PHYSICAL REVIEW B VOLUME 23, NUMBER 1 1 JANUARY 1981

Target dependence of effective projectile charge in stopping powers

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(Received 29 April 1980)

The theory of the effective ion charge, $Z_1^* e$, in stopping powers is tested for ion velocities v_1 near $v_0 = e^2/\hbar$ through measurements of relative stopping powers for He^+ and D^+ in foils of Au($r_s = 1.49$), C(1.66), Al(2.12), and Cs(5.88), where r_s is related to the conduction-electron density, n_e, as $r_s = (3/4\pi n_e a_0^3)^{1/3}$. One finds a strong material dependence of Z_1^*. It can be accounted for by referring to the relative ion-electron velocities, v_r, which include the Fermi velocity $v_F = 1.919 r_s^{-1}$ (a.u.) of the target electrons. The results may pertain to the stopping of charged particles in materials at very high pressures and temperatures.

The effective charge theory of the stopping power of solids for swift ions has become a powerful tool to correlate comprehensively a large body of experimental data.[1-5] The rapidly growing interest, moreover, in laser- and heavy-ion-induced inertial confinement fusion raises important questions as to the dependence of the effective charge of light and heavy ions and, hence, their rate of kinetic-energy loss on the properties of the medium under extreme conditions of temperature and pressure. It is the purpose of this letter to present experimental stopping-power data that pertain to these questions.

The basic tenet of the effective-charge theory is that ions of atomic number Z_1, moving with velocity $v_1 >> v_0 = e^2/\hbar$, interact with the electrons of the medium with an effective charge $Z_1^* = Z_1 f(v_1)$ where the function $f(v_1) \leq 1$ depends primarily on v_1 and only weakly on the properties of the substance in which the ions move. That is, one writes the stopping power, S, as

$$S(Z_1, Z_2; v_1) = [Z_1^*(v_1)]^2 S_0(Z_2; v_1) \quad , \tag{1}$$

where S_0 is the stopping power per unit ion charge of the material composed of the element with atomic number Z_2. If taken in the limit of vanishing ion charge, S_0 can be calculated exactly in the first Born approximation.[6] For real projectiles, S_0 contains Z_1-dependent contributions from Bloch terms[7] and Z_1^3 terms[8] which are negligibly small for light ions and largely cancel for heavy ions.[5,9] Therefore, they are neglected in the following.

As it stands, Eq. (1) has been demonstrated to apply on the average to stopping power data of solids at velocities $v_1 >> v_0$ of ions with Z_1 ranging from 1 to 92.[5,9] There is growing interest in the stopping powers of light ions such as ${}^1_1H$, ${}^2_1H$, ${}^3_1H$ ($Z_1 = 1$) and ${}^3_2He$, ${}^4_2He$ ($Z_2 = 2$) at low velocities, where the effective charge analysis has given first indications that protons have an effective $Z_H^* < 1$ as $v_1 \rightarrow v_0$.[5] Theoretical treatments[10] have attacked the effective-charge problem at very low velocities, $v_1 << v_0$, where $S_0 \propto v_1$.

One may loosely interpret Z_1^* as the number of electrons that, with regard to the stopping process, are effectively stripped off the swift ion by the medium. A stripping criterion is that the charge of the moving nucleus is screened by a coterie of electrons with orbital velocities higher than the ion velocity v_1. As v_1 approaches the mean velocity, v_e, of the valence electrons of the medium, the atomic cores in the medium can no longer be exited (inner-shell corrections to stopping powers), and the electronic stopping power, in particular of metals, is determined by the conduction electrons. The effective charge should then not merely depend on v_1 but, in fact, on the relative velocity $v_r = v_r(v_e, v_1)$ between the projectiles and the conduction electrons. It is the purpose of this investigation to test this new prediction of the effective charge theory experimentally.

Our experimental results are shown in Fig. 1. We find a strong material dependence of the relative energy losses $\Delta E_{He}/\Delta E_D$ as measured in the same foil at equal velocities, where $\Delta E \approx 10$ keV and $\Delta E/E \sim 0.1$ depending on the ion-target combination. The target materials were chosen to vary v_e as widely as possible relative to our accessible velocity range $0.7 v_0 \lesssim v_1 \lesssim 1.5 v_0$ common to D^+ and He^+, corresponding to energies ranging from 12.5 keV/nucleon to 50 keV/nucleon. The temperature, T, of the metals in the laboratory, with conduction electrons of density n_e and Fermi energy E_F, is always such that $\Theta \equiv kT/E_F << 1$. The electron velocity, v_e, is then comparable to the Fermi velocity $v_F = (2E_F/m)^{1/2}$ which can be expressed in terms of the one-electron radius, r_s, in the medium as $v_F = 1.919 r_s^{-1} v_0$, where

$$(\tfrac{4}{3}\pi)(r_s a_0)^3 n_e = 1$$

with $a_0 = \hbar^2/me^2 = 0.529$ Å and $v_0 = e^2/\hbar = 2.18 \times 10^8$ cm/sec. As indicated in Fig. 1 through the respective

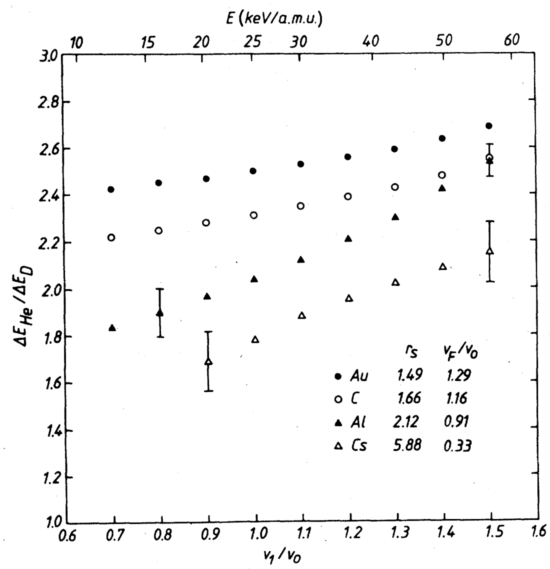

FIG. 1. Ratios of energy-loss data for He^+ ions ΔE_{He} and D^+ ions ΔE_D, measured in transmission at the same ion velocities v_1, or ion energies E_1 per nucleon, under otherwise identical conditions on foils of Au($r_s = 1.49$), C(1.66), Al(2.12), and Cs(5.88). The uncertainties in the data are indicated.

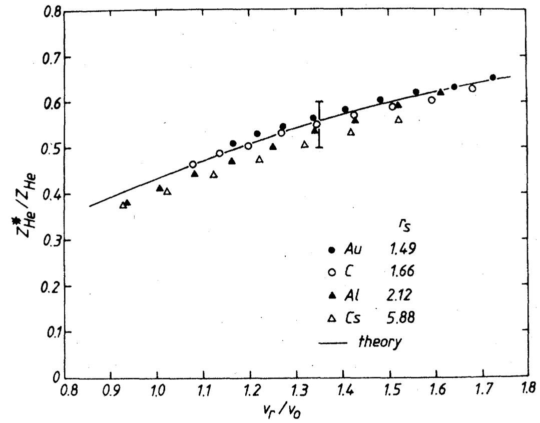

FIG. 2. Effective stopping-power charge of He projectiles, $Z_1^* = Z_{He}^*$, relative to $Z_1 = Z_{He} = 2$, as a function of the relative velocities between the ions and the electrons in the medium in atomic units, v_r/v_0. The data presented in Fig. 1 are plotted as given in Eq. (2). The relative velocities are calculated in terms of v_1 and r_s according to Eq. (4). Within the uncertainties indicated, the data agree in this representation with each other. The theoretical curve represents $Z_1^*(v_r)/Z_1$ evaluated for the Lenz-Jensen statistical model of the atom (Refs. 4 and 5). The strong material dependence displayed in Fig. 1 is manifest only in its effect of the conduction-electron velocities on $v_r = v_r(v_e(r_s), v_1)$.

r_s values, the conduction-electron velocities increase by a factor 4 in the series Cs($r_s = 5.88$), Al(2.12), C(1.66), and Au(1.49).[11]

Energy losses ΔE were measured by standard transmission techniques.[12] Details and the measured stopping powers will be published elsewhere. The preparation of Cs foils posed special problems.[13] After many trials, we settled on the evaporation *in situ* under ultra vacuum conditions (10^{-10} Torr) of approximately 30 $\mu g/cm^2$ of Cs on a cooled Au foil for which the energy loss had been measured beforehand. The Cs increased the energy loss in the composite foil typically by a factor of 2. The energy losses and ΔE_{He} and ΔE_D were determined from the centroid shifts of the distributions with and without foil. We estimate the combined uncertainties in ΔE to be smaller than $\pm 5\%$ for the C, Al, and Au foils, and $\pm 7\%$ for the Cs films on Au substrates.

To test whether $Z_1^*(v_r)$ can account for the pronounced material dependence displayed in Fig. 1, we have replotted the data in the form ($Z_D = 1$; $Z_{He} = 2$)

$$\frac{Z_{He}^*}{Z_{He}} = \frac{Z_D^*}{Z_{He}} \left[\frac{\Delta E_{He}}{\Delta E_D} \right]^{1/2} \tag{2}$$

vs $v_r(v_1, v_e)$. The effective deuteron charge, Z_D^*, at different v_1 was taken to be equal to the empirical values of Z_H^* given in Ref. 5. We calculated v_r by averaging over the difference between the electron velocity $\vec{v}_e$ and the ion velocity $\vec{v}_1$ assuming an isotropic distribution of the conduction-electron velocities, and found that

$$v_r = \langle |\vec{v}_1 - \vec{v}_e| \rangle = \frac{v_e}{6} \frac{(v_1/v_e + 1)^3 - |v_1/v_e - 1|^3}{v_1/v_e} . \tag{3}$$

Averaging over the Fermi sphere, $0 \leq v_e \leq v_F$, yields

$$v_r = \frac{v_F}{10} \frac{(v_1/v_F + 1)^3 - |v_1/v_F - 1|^3 + 4(v_1/v_F)^2 + \theta(v_1/v_F - 1)[\frac{3}{2} v_1/v_F - 4(v_1/v_F)^2 + 3(v_1/v_F)^3 - \frac{1}{2}(v_1/v_F)^5]}{v_1/v_F} , \tag{4}$$

where $v_F/v_0 = 1.919 r_s^{-1}$ and $\theta(x) = \frac{1}{2}(1 - x/|x|)$ denotes the step function. Equation (4) has the proper limit for $v_1 \ll v_F$. In the range of the validity of Eq. (1) for $v_1 \gtrsim v_F$ Eq. (3) agrees with Eq. (4) to $<1\%$ if we insert $v_e = (\frac{3}{5})^{1/2} v_F$ in Eq. (3) corresponding to the mean kinetic energy in the electron gas, $\frac{1}{2} m v_e^2 = \frac{3}{5} E_F$.

The results are shown in Fig. 2 and compared with the effective charge theory given in Ref. 4, calculated for the Lenz-Jensen model of the atom.[5,14] The large material dependence displayed in Fig. 1 has been resolved by attributing it to the change of v_e with r_s

through the relative ion-valence electron velocities. Whether there is a small residual trend in the data with r_s, just within our present experimental uncertainties, requires further study.

We conclude that effective charge theory describes the stopping power of metals for atomic projectiles even when $v_1 \sim v_F$, provided that the stripping criterion is applied to the relative velocities between the projectiles and the conduction-plasma electrons. Although demonstrated here for He ions, we expect this conclusion to hold for heavier ions as well. Based on these conclusions one may expect that the stopping power of materials at very high pressures and temperatures such that $\theta \sim 1$ can be predicted by calculating $Z_1^*(v_r)$. The stripping criterion is applied to v_r as given by Eq. (3), where now

$$v_e \simeq \left[\frac{2}{m}\left(\tfrac{3}{5}E_F + \tfrac{3}{2}kT\right)\right]^{1/2} = \left(\tfrac{3}{5}\right)^{1/2} v_F\left(1 + \tfrac{5}{2}\Theta\right)^{1/2} \quad (5)$$

interpolates between the degenerate ($\Theta \ll 1$) and the nondegenerate ($\Theta \gg 1$) state of the plasma.

ACKNOWLEDGMENTS

The work was supported by the Bundesministerium für Forschung und Technologie, Bonn, Germany. One of us (W.B.) is grateful for the hospitality extended him during the tenure of his Humboldt Award by Professor R. Sizmann at the Sektion der Physik, Universität München.

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[1]N. Bohr, Phys. Rev. 58, 654 (1940); 59, 270 (1941).

[2]W. E. Lamb, Phys. Rev. 58, 696 (1940).

[3]J. H. M. Brunig, J. K. Knipp, and E. Teller, Phys. Rev. 60, 657 (1941).

[4]W. Brandt, in *Atomic Collisions in Solids* (Plenum, New York, 1975), Vol. 1, pp. 261–288.

[5]B. S. Yarlagadda, J. E. Robinson, and W. Brandt, Phys. Rev. B 17, 3473 (1978).

[6]H. A. Bethe, Ann. Phys. (Leipzig) 5, 325 (1930); cf. also discussions in G. Basbas, W. Brandt, and R. Laubert, Phys. Rev. A 7, 983 (1973) and G. Basbas, W. Brandt, and R. H. Ritchie, *ibid.* 7, 1971 (1973), footnote 9.

[7]F. Bloch, Ann. Phys. (Leipzig) 16, 285 (1933).

[8]J. C. Ashley, R. H. Ritchie, and W. Brandt, Phys. Rev. B 5, 2393 (1972).

[9]R. H. Ritchie and W. Brandt, Phys. Rev. A 17, 2102 (1978).

[10]T. L. Ferrell and R. H. Ritchie, Phys. Rev. B 16, 115 (1977).

[11]D. Isaacson, *Compilation of r_s Values*, New York University, Document No. 02698 (National Auxiliary Publication Service, New York, 1975).

[12]We are grateful to Dr. H. J. Maier and his co-workers for the preparation and characterization of the foils.

[13]H. Jakusch, thesis (University of Karlsruhe, 1973) (unpublished); S. Kreussler, thesis (University of Munich, 1980) (unpublished).

[14]P. Gombás, *Die statistische Theorie des Atoms und ihre Anwendungen* (Springer Verlag, Wien, 1949), p. 71 ff.