# Collective modes of spatially separated, two-component, two-dimensional plasma in solids

S. Das Sarma and A. Madhukar
*Department of Physics, University of Southern California, Los Angeles, California 90007*
*and Department of Materials Science, University of Southern California, Los Angeles, California 90007*
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Longitudinal collective spectrum of a spatially separated, two-component, two-dimensional plasma is investigated using a generalized random-phase approximation. Such plasmas can be realized in semiconductor heterojunctions, superlattices, and inversion or accumulation layers. In general two modes are found to exist: the one with energy proportional to $\sqrt{q}$ at long wavelengths is the usual optical plasmon and the other with energy proportional to $q$ at long wavelengths is the acoustic plasmon. It is shown that the spatial separation between the two charge components makes it possible for the acoustic branch to move out of the electron-hole continua of both the components provided it exceeds a critical value. Consequently the acoustic-plasmon mode is totally undamped at long wavelength. The dynamic structure factor of the system is analyzed and the feasibility of observing an acoustic plasmon in GaAs-$Ga_xAl_{1-x}As$ double quantum well is discussed.

## I. INTRODUCTION

Recently there has been a great deal of interest in the properties of two-dimensional electronic systems.[1] The metal-oxide-semiconductor (MOS) system and the electron gas trapped by image potential at the liquid-He surface have, in the past decade, offered opportunity for the study of some new and novel areas of physics in fewer than three dimensions. Current interest in two-dimensionally-confined charge carriers has been enhanced by the recent advances[2,3] made in the experimental realization of such systems in quantum wells, heterojunctions, and superlattice structures made up of lattice-matched semiconductors. These new systems offer not simply another means of achieving two-dimensional electron and hole gas, but of creating situations with potentially new physics. The two systems on which most of the studies have been reported so far are the GaAs-$Ga_xAl_{1-x}As$ and InAs-GaSb structures. Both of these systems offer in very different ways possibilities of creating two-dimensionally-confined and spatially-separated charged multicomponent plasmas. The former system has been combined with the notion of modulated doping to create electrons confined within the GaAs layers due to the barriers provided by the adjacent $Ga_xAl_{1-x}As$ layers. In the InAs-GaSb system, a spatial separation of electrons and holes is found to exist in the heterojunction. This has been attributed to the transfer of charge (~$10^{17}$ $cm^{-3}$) from GaSb to InAs due to an unusual line-up of the bulk energy bands: At the $\Gamma$ point, a band line-up based upon the prevalent electron affinity rule shows that the valence band of GaSb lies approximately 140 meV above the conduction-band edge of InAs. Thin ($\lesssim$125 Å) superlattices of InAs-GaSb are found to be semiconducting and they also exhibit spatial separation and confinement of electrons (in InAs) and holes (in GaSb). However, superlattices of thicknesses between ~125 Å and the appropriate depletion length have been reported to be semimetallic in character normal to the interface. Various aspects of these superlattices in this intermediate thickness regime remain to be clarified. However, it is clear that the quantum well structures of GaAs-$Ga_xAl_{1-x}As$ and the InAs-GaSb heterojunctions offer systems which give rise to spatially-separated two-dimensionally-confined charged plasmas. In addition, one can obtain spatially-separated, multicomponent, two-dimensional plasmas in the MOS inversion layer by having more than one subband populated. This is easily achieved in InSb inversion layers at moderate densities.[4] In the inversion-layer situation, one is, however, limited in the freedom to manipulate the spatial separation between the charge components of the plasma. In this paper we report the results of a study of the collective electronic modes of such a multicomponent, spatially-separated, two-dimensional plasma. Even though in the true, repeated superlattice structure there would be very many charge components of such a plasma, we restrict our study to a two-component system to illustrate the essential physics involved. Thus our results will be strictly valid for the double quantum wells of GaAs-$Ga_xAl_{1-x}As$, heterojunctions of InAs-GaSb, and an inversion or accumulation layer with two subbands occupied by the charge carriers. Generalization to systems with more than two charge components is, however, straightforward within the scheme we develop here.

It has long been known[5] that a two-component plasma has two branches to its longitudinal oscillation spectrum. In the higher-frequency branch, the two carriers oscillate out of phase (in phase) with the long-wavelength limit at the square root of the sum of the squares of individual plasma frequencies of the two components, while in the lower branch the carriers oscillate in phase (out of phase), exhibiting at long wavelengths a linear dispersion like that of a sound wave, assuming the two carriers have opposite (same) charge. Pines[6] first discussed the relevance of these modes to multicomponent, solid-state plasmas as occurring, for example, in bulk semiconductors. The high-frequency branch has been called "optical plasmon" (OP) and the lower branch "acoustic plasmon" (AP), referring to its soundlike long-wavelength dispersion. Even though the mode corresponding to AP was observed[7,8] in ionic plasmas more than fifteen years ago, no such acoustic branch has been detected in solid-state plasmas in spite of considerable experimental efforts.[9] The reason lies primarily in the large damping associated with AP.[10] In bulk plasmas where all the experiments so far have been carried out, AP lies in the low-frequency regime and is inside the single-particle excitation spectrum of the faster moving charge carrier and is thus severely Landau damped (apart from any collisional damping associated with phonons and impurities, which can be reduced by working at low temperatures and by using sufficiently pure samples).

Results presented in this paper[11] indicate that the ideal solid-state systems to look for the AP are these new and novel two-dimensionally-confined, spatially-separated, multicomponent structures, in particular the quantum wells and the heterojunctions. We find that when the spatial separation between the charge components exceeds a critical distance ($d_c$), such a system is capable of supporting a mode of collective oscillation which exists in the high-frequency regime (outside the electron-hole continua of both the components), is undamped up to some critical wave vector, and has a frequency in the long-wavelength limit which is proportional to the wave vector $q$ with a proportionality constant (i.e., phase velocity $c$) which is higher than the Fermi velocities of both the components and also the sound velocity. In this regime of separation ($d>d_c$), there is no low-frequency collective mode of the system lying between the two electron-hole continua. Thus by virtue of the spatial separation between the two components, the AP becomes a high-frequency undamped mode at long wavelengths coming out of the single-particle regimes of both components. This is further borne out by an analysis at small separation where we find the usual AP lying within the electron-hole continua of the two components. However, the phase velocity of this low-frequency AP is greatly enhanced by the spatial separation, thus indicating that at large enough separation the phase velocity of the AP may exceed the Fermi velocity of the faster moving charge component, thus bringing the mode out of the Landau damping region. The OP, which goes as $q^{1/2}$ in two dimensions, remains unaffected by spatial separation in leading order and obviously is undamped at long wavelength. (See Fig. 1.)

The possibility of having a high-frequency mode with linear dispersion in a two-component, two-dimensional plasma was earlier anticipated by Equiluz *et al.*[12] in the context of silicon inversion layers. They numerically obtained the dispersion curves for the collective modes of a double-inversion-layer geometry for various values of the separation between the two layers. Our calculation, which is completely analytic, shows the connection of this linear mode to the usual acoustic plasmon. In addition, our analysis manifestly shows the importance of the spatial separation with respect to the damping of the AP. In particular, the fact that one can have an undamped AP mode when $d$ exceeds $d_c$ is emphasized in our work. Our analysis clearly suggests a possible experimental realization of the linear mode in the new and novel semiconducting heterojunctions, quantum wells, and superlattices. In this sense our work is a generalization of that in Ref. 12 to include collisional damping and arbitrary separation (including zero separation) between the two charge components.

An alternative way of visualizing this linear

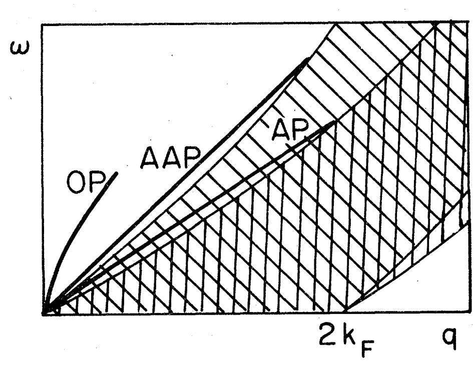


FIG. 1. Shows schematically the wave-vector dependence of the different collective modes with respect to the single-particle continua of the two charge components. OP refers to the optical plasmon, whereas AP is the damped acoustic plasmon in the normal situation. AAP is the undamped (anomalous) acoustic mode in the situation of large spatial separation between the two charge components. Note that for any given separation only one of the two acoustic plasmons can exist.

mode in the coupled, spatially-separated two-component, two-dimensional plasma is provided by the collective electronic modes of a thin metallic film. At large separation the OP and the AP are analogous to the symmetric and the antisymmetric coupled surface plasmons of a thin film localized at the two surfaces, respectively. For small separation or when $d=0$, these modes, however, become the usual OP and the damped AP of homogeneous two-component systems. We shall, however, refer to the high-frequency (going as $q^{1/2}$) mode as OP and the low-frequency mode (going as $q$) as AP in this paper for all separations.

The advantage of this new class of two-dimensional systems discussed above is the controllability of spatial separation and the possibility of having $d$ exceed $d_c$, so that the AP becomes undamped and the main difficulty in observing this acoustic branch is circumvented. There is a more subtle, but associated advantage as well. This has to do with the masses and the Fermi velocites of the charge components composing the plasma. In the bulk three-dimensional unseparated plasma, the Fermi velocities as well as the masses of the two charge components need to be very different[9] from each other for the existence of the AP. For the case of spatially-separated plasmas ($d>d_c$) such that the AP is undamped at long wavelengths, the acoustic branch exists as a well defined mode outside the single-particle spectra even when the two separated charge components are identical (i.e., same mass and Fermi velocity). This remarkable feature, which is totally absent in unseparated plasmas makes the GaAs-$Ga_xAl_{1-x}As$ structures perhaps the most suitable candidate for the experimental realization of the AP. In Fig. 2 a schematic representation of the energy diagram of the GaAs-$Ga_xAl_{1-x}As$ quantum-well structure is given. The formation of two-dimensionally confined bands arises from the band-edge discontinuities which provide potential barriers to motion normal to the interface between GaAs and AlAs. By doping AlAs layers only, it is possible to transfer electrons from the donor states in AlAs to the quantized two-dimensional bands in the GaAs layers. Two such structures side by side produce a double quantum well with the two two-dimensional electron gases separated spatially by the AlAs layer. The value of $d_c$ needed for the existence of undamped AP can be experimentally achieved in GaAs-$Ga_xAl_{1-x}As$ double-quantum-well structures. In the next section we give an analysis within the generalized random-phase approximation (RPA) to obtain the collective modes and their existence criteria. In Sec. III we discuss the results with particular

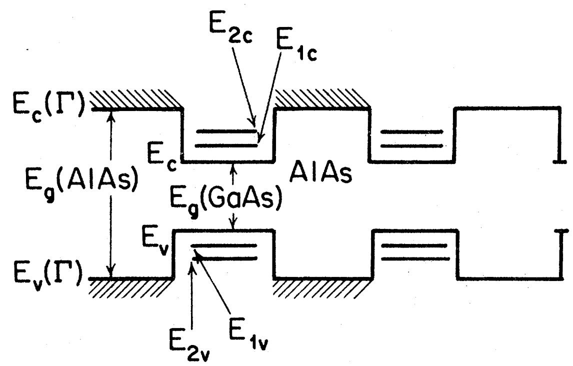


FIG. 2. Shows schematically double quantum well of GaAs-GaAlAs structure. The two-dimensional subbands in the conduction and valence bands of GaAs are shown. Modulation $p$-type doping of AlAs layers populates the conduction subbands in GaAs in both of the quantum wells, giving rise to spatially-separated, two-component, two-dimensional plasma.

emphasis on the experimental observability of the AP. We conclude in Sec. IV by summarizing our results.

## II. THEORY

For a one-component plasma the collective modes are given by the poles of the inverse dielectric function, or equivalently by the zeros of the dielectric function.[9] For a multicomponent system the collective modes are given by the poles of the inverse dielectric tensor $\epsilon^{-1}$, which is a straightforward generalization of the ordinary dielectric function of the one-component plasma. Denoting the two-dimensional bands in which the charge carriers move by $i,j$, etc. (as shown in Fig. 2) we can write the $i$-$j$ component of the dielectric tensor within RPA to be

$$\epsilon_{ij}(\vec{q},\omega)=\delta_{ij}-V_{ij}(\vec{q})\Pi^0_{jj}(\vec{q},\omega)\,. \quad (1)$$

In Eq. (1), $\vec{q}$ is the wave vector parallel to the two-dimensional plane of confinement (i.e., parallel to the interface) and $\omega$, the frequency, is implicitly understood to carry a small positive imaginary part so that all dynamic quantities are retarded functions. $\delta_{ij}$ is the usual Krönecker delta function and $\Pi^0_{jj}(\vec{q},\omega)$ is the intraband noninteracting polarizability function for the $j$th two-dimensional band. $V_{ij}(\vec{q})$ stands for the Coulomb interaction vertex $V_{iijj}(\vec{q})$ in which an electron in $i$th band interacts with another one in band $j$ with two-dimensional momentum exchange $\hbar\vec{q}$. In general, the Coulomb vertex $V_{ijlm}(\vec{q})$ is a fourth-rank tensor, allowing for the possibility of interband scattering of electrons. But in our analysis we make the so-called diagonal approximation, in which we neglect any interband component of

Coulomb interaction by assuming $V_{ijlm}$ to vanish unless $i=j$ and $1=m$. The diagonal approximation is exact in the limit of $q\to 0$ due to the orthonormality of wave functions describing the various bands. Even for finite $\vec{q}$, the off-diagonal elements are usually much smaller than the diagonal elements.[13] We should point out that it is a consequence of the diagonal approximation (along with the generalized RPA) that the dielectric tensor can be written in the simple form given by Eq. (1). In general, the dielectric function is a tensor of fourth rank and the general structure of a theory of collective modes in this multicomponent system including both interband and intraband contributions is much more complicated (Appendix A).

The actual bound-state wave functions for the quantization of motion along the $z$ direction (normal to the interface) in these systems are not known. We can thus only make qualitative remarks about the effect of the finite width of the wave functions on the properties of the collective modes discussed here. One-electron wave functions describing the carriers in these systems can, in general, be written as

$$\psi_i(\vec{r}) \sim e^{i\vec{q}\cdot\vec{r}_{11}}\xi_i(z)\,, \tag{2}$$

where $\vec{r}_{11}$ is a two-dimensional position vector in the plane of free motion of the carriers and $\xi_i(z)$ is the envelope wave function describing the confinement in $z$ direction for the $i$th band. For a two-component system we need consider only two bands and for algebraic simplicity, we assume $|\xi_i(z)|^2$ to be delta functions located at $z=a$ and $b$ for the two bands without any loss of generality. Changing $\xi_i(z)$ to more complicated localized functions does not change the conclusions of this paper in any qualitative fashion. The interaction vertex $V_{ij}(\vec{q})$ is given by

$$V_{ij}(\vec{q}) = \langle ii\,|\,V(\vec{q};z,z')\,|\,jj\rangle\,, \tag{3}$$

where $V(\vec{q},z,z')$ is the two-dimensional Fourier transform of Coulomb interaction. It is given by

$$V(q,z,z') = \pm\frac{2\pi e^2}{\bar{\kappa}q}\,e^{-q|z-z'|}\,, \tag{4}$$

where the + (−) sign refers to interaction between the same (opposite) types of charge. Since the static lattice dielectric constants $\bar{\kappa}_1$ and $\bar{\kappa}_2$ of the semiconductors used in the systems (InAs-GaSb, GaAs-AlAs) are very close,[14] we use an average dielectric constant $\bar{\kappa}$ in Eq. (4) to avoid unnecessary complication. From Eqs. (3) and (4) and using the delta-function forms for the envelope functions, we can write down the various components of Coulomb interaction as follows:

$$V_{11}(\vec{q}) = V_{22}(\vec{q}) = \frac{2\pi e^2}{\bar{\kappa}q}\,, \tag{5}$$

$$V_{12}(\vec{q}) = V_{21}(\vec{q}) = \pm\frac{2\pi e^2}{\bar{\kappa}q}\,e^{-q|a-b|}\,. \tag{6}$$

In writing Eqs. (5) and (6), we explicitly consider the two-component case, letting $i=1,2$ denote the two different charge carriers in the two bands.

We note that the spatial separation between the two carriers gives different strengths to different components of Coulomb interaction. In purely two-dimensional plasma, the magnitude of Coulomb interaction would be $2\pi e^2/\bar{\kappa}q$ for all the components. The separation $|a-b|$, in effect, reduces the strength of interaction between the components by the factor $\exp(-q|a-b|)$. Since this interaction between the two components is responsible for the AP, one can already anticipate drastic effects on the properties of AP by the introduction of the new parameter $qd$, where $d=|a-b|$ is the separation of the two charge components.

The noninteracting polarizability $\Pi^0_{ii}(\vec{q},\omega)$ in two dimensions has the following limiting forms[15] in the high- and low-frequency regimes:

$$\Pi^0_{ii}(\vec{q},\omega) \simeq \begin{cases} \alpha_i\dfrac{q^2}{\omega^2}+\beta_i\dfrac{q^4}{\omega^4} & \text{for } \omega \gg qv_{fi} \quad (7)\\[2ex] -\left(\dfrac{m_i}{\pi\hbar^2}\right)\left(1+i\dfrac{\omega}{qv_{fi}}\right) & \text{for } \omega \ll qv_{fi}\,, \quad (8)\end{cases}$$

where

$$\alpha_i = \frac{m_i v_{fi}^2}{2\pi\hbar^2} = \frac{N_i}{m_i}\,, \tag{9}$$

$$\beta_i = \frac{3m_i v_{fi}^4}{8\pi\hbar^2} = \frac{3\pi N_i^2\hbar^2}{2m_i^3}\,. \tag{10}$$

Here $m_i$, $v_{fi}$, and $N_i$ are, respectively, the band mass, Fermi velocity, and particle density for the two-dimensional motion parallel to the interface in the $i$th band. For our purpose, the limiting forms [Eqs. (7) and (8)] for the polarizability would be sufficient since we are interested in the leading-order wave-vector dependence of the collective modes of the system. We should remark that it is only in this long-wavelength limit ($q\to 0$) that RPA is known to be valid.

The condition for the existence of a collective mode is given by the pole of the function $\epsilon^{-1}$, which is equivalent to the vanishing of $|\epsilon|$, the determinant of the dielectric tensor defined by Eq. (1). Thus, collective modes are obtained by solving the determinantal equation

$$|\epsilon| = |\delta_{ij} - V_{ij}(\vec{q})\Pi^0_{jj}(\vec{q},\omega)| = 0\,. \tag{11}$$

For a two-component system, Eq. (11) becomes (Appendix B)

$$1 - V(\vec{q})[\Pi^0_{11}(\vec{q}, \omega) + \Pi^0_{22}(\vec{q}, \omega)] + V(\vec{q})^2(1 - e^{-2qd})\Pi^0_{11}(\vec{q}, \omega)\Pi^0_{22}(\vec{q}, \omega) = 0 , \quad (12)$$

where use has been made of Eqs. (5) and (6) to write $V_{11} = V_{22} = V$ and $V_{12} = V_{21} = \pm Ve^{-qd}$. Note that Eq. (12) indicates that the sign of the charge components forming the plasma has no effect on the dispersion relation of the collective modes of a two-component system. Using Eq. (7) or (8) in Eq. (12), we may seek the high- or low-frequency collective mode of the system. It is apparent from Eq. (8) that there could be no meaningful solution of Eq. (12) in the region $qv_{f1}, qv_{f2} > \omega$. This is understandable, since this region is within the electron-hole continua of both the components. We want to point out that Eq. (12) is completely equivalent to Eq. (9), derived in Ref. 12 under the approximation $\kappa_1 = \kappa_2 = \kappa$ and in the nonretarded limit (Appendix B).

Thus we shall seek solutions in two regimes: the high-frequency regime ($\omega \gg qv_{f1}, qv_{f2}$) and the low-frequency regime ($qv_{f1} \gg \omega \gg qv_{f2}$). We take $v_{f1} > v_{f2}$. If the two Fermi velocities are equal (as would be the case for GaAs-$Ga_xAl_{1-x}$As double quantum wells), we are left with only the high-frequency regime.

### A. High-frequency regime ($\omega \gg qv_{f1}, qv_{f2}$)

Using Eqs. (5)–(7) in Eq. (12), we get the following two solutions:

$$\omega_\pm^2 = \frac{q}{4}(Q_1 V_{f1}^2 - Q_2 V_{f2}^2)\left[1 \pm \left(1 - \frac{4[Q_1 Q_2 v_{f1}^2 v_{f2}^2 p - \frac{3}{2}q(Q_1 v_{f1}^4 + Q_2 v_{f2}^4)]}{(Q_1 v_{f1}^2 + Q_2 v_{f2}^2)^2}\right)^{1/2}\right] , \quad (13)$$

where

$$p = 1 - e^{-2qd} . \quad (14)$$

$Q_i$ is the Thomas-Fermi wave vector in the $i$th two-dimensional band and is given by

$$Q_i = 2m_i e^2/\bar{\kappa}\hbar^2 . \quad (15)$$

Equation (13) gives the symmetric and the antisymmetric modes of a two-component plasma in the long-wavelength limit, but with $qd$ as an arbitrary parameter. Since we are interested in the long-wavelength ($q \to 0$) behavior of the collective modes, we shall consider the long-wavelength limit of Eq. (13) in the intermediate- ($qd \ll 1$) and weak- ($qd \gg 1$) coupling limits.

#### 1. Intermediate-coupling limit ($qd \ll 1$)

In this situation, $p$ is given by

$$p \simeq 2qd . \quad (16)$$

Using Eq. (16) in Eq. (13) and looking at the leading order $q$ dependence (as we are interested in the $q \to 0$ limit only) of the collective modes, we get

$$\omega_+ \simeq C_+ q^{1/2} , \quad (17)$$

$$\omega_- \simeq C_- q . \quad (18)$$

The coefficients $C_\pm$ are given by

$$C_+ = \left(\frac{Q_1}{2} v_{f1}^2 + \frac{Q_2}{2} v_{f2}^2\right)^{1/2} \quad (19)$$

$$= \left[\frac{2\pi e^2}{\bar{k}}\left(\frac{N_1}{m_1} + \frac{N_2}{m_2}\right)\right]^{1/2} , \quad (20)$$

$$C_- = \left(\frac{Q_1 Q_2 v_{f1}^2 v_{f2}^2 - \frac{3}{4}(Q_1 v_{f1}^4 + Q_2 v_{f2}^4)}{Q_1 v_{f1}^2 + Q_2 v_{f2}^2}\right)^{1/2} . \quad (21)$$

The $\omega_+$ mode is the optical-plasmon mode with its frequency given by the square root of the sum of the squares of the individual plasma frequencies of the two components. In two dimensions it has the well known[12,15] $q^{1/2}$ behavior at long wavelengths. We point out that in the leading order, the optical plasmon is independent of the separation $d$ and always exists as the high-frequency symmetric mode of the two-component system.

The other mode, $\omega_-$, is the antisymmetric mode of the coupled system and is, in fact, the acoustic branch going as $q$ in long wavelengths. By virtue of having a separation $d$ between the components, the AP now lies in the high-frequency regime and is outside the electron-hole continua of both charge components. Clearly, for $d \to 0$, the $\omega_-$ mode is unphysical, since $C_-$ becomes pure imaginary. Thus AP can exist as an undamped, stable mode in the long wavelengths only for spatially-separated multicomponent plasma.

Since our analysis is valid only in the high-frequency regime ($\omega \gg qv_{f1}, qv_{f2}$), we must demand that the velocity $C_-$ of the acoustic branch exceed $v_{f1}$ (we assume $v_{f1} > v_{f2}$). This is the consistency criterion essential for the existence of $\omega_-$ mode in the high-frequency regime. We note, however, that no such problem of consistency arises for the optical branch $\omega_+$ which goes as $\omega_+ \simeq C_+ q^{1/2}$ in the long wavelengths and hence $\omega_+ > qv_{f1}$ for sufficiently small $q$. Thus the necessary and sufficient condition for the existence of the acoustic branch $\omega_-$ in the high-frequency regime is

$$C_- > v_{f1} \,. \tag{22}$$

Using Eq. (21) for $C_-$ we get

$$d > d_c = \left(\frac{\frac{7}{4}v_{f1}^2/v_{f2}^2}{Q_2} + \frac{(1+\frac{3}{4}v_{f2}^2/v_{f1}^2)}{Q_1}\right). \tag{23}$$

Thus the separation $d$ between the charge components must exceed a critical length $d_c$ determined by the screening lengths and the Fermi velocities of the system for the $\omega_-$ mode to be a physically meaningful mode in the high-frequency regime. We consider two special cases of interest:

(a) $v_{f1} = v_{f2}$. (24)

For this case we have $d_c = \frac{7}{2}a_s$, where

$$2a_s = 1/Q_1 + 1/Q_2 \tag{25}$$

is a Thomas-Fermi screening length.

(b) $v_{f1} \gg v_{f2}$ and $m_2 \gg m_1$.

Here

$$d_c \simeq \tfrac{7}{4}\, v_{f1}^2/v_{f2}^2 a_{s2} + a_{s1} \,, \tag{26}$$

with $a_{si} = Q_i^{-1}$ as the screening length of the individual charge components. We also emphasize that the above analysis is quantitatively valid for wavelengths $(1/q)$ greater than the separation $d$, since we must have $qd \ll 1$.

*2. Weak-coupling limit ($qd \gg 1$)*

This limit is mathematically achieved by letting $d$ become very large first and then we take the $q \to 0$ limit of the resulting dispersion relations to obtain the long-wavelength behavior of the solution. In this limit, the parameter $p$ of Eq. (14) becomes almost unity and using that in Eq. (13) and keeping only the leading-order $q$-dependent terms, we get

$$\omega_\pm^2 \simeq \frac{q}{4}\left[(Q_1 v_{f1}^2 + Q_2 v_{f2}^2) \pm (Q_1 v_{f1}^2 - Q_2 v_{f2}^2)\right], \tag{27}$$

$$\omega_+ \simeq \left(\frac{Q_1}{2}\right)^{1/2} v_{f1}\, q^{1/2} \,, \tag{28}$$

and

$$\omega_- \simeq \left(\frac{Q_2}{2}\right)^{1/2} v_{f2}\, q^{1/2} \,. \tag{29}$$

Thus we obtain the intuitively very satisfying result that in the weak-coupling limit the $\omega_\pm$ modes are simply the respective two-dimensional plasma frequencies of the two components.

**B. Low-frequency regime ($qv_{f1} \gg \omega \gg qv_{f2}$)**

Using the appropriate limiting forms for the two polarizability functions [Eqs. (7) and (8)] in this regime, we find that there could be just one solution of Eq. (12) satisfying $qv_{f1} > \omega > qv_{f2}$. This solution is necessarily complex (indicating the mode to be a damped one), since $\Pi_{11}^0(\vec{q}, \omega)$ has an imaginary part in this regime due to single-particle excitations. For the collective mode to be physically meaningful, damping has to be small and we shall assume that in our analysis. We write the solution to Eq. (12) in this low-frequency regime in the form

$$\omega = \omega_A - i\delta_A \,, \tag{30}$$

where $\omega_A$ and $\delta_A$ are given by

$$\omega_A \simeq (qv_{f2}^2)^{1/2}\left(\frac{m_2}{2m_1}q + \tfrac{1}{2}pQ_2\right)^{1/2} \tag{31}$$

and

$$\delta_A \simeq \frac{1}{2\omega_A(q+Q_1)}\left(-\frac{pQ_1Q_2v_{f2}^2\omega_A}{2v_{f1}} + \frac{Q_1\omega_A^3}{qv_{f1}^2}\right). \tag{32}$$

We again consider the intermediate- and weak-coupling limits.

*1. Intermediate-coupling limit ($qd \ll 1$)*

In this limit ($p \simeq 2qd$), the long-wavelength behavior of $\omega_A$ and $\delta_A$ are given by

$$\omega_A \simeq C_A q \,, \tag{33}$$

where

$$C_A = \left(\frac{m_2}{2m_1} + Q_2 d\right)^{1/2} v_{f2} \tag{34}$$

and

$$\delta_A \simeq \frac{m_2}{4m_1}\frac{v_{f2}^2}{v_{f1}}q \,. \tag{35}$$

Equations (33)–(35) give the leading-order energy dispersion and damping of the ordinary AP, which lies within the single-particle excitation spectrum of the faster moving charge component and is thus damped. Some of these results for the ordinary AP were earlier obtained[16] by Takada in the context of inversion layers.

The consistency criterion that must be satisfied is that $qv_{f2} < \omega_A < qv_{f1}$. From Eq. (33) we find that we must have

$$v_{f2} < C_A < v_{f1} \,. \tag{36}$$

Using Eq. (34) in the inequality (36), we get

$$1 < \frac{m_2}{2m_1} + Q_2 d < \frac{v_{f1}^2}{v_{f2}^2} \,. \tag{37}$$

If $m_2 > 2m_1$, the lower limit is always satisfied, even for an unseparated plasma ($d=0$). Otherwise, there would be a lower limit on $d$ below which the low-frequency AP does not exist.

Thus,

$$d > d_{\min} = \frac{1}{Q_2}\left(1 - \frac{m_2}{2m_1}\right) \tag{38}$$

must be satisfied for the existence of AP. This condition is not very important, since one always looks for ordinary low-frequency AP in a system with $m_2 \gg m_1$, $v_{f1} \gg v_{f2}$, so as to make damping [cf. Eq. (35)] very small. It is to be noted that the ordinary AP exists even when $m_1 = m_2$, provided $d > 1/2Q_2 = a_{s2}/2$ (obviously, $v_{f1}$ still has to be greater than $v_{f2}$ to define the mode). In three-dimensional bulk systems this would have been impossible.

The upper limit on $d$ is imposed by requiring that $C_A < v_{f1}$ for the low-frequency condition. From (37) we obtain

$$d < d_{\max} = \frac{1}{Q_2}\left(\frac{v_{f1}^2}{v_{f2}^2} - \frac{m_2}{2m_1}\right). \tag{39}$$

From Eq. (34) we note that the effect of the separation is to increase the phase velocity or the slope of the acoustic branch. Thus a situation arises when the phase velocity tries to exceed $v_{f1}$, which is not allowed within this low-frequency analysis ($\omega_A < qv_{f1}$, implying $C_A < v_{f1}$). Thus ordinary AP (lying within the electron-hole continua of one of the two carriers) exists only for $d < d_{\max}$.

However, as noted in the high-frequency analysis for $d > d_c$, there is an undamped acoustic branch at long wavelengths lying outside the single-particle excitations of either charge component. We may point out that $d_c > d_{\max}$ and hence the two acoustic branches (damped and undamped) never exist together.

#### 2. *Weak-coupling limit ($qd \gg 1$)*

We take the weak-coupling limit as before by setting $p = 1$ in Eqs. (31) and (32) and then retaining only the leading-order terms to obtain

$$\omega_A \simeq \left(\frac{Q_2}{2}\right)^{1/2} v_{f2} q^{1/2} \tag{40}$$

and

$$\delta_A \simeq 0. \tag{41}$$

Thus, as before, the acoustic branch goes into the decoupled plasma mode of the slower moving charge component in the weak-coupling limit.

## III. DISCUSSION

The analysis of Sec. II demonstrates that the spatial separation greatly enhances the possibility of observing AP in the new class of two-dimensional materials, since one of the main experimental obstacles, namely damping, can be altogether eliminated. Another subtle advantage is that the acoustic branch exists in the high-frequency regime independent of whether $m_1$, $m_2$ and $v_{f1}$, $v_{f2}$ are equal or not, provided, of course $d$ is greater than $d_c$. This makes the two-component electron plasma in the modulation-doped double quantum well of GaAs-$Ga_xAl_{1-x}$As structure a good candidate for the experimental realization of the AP.

Utilizing the appropriate values[14] of the relevant parameters of the two systems discussed so far, we obtain the following values for $d_c$:

$$d_c \simeq \begin{cases} 150\ \text{Å} & \text{for the GaAs-Ga}_x\text{Al}_{1-x}\text{As system} \\ 400\ \text{Å} & \text{for the InAs-GaSb system.} \end{cases}$$

A separation of 150 Å can be achieved in GaAs-$Ga_xAl_{1-x}$As structures and hence we suggest this as a suitable system for the experimental realization of the AP.

The actual experimental observability depends on the oscillator strength or the spectral weight carried by the particular collective mode. The spectral weight of the modes are given by the dynamical structure factor[9] of the system defined by

$$S(\vec{k}, k_z; \omega) = \int_{-\infty}^{+\infty} dt\, e^{i\omega(t-t')} \int d\vec{r} \int d\vec{r}\,' e^{-i\vec{k}(\vec{r}_{11}-\vec{r}\,'_{11})} e^{-ik_z(z-z')} \langle\langle n(\vec{r}, t) n(\vec{r}\,', t')\rangle\rangle, \tag{42}$$

where $(\vec{k}, k_z)$ is the three-dimensional wave vector associated with the external probe and $n(\vec{r}, t)$ is the charge-density operator with $\vec{r} \equiv (\vec{r}_{11}, z)$ as the three-dimensional wave vector. Using standard methods, we can express Eq. (42) as

$$S(\vec{k}, k_z; \omega) = 2\sum_{ijlm} \int dz \int dz'\, e^{-ik_z(z-z')} \xi_i^*(z)\xi_j(z)\xi_l^*(z')\xi_m(z')\, \text{Im}X_{ijlm}(\vec{k}, \omega), \tag{43}$$

where $i, j, l, m$ are band indices and $X_{ijlm}(\vec{k}, \omega)$ is the true density-density response function or the reducible polarizability function of the two-dimensional system. Like the interaction function $V_{ijlm}$ and the dielectric function $\epsilon_{ijlm}$ it is also a tensor of fourth rank in the most general case and is given within RPA by the tensor equation

$$X = \Pi^0 + \Pi^0 V X. \tag{44}$$

Within the diagonal approximation for $V$, $X$ is also diagonal (i.e., $X_{ijlm} \propto \delta_{ij}\, \delta_{lm}$) and is given by

$$X_{ij} = (\epsilon^{-1}\Pi^0)_{ij}. \tag{45}$$

In the systems we are considering, it is sensible to assume that $\Pi^0$ is completely diagonal and then we get

$$X_{ij} = \epsilon_{ij}^{-1} \Pi_j^0. \quad (46)$$

Using Eq. (46) in Eq. (43), one can estimate the strength of the dynamical structure factor at the various collective modes of the system. Doing that within our simplified model, we find that in the long-wavelength limit ($q \to 0$), the spectral weight is mostly carried by the OP [OP goes as $O(q^{1/2})$, whereas the AP goes as $O(q)$]. On the other hand, for $q$ not vanishingly small (but still within the long-wavelength approximation), the spectral weight associated with the acoustic branch when it is outside the single-particle continua may increase considerably even though OP is still the dominant one. The spectral weight of the usual damped AP (inside the single-particle excitations of the faster component) is rather small and is one of the main reasons (along with the damping) why this mode has never been observed in the bulk.

However, the undamped acoustic branch being separated from the OP in frequency, it appears likely that light scattering experiments may be the best available method for the observation of the acoustic branch. The mode (AP) being a longitudinal excitation in the two-dimensional plane parallel to the interface, the probe must have an electric field component parallel to the interface.

We have also investigated the effect of a width in the electronic envelope functions $\xi(z)$. We find that allowing for a width in the wavefunctions, while maintaining the same average separation, tends to decrease the oscillator strength of the acoustic branch. In that situation, the best coupling with the external probe is expected to occur for $q_z \simeq \Delta^{-1}$, where $q_z$ is the $z$ component of the probe wave vector and $\Delta$ is the average spread of the wave functions.

Finally, two observations about the present analysis and approximations are in order. First, we have used a long-wavelength ($q \to 0$) version of generalized RPA to obtain the dispersion relation in the high- and low-frequency regimes. This gives the leading-order wave-vector dependence of the collective modes correctly, but is inadequate for the higher-order corrections to the dispersion relation. However, RPA is, in general, known to be good only in the $q \to 0$ limit. As a matter of fact, it has been shown[17] that in two-dimensional single-component plasma, RPA gives only the leading term [i.e., the $O(q^{1/2})$ term] of the plasmon dispersion relation correctly. Vertex corrections are important for the next-order term in dispersion. Thus the leading-order analysis for the dispersion relation is consistent with our use of RPA. However, a manifestation of its limitation is the fact that $d_c > d_{\max}$. In a more sophisticated analysis one would expect $d_c$ to be the same as $d_{\max}$.

The other point is that we keep terms up to $O(q^4/\omega^4)$ in the high-frequency expansion of $\Pi(\vec{q}, \omega)$ in obtaining the undamped acoustic branch. This is needed for the sake of mathematical consistency as can be realized by analyzing Eq. (12). In order to retain the last term [which is of $O(q^4/\omega^4)$] on the left-hand side of Eq. (12), we must retain quantities of $O(q^4/\omega^4)$ in the second term as well. This is important since the last term is crucial in giving the acoustic branch. However, as one can easily verify, there are two branches of the solution in the high-frequency regime even if one retains terms only up to $O(q^2/\omega^2)$ in the expansion of the polarizability (Appendix C).

Finally, we want to remark that our analysis, even though it includes Landau damping in leading order, still neglects any collisional damping from background impurities and phonon scattering. One can introduce a phenomenological collision time $\tau$ in the analysis to account for the collisional damping. We consider that the condition $\omega\tau > 1$ is well satisfied (by working at very low temperatures to reduce phonon scattering and by having very pure samples to reduce impurity scattering) so that collisional damping can be neglected. Consideration of collisional damping also indicates that the GaAs-$Ga_xAl_{1-x}As$ double-quantum-well structure is perhaps the most suitable system for the observability of the collective modes discussed in this paper. This follows from the fact that in these structures, modulation doping of $Ga_xAl_{1-x}As$ layers allows[3] for very high mobility (even higher than the Brooks-Herring limit in the bulk) in the GaAs side and consequently $\tau$ is large, minimizing the effect of collisional damping. In fact, a typical value of $\tau$ in these systems (as extracted from low-temperature-mobility measurements) is of the order of $10^{-12}$ sec. This restricts the frequency to the far-infrared region for the condition $\omega\tau > 1$ to be well satisfied. Thus the present theory (with its neglect of collisional damping) is applicable in the infrared region. Experimental techniques for observing collective modes in this region are, however, well developed.[18]

## IV. CONCLUSIONS

In this paper, we have generalized the work[12] of Eguiluz *et al.* and have demonstrated the possible existence of a high-frequency, undamped acoustic mode in spatially-separated, two-component, two-dimensional plasma. This mode lies outside the electron-hole single-particle continua of both charge components and is thus stable (not Landau damped). We also find its existence depends critically on having the spatial separation between the two charge components exceed a critical distance, determined by the screening lengths of the system. The regular AP, which lies within the electron-hole continuum of the faster moving component and is thus severely Landau-damped, exists only at small separation. So we infer that in spatially-separated multicomponent plasma, the acoustic branch of plasma oscillation may come out of the Landau-damping region above a certain critical separation, and may thus become stable. As a suitable candidate for observing the AP, we suggest light scattering experiments on GaAs-$Ga_xAl_{1-x}As$ double-quantum wells with separation (layer thickness) of the order of 150 Å or more.

We note that the ordinary interpretation of the AP as the mode in which the two plasma components undergo in-phase or out-of-phase oscillations, depending on whether they have the opposite or same charges, applies more strongly to the acoustic branch lying outside the single-particle regions of both the components. In this case, the idea of individual collective motion of the two components as an antisymmetric coupled mode of vibration exists. On the other hand, when the AP lies within one of the single-particle excitation regions (as in the three-dimensional bulk materials), such a concept is invalid due to Landau damping from the single-particle excitations in the faster charge component.

Recently, there has been a notable interest in the collective properties of two-dimensional systems and we believe that this paper shows that the new class of systems[2,3] (e.g., GaAs-$Ga_xAl_{1-x}As$ double quantum wells) with spatially well separated, two-component, two-dimensional plasma may be suitable for the experimental realization of the linear mode discussed[12] first by Eguiluz *et al.* in the context of inversion layers and shown in this work to be the large-separation manifestation of the usual acoustic plasmon.

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## APPENDIX A

We consider the general dielectric reponse of a multiband (multicomponent) two-dimensional electronic system in this appendix without invoking any diagonal approximation.

Dyson's equation for the dynamically screened interaction in such a system can be written as[19]

$$U_{ijlm}(\vec{q},\omega)=V_{ijlm}(\vec{q})+\sum_{n_1n_2}V_{ijn_1n_2}(\vec{q})\,\Pi_{n_1n_2}(\vec{q},w)\,U_{n_2n_1lm}(\vec{q},\omega)\,, \tag{A1}$$

where $V$ is the fourth-rank tensor describing dynamically screened interaction between two electrons in a multiband two-dimensional system, one of which gets scattered from band $i$ to $j$ and the other one from $l$ to $m$ with a two-dimensional momentum exchange of $\hbar q$. RPA consists of using the noninteracting polarizability $\Pi^0$ for $\Pi$ in Eq. (A1). We shall use RPA here. Equation (A1) can be solved for $V$ by introducing a dielectric tensor $\epsilon$ defined by

$$\epsilon_{ijlm}(\vec{q},\omega)=\delta_{il}\,\delta_{jm}-V_{ijlm}(\vec{q})\,\Pi^0_{lm}(\vec{q},\boldsymbol{\omega})\,. \tag{A2}$$

Then Eq. (A1) gives

$$\epsilon U=\boldsymbol{V}\,. \tag{A3}$$

Thus,

$$U=\epsilon^{-1}V. \tag{A4}$$

The tensor product in (A3) has been defined as

$$(\epsilon V)_{ijlm}=\sum_{n_1n_2}\epsilon_{ijn_1n_2}V_{n_1n_2lm}\,. \tag{A5}$$

In writing Eqs. (A2)–(A5) we have explicitly used the reality of the bound-state wave functions describing the two-dimensional bands $i, j, l, m$, etc. Equations (A2)–(A4) are tensor generalizations of the equivalent relationships between bare interaction, effective interaction, and the dielectric function for the one-component plasma.

The collective modes are given by the poles of $\epsilon^{-1}$. Thus, if for any value of $(q,\omega)$ a particular component of $\epsilon^{-1}$ becomes singular, that describes the onset of a collective mode. The formalism described above gives both the intraband collective modes in which carriers oscillate in the two-dimensional plane and also the interband collective modes (which are always associated with interband excitations) in which the carriers oscillate normal to the two-dimensional plane. Obviously, the interband modes are possible only when the wave functions $\xi_i(z)$'s have a certain width in $z$ direction and they do not exist for the purely two-dimensional delta-function models of charge density employed in the main text of the paper. The delta-function approximation automatically incorporates the diagonal approximation in Eq. (A1).

The significance of the interband collective modes in the context of these inhomogeneous systems was first realized by Chen, Chen, and Burstein.[20] There is a considerable amount of literature[21–24] detailing various aspects of these excitations. In this appendix we just depicted how all these collective modes can be understood within a unified formalism of generalized dielectric response.

## APPENDIX B

The dispersion relation used by Equiluz *et al.*[12] in their numerical analysis (in the high-frequency regime only) is [Eq. (9) of Ref. 12]

$$\left(4\pi\chi_1 + \frac{\epsilon_1}{\beta_1} + \frac{\epsilon_0}{\beta_0}\coth\beta_0 d\right)\left(4\pi\chi_2 + \frac{\epsilon_2}{\beta_2} + \frac{\epsilon_0}{\beta_0}\coth\beta_0 d\right) = \left(\frac{\epsilon_0}{\beta_0}\right)^2 \operatorname{csch}^2\beta_0 d\,. \quad \text{(B1)}$$

For our system, $\epsilon_0=\epsilon_1=\epsilon_2=\bar{\kappa}$ and $\beta_1=\beta_0=q$. Also,

$$\chi_i = -\frac{\bar{\kappa}V(q)}{2\pi q}\Pi^0_{ii}(q)\,. \quad \text{(B2)}$$

Then (B1) becomes

$$[-2V(q)\Pi^0_{11}(q) + 1 + \coth qd] \times[-2V(q)\Pi^0_{22}(q) + 1 + \coth qd] = \operatorname{csch}^2 qd\,. \quad \text{(B3)}$$

(B3) simplifies to

$$1 - V(q)[\Pi^0_{11}(q) + \Pi^0_{22}(q)] + \left(\frac{e^{qd}-e^{-qd}}{e^{dq}}\right)V(q)^2\Pi_{11}(q)\Pi^0_{22}(q) = 0\,. \quad \text{(B4)}$$

The term within the large parentheses is simply $(1-e^{-2qd})$, whence (B4) reduces to Eq. (12) of the text showing the equivalence of our procedure to that of Ref. 12.

## APPENDIX C

We shall show here that retaining terms up to $O(q^2/\omega^2)$ in $\Pi^0(\vec{q},\omega)$ would still give us a high-frequency acoustic branch in solving Eq. (12). Keeping only the first term in the expansion of $\Pi^0(\vec{q},\omega)$ in Eq. (7) and using that in Eq. (12) we get the following for the $\omega_-$ solution of Eq. (12):

$$\omega_- \simeq \left(\frac{Q_1 Q_2 v_{f2}^2 d}{Q_1 v_{f1}^2 + Q_2 v_{f2}^2}\right)^{1/2} v_{f1}\, q\,. \quad \text{(C1)}$$

The consistency criterion $(\omega \gg qv_{f1} > qv_{f2})$ gives

$$d > d_c = \frac{1}{Q_2}\left(\frac{v_{f1}^2}{v_{f2}^2} + \frac{Q_2}{Q_1}\right). \quad \text{(C2)}$$

We note that the velocity of the acoustic branch is higher than that given by Eq. (21) of the main text, and as a result $d_c$ given by Eq. (C2) is smaller than that in the main text. One still, however, has $d_c > d_{max}$, where $d_{max}$ [given by Eq. (39) of the main text] is the separation above which the regular damped AP cannot exist.

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