 \times 10^{-3}$ with the coupling constant left unchanged, curve (c) for same as (b) but with coupling constant to spin waves enhanced by 25% compared to that for elastic processes.

a decreased pairbreaking in the antiferromagnetic phase. These observations would be consistent with the theory presented here if in the first two compounds antiferromagnetism is driven primarily by a nesting instability of the Fermi surface, which would lead to large superconducting magnetic coupling constants for spin-wave scattering. If in the third compound antiferromagnetism arises primarily from dipolar interaction, coupling constants for spin-wave scattering need not be enhanced. Note that the magnitude of the effects, of order 10% is what one gets with reasonable parameters in the calculations shown in Fig. 2. A detailed comparison is not possible since the $\vec{q}$ dependence of coupling constant depends upon myriad details of the conduction-electron structure.

Previous calculations of the effect of antiferromagnetism on pairbreaking have ignored the important question of the $\vec{q}$ dependence of the coupling and in one instance ignored the sum rule on the susceptibility.

Note added in proof. We briefly discuss the effect of longitudinal spin correlation on H_{c2}, well below T_N. There are two effects which compete and we believe, nearly cancel each other. There is a third which, depending on the shape of the Fermi surface, i.e., on whether $\chi(q)$ peaks near $\vec{G}$ or not, will increase or decrease H_{c2}. The first two effects arise from the fact that what affects the longitudinal spin fluctuations above T_N is partly sublattice magnetization below T_N. The effect of this new periodic potential on density of states, BCS pairing, etc., is most easily worked out in terms of the paramagnetic Bloch states, and in the clean limit. It leads to a reduction in H_{c2} (Nass, Levin, and Grest[19]). However, this same freezing eliminates the longitudinal part of spin disorder scattering. Of the Abrikosov-Gorkov pairbreaking, the part amounting to S^2 at $T=0$ in the sum rule, Eq. (26) is no longer present. This tends to raise H_{c2}, and has not been considered by Nass, Levin, and Grest. We argue that these two effects cancel in the dirty limit, where the momentum coherence length is short, shorter than any length associated with coupling to spins. In such a situation, *any* static arrangement of spins which has no net moment will have the same pairbreaking effect. It is difficult to see this in an approach starting from the opposite or clean limit. However, a hint of this is the finding of Nass, Levin, and Grest that in the presence of sublattice magnetization, turning on weak random potential scattering leads to pairbreaking. The third effect is due to the longitudinal correlation function peaking around $\vec{G}$ well below T_N, while it is flat well above T_N. If the joint density-of-states factor $\phi(q)$ [Eq. (2.11)] also peaks at $\vec{G}$, this will increase H_{c2}, an effect whose size is difficult to calculate as discussed above.

[1]O. Fischer, A. Treyvaud, R. Chevrel, and M. Sargent, Solid State Commun. 17, 721 (1975); W. A. Fertig, D. C. Johnson, L. E. DeLong, R. W. McCallum, M. B. Maple, B. T. Matthias, E. Corenzwit, J. M. Vandenberg, and H. E. Barz, Proc. Nat. Acad. Sci. U.S.A. 74, 1334 (1977).

[2]D. E. Moncton, D. B. McWhan, J. Eckert, G. Shirane, and W. Thomlinson, Phys. Rev. Lett. 39, 1164 (1977).

[3]See the review article by O. Fischer, M. Ishikawa, M. Peliz-zone, and A. Treyvaud, J. Phys. (Paris) Colloq. 40, C5-89 (1979); M. B. Maple, *ibid.* 40, C5-1374.

[4]A. A. Abrikosov and L. P. Gorkov, Sov. Phys. JETP 12, 1243 (1961) [Zh. Eksp. Teor. Fiz. 39, 1781 (1960)].

[5]H. C. Hamaker, L. D. Woolf, H. B. Mackay, Z. Fisk, and M. B. Maple, Solid State Commun. 32, 289 (1979).

[6]D. Rainer, Z. Phys. 252, 174 (1972).

[7]S. Alexander, J. S. Helman, and I. Balberg, Phys. Rev. B 13, 304 (1976).

[8]D. J. Bergmann and D. Rainer, Z. Phys. 263, 59 (1973).

[9]P. G. deGennes, *Superconductivity of Metals and Alloys* (Benjamin, New York, 1965).

[10]O. Fischer, Helv. Phys. Acta 45, 329 (1972).

[11]R. F. Hassing and J. W. Wilkins, Phys. Rev. B 7, 1890 (1973).

[12]C. M. Varma, in *Superconductivity in d and f band Metals,* edited by H. Suhl and M. B. Maple (Academic, New York, 1980).

[13]K. Machida and T. Matsubara, Solid State Commun. 31, 791 (1979).

[14]A. M. Toxen, P. C. Kwok, and R. J. Gambino, Phys. Rev. Lett. 21, 792 (1968).

[15]S. Maekawa and M. Tachiki, Phys. Rev. B 18, 4688 (1978).

[16]K. Machida, J. Low Temp. Phys. 37, 583 (1979).

[17]C. Soukoulis and G. Grest, Phys. Rev. B 21, 5119 (1980); M. J. Nass, K. Levin, and G. Grest, *ibid.* 23, 1111 (1981).

[18]K. Machida, K. Nokura, and T. Matsubara, Phys. Rev. Lett. 44, 821 (1980).

[19]M. J. Nass, K. Levin, and G. S. Grest, Phys. Rev. Lett. 46, 614 (1981).