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Comments

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Interference effect in the theory of magnetotransport in a transverse configuration

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Comments on theories of magnetotransport in the Born approximation are presented. It is shown that the "interference effect" appears when the density-matrix expression in the dressed Born approximation obtained by us is expanded to second order in the scattering interaction.

We have presented a theory[1,2] of magnetotransport in the dressed Born approximation (DBA) where the divergence difficulty encountered in previous theories (see Refs. 3 and 4 for other references) has been removed by a Breit-Wigner-type collision broadening present in the density matrix. The theories[3,4] in the strict Born approximation (SBA) interpret the magnetoresistance in terms of the migration of centers of cyclotron orbits and their subsequent scattering by imperfections in solids. This is attributed to the effect of interference between the electric field and the scattering potentials. A divergence difficulty encountered in SBA theories has now apparently been removed by the assumed existence of a cutoff mechanism suitable for a particular problem. For example, Cassiday and Spector[5] have considered taking into account the inelasticities in the electron-photon scattering due to the finite energy of the phonons involved. Govind and Miller,[6] by using the "classical cutoff," have applied the theory based on the interference effect to the many-valley model of n-type germanium and found strong disagreement between the theory[6] and experiment.[7] A quantum-limit analysis of the theoretical results[6] indicates a quadratic behavior of the transverse magnetoresistance on the magnetic field, whereas the experimental results show an approximately linear behavior in the regime where electron—acoustic-phonon scattering is considered to be the predominant mechanism of scattering. Our theory[1] in DBA, when applied[8] to the band structure appropriate to n-type germanium, appears to be in good agreement with the experimental results.[7] A quantum-limit analysis[9] of the theory[1] indicates a linear behavior of magnetoresistance on a magnetic field at high fields.

Barker and Hajdu[4] have attempted to resolve the controversies regarding quantum transport theories in crossed electric and magnetic fields. They indicate that the inclusion of the interference effect produces significant modifications at intermediate and high field strengths. In reviewing the present status of magnetotransport, they support the Titeica formula,[10] which is rigorously rederived in SBA theories. Unfortunately, no relationship of the outcome of SBA theories with those of the high-field experiments is discussed in these works.[3,4] Also, no quantum-limit analysis is presented to predict the behavior at strong magnetic fields. The DBA theory presented by us is criticized[3,4] on the grounds that it does not include in it the effect of initial-state correlations (equivalent to interference effect). In a recent communication,[11] we have shown that the neglect of a Breit-Wigner-type broadening which is present in DBA gives no magnetoresistance when all terms are included in the SBA; only the Hall current is shown to prevail. This is due to the fact that the interference effect in SBA has a destructive effect on scattering transport and cancels it exactly. If either of two terms present is neglected, the usual description[10] emerges in terms of the migration of the centers of cyclotron orbits.

In most of the theoretical framework on magnetotransport, a Landau gauge is used. In this gauge, the magnetic potential is $\vec{A} = (0,Bx,0)$, where B is the magnetic field. The harmonic-oscillator wave functions so obtained are centered at $x_k = -\lambda^2 k_y$ in the x direction, where $\lambda = (\hbar c/eB)^{1/2}$ is the radius of the cyclotron orbit and k_y is the y component of the momentum and is a constant of motion in this particular gauge. When an electric field is applied in the x direction, these centers are shown (by SBA theories) to drift and to interact at the same time with the scattering centers.[3] This description thus depends, to some extent, on the choice of the gauge. If, in the Landau gauge, a transverse electric field $\vec{E}$ is applied in the y direction instead, i.e., $\vec{E} \| \hat{y}$, or a gauge in which $\vec{A} = (-By,0,0)$ can be used, a difficulty appears in the interpretation of the motion of the centers drifting along the direction of an electric field. In the gauge where $\vec{A} = (-By,0,0)$, the wave function is that of a harmonic oscillator centered at $y_k = -\lambda^2 k_x$ in the y direction,[12] while it is a plane wave in the x direction. No consistent description of the center's migration in the direction of an electric field is possible in the above setup when $\vec{E} \| \hat{x}$. This description is even more difficult if a symmetric gauge[13] $\vec{A} = (-\frac{1}{2}By, \frac{1}{2}Bx, 0)$ is used. From this point of view, the interference effect may be considered as a "gauge effect" since it depends on the gauge chosen, namely the Landau gauge.

In the DBA,[1,2] where an electric field is treated strictly as a perturbation, no drifting centers appear in the theoretical framework and hence magnetoconductivity is independent of the choice of the gauge, as it should be. These results, which are valid for all magnetic fields, give a quadratic behavior of magnetoresistance at low magnetic fields and a linear behavior at high magnetic fields for acoustic-phonon scattering, which is shown to be predominant at high magnetic fields.[9] When plotted on a log-log scale, the results should indicate a slope change from 2 to 1 to be consistent with actual observations.[14] This transition is attributed to the Wigner crystallization. In light of our analysis, this may be interpreted as a transition from the classical to the quantum regime.

In the following, in order to expose clearly the relationship between SBA and DBA, we expand the density matrix in DBA to second order in the scattering interaction to obtain the density matrix in SBA. This expansion may be considered as equivalent to expansion in terms of $(\omega_c \tau)^{-1}$ under the assumption $\omega_c >> \tau^{-1}$. This expansion is not strictly correct as τ^{-1} diverges for slowly moving electrons in the direction of the magnetic field, violating the condition $\omega_c >> \tau^{-1}$. The divergence difficulty which is not present in the DBA thus reappears when this expansion is made, but the results so obtained are comparable to those in SBA theories.

The expectation value $\langle \vec{j} \rangle$ of the current $\vec{j}$ is obtained from $\vec{J} = \mathrm{Tr}(\rho \vec{j})$, where the density matrix ρ is given by[2]

$$\rho = \rho_0(H_0) - R\hat{F}\rho_0 \ , \tag{1}$$

with

$$R = [\hat{H}_0 - \Sigma^0(s) - i\hbar s]^{-1}, \quad s \to 0+ \tag{2}$$

$$\Sigma^0(s) = \hat{V} R_0 \hat{V} \ , \tag{3}$$

$$R_0 = (\hat{H}_0 - i\hbar s) \ , \tag{4}$$

where H_0 is the unperturbed Hamiltonian of an electron in a magnetic field, ρ_0 is the equilibrium density matrix for the electron-phonon system, F is the interaction of an electron with an electric field $(F = e\vec{E} \cdot \vec{r})$, and V is the electron-lattice interaction. The carets indicate commutator-generating superoperators defind by $\hat{A}B \equiv [A,B]$. The superoperator resolvent R, which is averaged over lattice sites, can expanded to second order in V by using the Dyson equation:

$$R = R_0 - R_0 \Sigma^0(s) R \ , \tag{5}$$

with the result

$$R \approx R_0 - R_0 \Sigma^0(s) R_0 + \cdots \ . \tag{6}$$

The difference between Eqs. (5) and (6) is noteworthy. The last resolvent in Eq. (5) is R while it is R_0 in Eq. (6). This is the major difference between DBA and SBA.

In DBA, R_0 of SBA is dressed by the Breit-Wigner-type collision broadening given by the collision operator $\Sigma^0(s)$, hence the name DBA. A better approximation than DBA is the generalized Born approximation (GBA) in which $\Sigma^0(s)$ is further dressed. This is necessary in degenerate systems where the quasiparticle effect discussed by Lodder and Fujita[15] is considered important. The collision damping in the GBA is useful in interpreting low-temperature oscillatory quantum effects in degenerate systems and is normally included in a phenomenological way.[16] But for nondegenerate systems, this collision damping can be neglected. The expansion of R as in Eq. (6) gives for ρ an expression in SBA:

$$\rho = \rho_0(H_0) - R_0 \hat{F} \rho_0 + R_0 \Sigma^0(s) R_0 \hat{F} \rho_0 \ . \tag{7}$$

The first equilibrium term $\rho_0(H_0)$ does not give any contribution to the electric current. The second scattering-independent term gives the Hall current. The last term in Eq. (7) which gives scattering transport may be considered equivalent to the interference term, which appears as a result of the expansion of the DBA expression as given by Eq. (1). The matrix elements of this term in the Laudau representation [1,2] $|\alpha\rangle$, where the Landau gauge is used, are given by

$$\langle\alpha'|R_0\Sigma^0(s)R_0\hat{F}\rho_0|\alpha\rangle = -\frac{i\pi eE}{\epsilon_{\alpha'\alpha}}\sum_\beta \frac{df}{d\epsilon_\beta}V_{\alpha'\beta}V_{\beta\alpha}(x_\alpha - x_\beta)[\delta(\epsilon_{\beta\alpha})+\delta(\epsilon_{\beta\alpha'})] - i\hbar\frac{f_{\alpha'\alpha}}{\epsilon_{\alpha'\alpha}}\frac{F_{\alpha'\alpha}}{\epsilon_{\alpha'\alpha}}\tau^{-1}_{\alpha'\alpha}\ , \quad (8)$$

with

$$\tau^{-1}_{\alpha'\alpha} = \frac{\pi}{\hbar}\sum_\beta[|V_{\alpha'\beta}|^2\delta(\epsilon_{\beta\alpha}) + |V_{\alpha\beta}|^2\delta(\epsilon_{\alpha'\beta})]\ . \quad (9)$$

Here we have used the properties of the isotropic scattering interactions. $\alpha' = (n+1,k)$ and $\alpha = nk$, where n and $k = (k_y,k_z)$ are quantum numbers[1,2] in the Landau representation. All other notations are the same as used earlier.[1,2] The electric field is assumed to be applied in the x direction ($F = eEx$). The first term, in Eq. (8) appears from the diagonal matrix elements of x which has diagonal as well as nondiagonal elements in the Landau gauge [17]:

$$\langle\alpha'|x|\alpha\rangle = -x_k\delta_{n'n}\delta_{k'k} + \frac{\lambda}{\sqrt{2}} \times [(n+1)^{1/2}\delta_{n',n+1} + n^{1/2}\delta_{n',n-1}]\delta_{k'k}\ . \quad (10)$$

In the SBA, there are terms which involve interference of the Hall term $R_0\hat{F}\rho_0$, with the collision operator $\Sigma^0(s)$ [see the last term in Eq. (7)]. the matrix elements of $R_0\hat{F}\rho_0$ are

$$\begin{aligned}\langle\alpha'|R_0\hat{F}\rho_0|\alpha\rangle &= \frac{f_{\alpha'\alpha}}{\epsilon_{\alpha'\alpha}}eE\langle\alpha'|x|\alpha\rangle \\ &= \frac{df}{d\epsilon_\alpha}eEx_k + \frac{f_{n+1}-f_n}{\hbar\omega_c}\frac{\lambda}{\sqrt{2}}(n+1)^{1/2}\delta_{n',n+1}\delta_{k'k} - \frac{f_{n-1}-f_n}{\hbar\omega_c}\frac{\lambda}{\sqrt{2}}n^{1/2}\delta_{n',n-1}\delta_{k'k}\ .\end{aligned} \quad (11)$$

The interference of the first term in Eq. (11) with $\Sigma^0(s)$ gives the first term of Eq. (8). This occurs as a result of the neglect of the Breit-Wigner-type collision broadening in the Hall term. If the Hall term is dressed as in the DBA ($R_0\hat{F}\rho_0 \rightarrow R\hat{F}\rho_0$), the diagonal elements disappear. This is due to the fact that the matrix elements of this term $f_{\alpha'\alpha}(\epsilon_{\alpha'\alpha} - i\hbar\tau^{-1}_{\alpha'\alpha})^{-1}$ have no diagonal part ($\alpha' = \alpha$), but when $\hbar\tau^{-1}_{\alpha'\alpha} \rightarrow 0$, $f_{\alpha'\alpha}\epsilon^{-1}_{\alpha'\alpha} \rightarrow df/d\epsilon_\alpha$ for $\alpha' = \alpha$. This is the reason why centers appear in SBA, but do not show up in the DBA. In the DBA $R_0\Sigma^0(s)R_0\hat{F}\rho_0$ is replaced by $R_0\Sigma^0(s)\rho'$ in the kinetic equation, where ρ' is the nonequilibrium density matrix, whose self-consistent solution gives Breit-Wigner-type collision broadening. if one attempts to calculate magnetoconductivity by taking $\vec{E}\,\|\,\hat{y}$, no centers will appear even in SBA. We thus conclude that the interference effect is present because of the special choice of gauge and direction of the electric field chosen in the SBA.

The first term in Eq. (8), when used for finding the expectation value of the current by using $\vec{J} = \mathrm{Tr}(\rho\vec{j})$, gives the Titeica formula, while the second term will yield σ_{xx} of the DBA expanded to first order in $(\omega_c\tau)^{-1}$. Equation (8) vanishes for the classical case of large quantum numbers ($\alpha' \approx \alpha$) at low magnetic fields and hence is described as a quantum-mechanical effect.[18].

We summarize the above arguments by stating that the DBA gives results consistent with the theoretical framework as well as the experiment observations, which are free from the divergence difficulty, and can be usefully exploited for the correct interpretation of experimental results if appropriate scattering mechanisms are included.

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