Semiclassical approach to quantum-electromagnetic excitations in metals

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(Received 2 July 1980)

A semiclassical approach to quantum-electromagnetic excitations for metals in the collisionless-damping regime at strong magnetic fields is carried out. The quantization of energy levels results in different values of the electromagnetic wave vector, whose numbers are increased by an increase in the magnetic field. The theory, specialized for bismuth, explains correctly most of the experimental results found in a previous paper.

I. INTRODUCTION

In a recent paper (in the following referred to as I),[1] a set of experimental results of microwave absorption as a function of an external magnetic field has been presented. The existence of absorption resonances due to the Landau levels has been found and, in particular, it has been shown that these resonances are analogous to the giant quantum oscillations of ultrasound. However, some peculiar effects are present, the most important of which is the disappearance of resonance above a certain value of the magnetic field. To explain such experimental results, in this paper we present a theory of electromagnetic excitations in metals at strong magnetic fields. We consider a plane monochromatic electromagnetic wave with frequency ω propagating in an infinite metal with wave vector $\vec{k}$ forming an angle ϑ to the direction of the static magnetic field. To obtain the spectrum and the damping of the electromagnetic wave we assume "quantum" conditions, that is to say, the conditions $\Omega \gg \omega$, $\omega\tau \gg 1$, $\epsilon_F \gg kT$ (where Ω is the cyclotron frequency, τ is the relaxation time, ϵ_F is the Fermi level, T is the absolute temperature and k the Boltzmann constant).

Because of the quantization of the energy levels for the carriers in a magnetic field, different values of the wave vector $\vec{k}$ are found, each of them connected to a particular Landau level. When the magnetic field is increased, the number of the possible $\vec{k}$ values increases. On the basis of this result, as will be seen, a possible explanation of the disappearance of the quantum oscillations in the microwave absorption is found.

The present theory, beginning with general considerations, is able to point out the right approximations which allow us to obtain analytic results suitable for explaining most of the measurements reported in I for bismuth. In particular, the semiclassic approach is used with $kR < 1$, where R is the Larmor's radius. The return to this argument follows from the fact that the current theories are not specific and are difficult to use in looking into the actual physical situations.[2,4]

In Sec. II we present the basic equations of the theory, in Sec. III the kinetic equation solution, in Sec. IV the conductivity tensor calculation, and in Sec. V the dispersion law and absorption coefficient for the electromagnetic field in bismuth semimetal.

II. BASIC EQUATIONS

The spectrum of the electromagnetic excitations in metals in a static magnetic field will be looked for in the frame of the semiclassical quantum theory under the anomalous skin-effect condition. The solution will be found in working out, simultaneously, the Maxwell and the transport kinetic equations according to Azbel'.[2,3] Two reference systems will be used: the reference x, η, ζ with $x\eta$ plane coinciding with the surface of the metal which is thought to fill the semispace $\zeta \geq 0$, and the reference xyz with z axis along the static magnetic field $\vec{H}$ (Fig. 1). $x\eta\zeta$ is the natural frame for the electromagnetic field propagation because

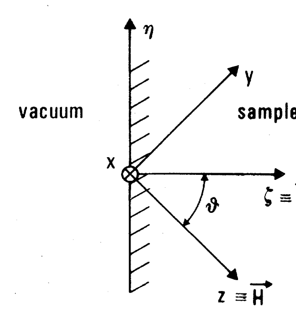

FIG. 1. Sketch of the two reference frames used. $\vec{k}$ is the electromagnetic wave vector, $\vec{H}$ is the magnetic field rotating in $\eta\zeta$ plane. The plane $x\eta$ is the sample surface.

the z axis is directed along the wave vector $\vec{k}$ of the electromagnetic field, while xyz is the natural frame for the study of the energy levels of carriers. With reference to Fig. 1, ϑ is the angle between $\vec{H}$ and $\vec{k}$ with $\vec{H}$ rotating in the $\eta\xi$ plane. The kinetic problem will be solved in the xyz frame. Once the conductivity tensor is found, we shall go to the $x\eta\xi$ frame and, by means of the Maxwell equations, the spectrum and absorption of the electromagnetic field will be calculated.

With regard to the kinetic equation for the collision integral, the relaxation time approximation will be used. This approximation is well stated at the high-frequency regime, $\omega\tau \gg 1$.[3] The Boltzmann equation, linearized in the wave electric field which is dependent on space and time as $E\exp(i\vec{k}\cdot\vec{r}-i\omega t)$, is

$$[i(\vec{k}\cdot\vec{v}-\omega)+\nu]f+\frac{e\vec{H}}{c}\cdot(\vec{v}\times\vec{\nabla}_p f)=e\vec{E}\cdot\vec{v}\,\frac{\partial f_0}{\partial\epsilon}, \quad (1)$$

where $f=f(p_x,p_y,p_z)$ is the Fourier component of the nonequilibrium part of the distribution function, f_0 is the Fermi distribution, $\nu=1/\tau$ is the collision frequency for the carriers, and $\vec{v}=\vec{\nabla}_p\epsilon(p_x,p_y,p_z)$.

When the static magnetic field is present, it is useful to specify the carrier state by means of the motion constants ϵ, p_z, and the quantity $\varphi=\Omega t$, instead of the momentum components p_x,p_y,p_z. The quantity φ is the angle that locates the point P representative of the carrier state on the orbit Γ given by the intersection of the energy constant surface $\epsilon(p_x,p_y,p_z)=\epsilon$ with the plane $p_z=\text{const}$ orthogonal to the magnetic field $\vec{H}$. Assuming for the carriers an ellipsoidal dispersion law centered in the origin of the momentum space (Fig. 2),

$$\epsilon=(a_{11}p_x^2+a_{22}p_y^2+a_{33}p_z^2+2a_{12}p_xp_y+2a_{13}p_xp_z+2a_{23}p_yp_z)\frac{1}{2m_0}, \quad (2)$$

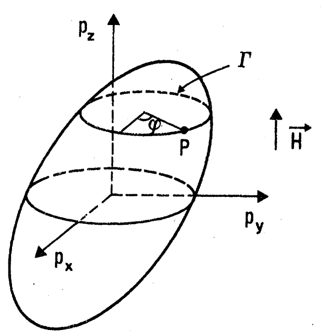

FIG. 2. P is the representative point of the carrier state for an ellipsoidal dispersion law, both in the reference p_x,p_y, and p_z and in the reference ϵ, p_z, and φ.

it is possible to write the second term on the left-hand side of Eq. (1) as follows:

$$\frac{e\vec{H}}{c}\cdot(\vec{v}\times\vec{\nabla}_p f)=\frac{e\vec{H}}{c}\,\frac{(\overline{\alpha}_x\overline{\alpha}_y)^{1/2}}{m_0}\,\frac{\partial f}{\partial\varphi}=\frac{eH}{cm_c}\,\frac{\partial f}{\partial\varphi}=\Omega\,\frac{\partial f}{\partial\varphi},$$

where $\Omega=e\vec{H}/(cm_c)$ is the cyclotron frequency, $m_c=m_0/(\overline{\alpha}_x\overline{\alpha}_y)^{1/2}$ is the cyclotron mass, and the coefficients $\overline{\alpha}_x$, $\overline{\alpha}_y$ are given in (a) of the Appendix. Then, Eq. (1) becomes

$$\left(\Omega\frac{\partial}{\partial\varphi}+i(\vec{k}\cdot\vec{v}-\omega)+\nu\right)f(\epsilon,p_z,\varphi)=e\vec{E}\cdot\vec{v}\,\frac{\partial f_0}{\partial\epsilon},$$

whose formal solution is

$$f(\epsilon,p_z,\varphi)=\frac{df_0}{d\epsilon}\int_{-\infty}^{\varphi}d\varphi'\,\frac{e\vec{E}\cdot\vec{v}(\varphi')}{\Omega}\times\exp\left(\frac{\nu-i\omega}{\Omega}(\varphi-\varphi')+i\frac{\vec{k}}{\Omega}\cdot\int_{\varphi}^{\varphi'}d\varphi''\,\vec{v}(\varphi'')\right). \quad (3)$$

III. KINETIC EQUATION SOLUTION

To solve Eq. (3), the velocity components as a function of ϵ, p_z, and φ are calculated taking into account Eq. (2). After cumbersome calculations one obtains

$$\begin{aligned} v_x&=u_x\cos\alpha\cos\varphi-u_y\sin\alpha\sin\varphi,\\ v_y&=u_x\sin\alpha\cos\varphi+u_y\cos\alpha\sin\varphi,\\ v_z&=u_xu_1\cos\varphi+u_2u_y\sin\varphi+u_z, \end{aligned} \quad (4)$$

where $u_x,u_y,u_1,u_2,u_z=\overline{\alpha}_zp_z/m_0$, and the angle α are dependent on effective mass coefficients and the angle ϑ, as specified in (a) of the Appendix. In Eq. (3), having the wave vector $\vec{k}$ components $k_x=0$, $k_y=k\sin\vartheta$, and $k_z=k\cos\vartheta$, the second term of the exponential becomes

$$i\frac{\vec{k}}{\Omega}\cdot\int_{\varphi}^{\varphi'}d\varphi''\,\vec{v}(\varphi'')=iA(\sin\varphi'-\sin\varphi)+iB(\cos\varphi-\cos\varphi')+i\frac{\gamma_z}{\Omega}p_z(\varphi'-\varphi), \quad (5)$$

where $\gamma_z=\overline{\alpha}_zk_z/m_0$,

$$A=k(u_x\sin\vartheta\sin\alpha+u_xu_1\cos\vartheta)/\Omega,$$

and

$$B=k(u_y\sin\vartheta\cos\alpha+u_yu_2\cos\vartheta)/\Omega.$$

One can see that in the exponential of Eq. (3), as given by Eq. (5), the terms expressing functions of $\cos\varphi$ and $\sin\varphi$ are present. That results in very complicated formal equations where the exponentials will be developed as a series of Bessel functions. However, both $\cos\varphi$ and $\sin\varphi$ terms must be retained if the magnetic field direction is left variable with respect to the sample surface. So the following developments are used:

$$\exp(iD\sin\varphi)=\sum_{s=-\infty}^{\infty} J_s(D)\exp(is\varphi),$$
$$\exp(iD\cos\varphi)=\sum_{s=-\infty}^{\infty} i^s J_s(D)\exp(is\varphi), \tag{6}$$

with J_s the integer-order Bessel function.

Putting in Eq. (3) the values v_x, v_y, and v_z given by Eq. (4) and the Bessel-function developments (6), one obtains the expression of $f(\epsilon, p_z, \varphi)$. Now, to obtain the conductivity tensor σ_{ik} (i,k stand for x,y,z), the current density $\vec{j}$ must be calculated. This, among other things, involves the calculation of integrals similar to $\int_0^{2\pi} d\varphi\, v_i(\varphi) f(\varphi)$. Then it is necessary to compute terms of the type

$$\mathcal{J}=\int_0^{2\pi} d\varphi \begin{bmatrix}\cos\varphi\\ \sin\varphi\\ K_1\end{bmatrix}\exp(-\beta\varphi+iB\cos\varphi-iA\sin\varphi)$$
$$\times\int_{-\infty}^{\varphi} d\varphi' \begin{bmatrix}\cos\varphi'\\ \sin\varphi'\\ K_2\end{bmatrix}\exp(\beta\varphi'+iA\sin\varphi-iB\cos\varphi'), \tag{7}$$

where $\beta=[\nu-(i\omega-\gamma_z p_z)]/\Omega$ and K_1, K_2 are quantities independent of φ. Equation (7) is a synthetic form to represent nine integrals obtained by combining the terms in the square brackets.

Cohen *et al.*,[5] using the ultrasound-absorption calculation, have found expressions (used also by Platzman and Wolff in the microwave absorption[6]) similar to Eq. (7) [see Eq. (3.8) of their paper]. However, unlike the previous authors who have developed the calculation in the classic case (low magnetic fields and $\omega\tau \ll 1$), in this paper we consider the opposite limiting physical conditions: high magnetic fields ($\Omega \gg \omega$ and $kR<1$), low temperature, and high purity of samples ($\omega\tau \gg 1$). In these conditions, as will be shown in the following, each conductivity coefficient can be divided into two terms. The first term gives the contribution of the electrons which do not change the Landau level and, in the absorption or emission of the electromagnetic field, conserve the energy and momentum as in the interaction of the electron-photon quasiparticles. This term assumes an essential rule in the diagonal elements of the conductivity tensor. The second term gives the contribution of all the Landau levels and furnishes the Hall part in the nondiagonal elements of the conductivity tensor.

In the calculations of Eq. (7), by means of Eq. (6), we will stress, in the following, only the essential points and the approximations used:

(i) Let us consider the combination $\cos\varphi\cos\varphi'$; then Eq. (7) becomes

$$\mathcal{J}=\sum_{s,s',s'',s'''=-\infty}^{\infty}\pi(-i)^{s'+s'''}(-1)^{s'+s''}\times\frac{J_s(A)J_{s'}(B)J_{s''}(A)J_{s'''}(B)}{[\beta+i(s+s')]^2+1}[\beta+i(s+s')]. \tag{8}$$

Here, and throughout the following, the summation indices s, s', s'', and s''' fulfill the equation $s+s'+s''+s'''=0$. If, in the condition $\omega\tau \gg 1$, the factor

$$[\beta+i(s+s')]/\{[\beta+i(s+s')]^2+1\}$$

is calculated by means of the identity

$$\lim_{\nu\to 0}\frac{1}{x-i\nu}=\mathrm{P}\left(\frac{1}{x}\right)+i\pi\delta(x),$$

Eq. (8) becomes the sum of a quantum part $\mathcal{J}_q$ following from the delta function $\delta(x)$ and a classic part $\mathcal{J}_c$ following from the principal part $\mathrm{P}(1/x)$. The word "quantum" is used because the δ function, whose argument is $(\omega-\gamma_z p_z)$, gives the momentum and energy conservation laws in the photon absorption by electrons when transitions between Landau levels are forbidden ($\Delta n=0$), as follows from our physical condition $\Omega \gg \omega$. So, Eq. (8) becomes

$$\mathcal{J}=\mathcal{J}_c+\mathcal{J}_q=\sum_{s,s',s'',s'''=-\infty}^{\infty}(-1)^{s'+s''}(i)^{s'+s'''}(-i)\pi\Omega\frac{\omega-\gamma_z p_z-(s+s')\Omega}{\Omega^2-[\omega-\gamma_z p_z-(s+s')\Omega]^2}J_s(A)J_{s'}(B)J_{s''}(A)J_{s'''}(B)$$
$$+(-i)\frac{\pi^2}{\gamma_z}\Omega\delta\left(\frac{\omega}{\gamma_z}-p_z\right)\sum_{s',s'''=-\infty}^{\infty}(i)^{s+s'}J_{s'+1}(A)J_{s'}(B)J_s(A)J_{s+1}(B). \tag{9}$$

Using Eq. (2.27) of the book by Tranter,[7] the quantum part is

$$\mathcal{J}_q=\pi^2\frac{\Omega}{\gamma_z}\delta\left(\frac{\omega}{\gamma_z}-p_z\right)J_1^2((A^2+B^2)^{1/2}). \tag{10}$$

(ii) Equation (9) is also valid for the term $\sin\varphi \sin\varphi'$.

(iii) The quantum part $\mathcal{J}_q$ coming from the term $\sin\varphi\cos\varphi'$ is zero, while the classic one $\mathcal{J}_c$ is still given by the first part of Eq. (9), provided that the quantity $[\omega - \gamma_z p_z - (s+s')\Omega]$ in the numerator is substituted by Ω.

(iv) The integral $\mathcal{J}$ coming from the term $\cos\varphi\sin\varphi'$ is the same as $\mathcal{J}$ coming from $\sin\varphi\cos\varphi'$ with a negative sign, as is necessary because of Onsager's relations for the conductivity tensor. In fact, these terms give the contributions versus $1/H$ of nondiagonal elements, and changing from $\sin\varphi\cos\varphi'$ to $\cos\varphi\sin\varphi'$ is equivalent to considering σ_{ik} and σ_{ki}, respectively.

(v) The term K_1K_2, which contributes to the element σ_{zz}, gives for $\mathcal{J}$ the expression

$$\mathcal{J} = \sum_{s,s',s'',s'''=-\infty}^{\infty} \frac{i2\pi\Omega}{\omega - \gamma_z p_z - (s+s')\Omega} (-1)^{s'+s''} (i)^{s'+s'''} \times J_s(A) J_{s'}(B) J_{s''}(A) J_{s'''}(B) + 2\pi^2 \frac{\Omega}{\gamma_z} \delta\left(\frac{\omega}{\gamma_z} - p_z\right) J_0^2((A^2+B^2)^{1/2}). \tag{11}$$

(vi) The terms $K\cos\varphi$ and $K\sin\varphi$ in Eq. (7) make $\mathcal{J}=0$.

IV. CONDUCTIVITY TENSOR

Using the variables ϵ, p_z, and φ, the current density is given by

$$\vec{j} = \frac{2e}{(2\pi\hbar)^3} \iiint \vec{v} F m_c \, d\epsilon \, dp_z \, d\varphi, \tag{12}$$

taking into account that the integral in the variable φ has been carried out in the preceding section. As is well known, the semiclassical method considers the carrier energy-level function of the Landau quantum number n and the quasi-momentum p_z. For an ellipsoidal dispersion law for the carriers, as in Eq. (2), the energy is $\epsilon = (n+\frac{1}{2})\hbar\Omega + p_z^2/(2m_z)$ (with $m_z = m_0/\overline{\alpha}_z$) and the integral in the variable ϵ in Eq. (12) is replaced by a summation

$$\int_0^\infty d\epsilon \cdots = \hbar\Omega \sum_{n=0}^{\infty} \cdots .$$

As we have said, we assume that the magnetic field H is so large that $kR < 1$. In these conditions we shall maintain only terms up to $(k/\Omega)^2$. In particular, as the argument of the Bessel function, in Eqs. (10) and (11), is $(A^2+B^2)^{1/2} = krf(\vartheta)/\Omega$, where $f(\vartheta)$ and r are given in (b) of the Appendix, we can set

$$J_0^2 = 1 - \frac{r^2k^2}{2\Omega^2} f^2(\vartheta),$$

$$J_1^2 = \frac{k^2r^2}{4\Omega^2} f^2(\vartheta).$$

Starting from Eqs. (9), (10), and (11), one can see that the conductivity-tensor elements are linear combinations, with coefficients depending on the angle ϑ and effective masses of the following terms (for each of them the behavior versus H is stressed):

$$\Gamma_q(\sim 1/H^2) = -\frac{2e^2}{(2\pi\hbar)^3} \int d\epsilon \frac{df_0}{d\epsilon} \int dp_z \left(\epsilon - \frac{p_z^2}{2m_z}\right) 2m_0 \frac{m_c}{\gamma_z} \pi^2 \times \delta\left(\frac{\omega}{\gamma_z} - p_z\right) \frac{k^2r^2}{4\Omega^2} f^2(\vartheta), \tag{13}$$

$$\Gamma_1(\sim 1/H^2) = -\frac{2e^2}{(2\pi\hbar)^3} \int d\epsilon \frac{df_0}{d\epsilon} \int dp_z \left(-i\frac{\omega}{\Omega^2}\right) \times m_c \left(\epsilon - \frac{p_z^2}{2m_z}\right) 2m_0, \tag{14}$$

$$\Gamma_0(\sim 1/H) = -\frac{2e^2}{(2\pi\hbar)^3} \int d\epsilon \frac{df_0}{d\epsilon} \times \int dp_z \frac{m_c}{\Omega} \left(\epsilon - \frac{p_z^2}{2m_z}\right) 2\pi m_0, \tag{15}$$

$$\Gamma_3 = -\frac{2e^2}{(2\pi\hbar)^3} \int d\epsilon \frac{df_0}{d\epsilon} \times \int dp_z \left(\frac{\overline{\alpha}_z}{m_0} p_z\right)^2 \frac{2\pi^2 m_c}{\gamma_z} \delta\left(\frac{\omega}{\gamma_z} - p_z\right). \tag{16}$$

Γ_3 is not dependent on H. In fact, coming from Eq. (11), it gives the conductivity σ_{zz} along the external magnetic field. Γ_0, coming from the term $\sin\varphi\cos\varphi'$ of Eq. (7), gives the Hall contribution. Γ_q and Γ_1 come from the term $\cos\varphi\cos\varphi'$.

In conclusion, the components σ_{ik} are

$$\sigma_{xx}=\sum_i m_0^{-2}(\bar\alpha_x\cos^2\alpha+\bar\alpha_y\sin^2\alpha)(\Gamma_q+\Gamma_1)\,,$$

$$\sigma_{yy}=\sum_i m_0^{-2}(\bar\alpha_x\sin^2\alpha+\bar\alpha_y\cos^2\alpha)(\Gamma_q+\Gamma_1)\,,$$

$$\sigma_{zz}=\sum_i [m_0^{-2}(\bar\alpha_x u_1^2+\bar\alpha_y u_2^2)(\Gamma_q+\Gamma_1)+\Gamma_3]\cong\sum_i\Gamma_3\,,$$

$$\sigma_{xy}=\sum_i m_0^{-2}(\bar\alpha_x\bar\alpha_y)^{1/2}\Gamma_0\,,$$

$$\sigma_{yx}=-\sum_i m_0^{-2}(\bar\alpha_x\bar\alpha_y)^{1/2}\Gamma_0\,, \tag{17}$$

$$\sigma_{xz}=\sum_i \gamma_y m_0^{-2}(\bar\alpha_x\bar\alpha_y)^{1/2}\Gamma_0\,,$$

$$\sigma_{zx}=-\sum_i \gamma_y m_0^{-2}(\bar\alpha_x\bar\alpha_y)^{1/2}\Gamma_0\,,$$

$$\sigma_{yz}=\sum_i \gamma_x m_0^{-2}(\bar\alpha_x\bar\alpha_y)^{1/2}\Gamma_0\,,$$

$$\sigma_{zy}=-\sum_i \gamma_x m_0^{-2}(\bar\alpha_x\bar\alpha_y)^{1/2}\Gamma_0\,,$$

where the i index varies with different carriers. In the expressions of the nondiagonal elements we have neglected the terms dependent on $1/H^2$ with respect to Γ_0. The same has been done for σ_{zz}.

The explicit expression for the coefficient Γ_0 is

$$\Gamma_0=-\frac{2e^2}{(2\pi\hbar)^3}\,\frac{m_c}{\Omega}\int d\epsilon\,\frac{df_0}{d\epsilon}\int dp_z(\bar\alpha_x\bar\alpha_y)^{1/2}S(\epsilon,p_z)\,,$$

where

$$S(\epsilon,p_z)=2\pi m_0[\epsilon-p_z^2/(2m_z)]/(\bar\alpha_x\bar\alpha_y)^{1/2}\,.$$

Assuming $(df_0/d\epsilon)=-\delta(\epsilon-\epsilon_F)$ for $kT\ll\epsilon_F$, one obtains

$$\Gamma_0=\frac{2e}{(2\pi\hbar)^3}\,\frac{m_0m_cc}{H}\,V(\epsilon_F)\,,$$

where $V(\epsilon_F)$ is the volume inside the Fermi surface. As $N=2V(\epsilon_F)/(2\pi\hbar)^3$ is the carrier number, we have the Hall term

$$\frac{ecN}{H}=\frac{(\bar\alpha_x\bar\alpha_y)^{1/2}}{m_0^2}\,\Gamma_0\,. \tag{18}$$

For the coefficients Γ_1 and Γ_q we have

$$\Gamma_1=-i\frac{\omega}{H^2}\,\frac{m_0^3c^2N}{\pi\bar\alpha_x\bar\alpha_y}\,, \tag{19}$$

$$\Gamma_q=\frac{kH}{\cos\vartheta}\,\frac{f^2(\vartheta)}{\bar\alpha_z}\,\frac{e^3m_0^3}{16c\pi kT}\times\sum_{n=0}^{\infty}(n+\tfrac12)^2\cosh^{-2}\frac{\epsilon_F-(n+\frac12)\hbar\Omega-\dfrac{1}{2m_z}\left(\dfrac{\omega}{\gamma_z}\right)^2}{2kT}\,. \tag{20}$$

This equation is similar to Eq. (4.3) of the paper by Kaner and Skobov[8] and other similar expressions found for quantum ultrasound absorption[9] that show the amplitude of absorption resonances increasing with the magnetic field. However, in the argument of cosh of Eq. (20) is present the term ω/γ_z which, as we shall show, is a very complicated function of k and H. As a consequence the absorption will be a function of Γ_q.

Finally, Γ_3 is given by

$$\Gamma_3=\frac{\omega^2H}{k^3\cos^3\vartheta}\,\frac{e^3m_0}{8\pi\hbar^2kTc\bar\alpha_z}\times\sum_{n=0}^{\infty}\cosh^{-2}\frac{\epsilon_F-(n+\frac12)\hbar\Omega-\dfrac{1}{2m_z}\left(\dfrac{\omega}{\gamma_z}\right)^2}{2kT}\,. \tag{21}$$

In the quantum terms Γ_q and Γ_3 we have set

$$\frac{df_0}{d\epsilon}=-\frac{1}{kT}\cosh^{-2}[(\epsilon-\epsilon_{n,p_z})/(2kT)]\,.$$

V. DISPERSION LAW AND ABSORPTION COEFFICIENT FOR THE ELECTROMAGNETIC FIELD IN BISMUTH SEMIMETAL

To find the dispersion relation and absorption of the electromagnetic field, it is suitable to pass to the $x\eta\zeta$ frame with the $x\eta$ plane coinciding with the surface of the sample and the ζ axis along the $\vec k$ vector. Following the works by Kaner and Skobov,[4,8] the Maxwell equations become

$$\begin{aligned}&k^2E_x-\frac{4\pi i\omega}{c^2}(\tilde\sigma_{xx}E_x+\tilde\sigma_{x\eta}E_\eta)+2E_x'(0)=0\,,\\&k^2E_\eta-\frac{4\pi i\omega}{c^2}(\tilde\sigma_{\eta x}E_x+\tilde\sigma_{\eta\eta}E_\eta)+2E_\eta'(0)=0\,,\end{aligned} \tag{22}$$

with

$$\tilde\sigma_{\alpha\beta}=\sigma_{\alpha\beta}-\frac{\sigma_{\alpha\zeta}\sigma_{\zeta\beta}}{\sigma_{\zeta\zeta}}\,, \tag{23}$$

where $\sigma_{\alpha\beta}$ are the conductivity tensor elements in the $x\eta\zeta$ frame. The quantities $E_x'(0)$ and $E_\eta'(0)$ are a consequence of the fact that the metal fills a semispace.[3]

The dispersion $\omega(k)$ and the absorption are obtained by setting equal to zero the determinant of Eq. (22). Changing σ_{ik} of Eq. (17) (determined in xyz frame) into $\sigma_{\alpha\beta}$ in the $x\eta\zeta$ frame, the calculation of $\tilde\sigma_{\alpha\beta}$ and, subsequently, $\omega(k)$ is extremely complicated.

If we consider the bismuth semimetal, taking into account that it has an equal number of holes and electrons, from Eqs. (17) and (18) we have $\sigma_{xy}=0$. As a first solution, because the meaningful experimental results found in Bi (Ref. 1) are present for angles ϑ near $\pi/2$, we will neglect $\cos^2\vartheta$ with respect to $\cos\vartheta$. In this case $\tilde\sigma_{\alpha\beta}$ become

$$\tilde{\sigma}_{xx} = \sigma_{xx}\,,$$
$$\tilde{\sigma}_{\eta\eta} = 2\sigma_{yy}\sin^2\vartheta\,, \qquad (24)$$
$$\tilde{\sigma}_{x\eta}\tilde{\sigma}_{\eta x} = \frac{\sigma_{xz}^2\sigma_{zy}^2}{\sigma_{zz}^2}\,.$$

In addition, as in Bi for a large range of the angle ϑ the resonances due to the term cosh occur at different ranges of the magnetic field H, we can separate the quantum contribution of the electrons from the holes.

Let us introduce in Eq. (17) the complex variable $\omega/(kv_a) = \alpha + i\beta$, where $v_a = [H^2/(4\pi^2 m_0\bar{\delta}N)]^{1/2}$ is the Alfvén velocity of the plasma of the semimetal ($\bar{\delta}$ is a function of the electron and hole effective masses and the angle ϑ, see the Appendix). Setting equal to zero the coefficient determinant in Eq. (22), we obtain for the quantities α and β in the case $\alpha < 1$

$$1 - (\alpha^2 - \beta^2)[1 + c_2 G^2(\alpha,\beta)] + c_1\beta G(\alpha,\beta) = 0\,,$$
$$2\beta[1 + c_2 G^2(\alpha,\beta)] + c_1 G(\alpha,\beta) = 0\,, \qquad (25)$$

where

$$c_1 = \frac{m_0^{1/2}e^3H^2}{8c^2kTN^{1/2}}\;\frac{f^2(\vartheta)(2a_y\sin^2\vartheta + a_x)}{\bar{\alpha}_z\bar{\delta}^{1/2}\cos\vartheta}\,, \qquad (26)$$

$$c_2 = c_1^2\,\frac{2a_xa_y\sin^2\vartheta + a_x}{(2a_y\sin^2\vartheta + a_x)^2}\,, \qquad (27)$$

$$G(\alpha,\beta) = \sum_{n=0}^{N}(n+\tfrac{1}{2})^2\,\frac{\cos^2(2\alpha\beta a)\cosh^2 a(x_n^2 - \alpha^2 + \beta^2) - \sin^2(2\alpha\beta a)\sinh^2 a(x_n^2 - \alpha^2 + \beta^2)}{[\cos^2(2\alpha\beta a)\cosh^2 a(x_n^2 - \alpha^2 + \beta^2) + \sin^2(2\alpha\beta a)\sinh^2 a(x_n^2 - \alpha^2 + \beta^2)]^2}\,, \qquad (28)$$

$$x_n^2 = \frac{4\bar{\alpha}_z\bar{\delta}N\cos^2\vartheta}{H^2}\left(\epsilon_F - (n+\tfrac{1}{2})\frac{\hbar eH}{m_0c}(\bar{\alpha}_x\bar{\alpha}_y)^{1/2}\right)\,, \qquad (29)$$

$$a = \frac{m_0v_a^2}{2kT\bar{\alpha}_z\cos^2\vartheta}\,. \qquad (30)$$

x_n is the electron velocity at $\epsilon = \epsilon_F$ and nth Landau level, measured in v_a units: $\bar{\delta}$, a_x, and a_y are given in (b) of the Appendix.

Substituting the second of Eqs. (25) in the first, one obtains an equation $F(\xi) = 0$ of the variable $\xi = \alpha^2 - \beta^2$, whose solutions give the permitted $\vec{k}$ values. We shall find a graphic solution with this procedure: For fixed values of H and ϑ, the function $F(\xi)$ is tabulated and the values for which $F(\xi) = 0$ are found. Then the absorption β, by means of Eq. (25), can be calculated.

In Fig. 3 a set of $F(\xi)$ for a fixed value of ϑ and for different values of H are reported. The angle ϑ is rotating in the trigonal-bisector plane for Bi. The effective mass coefficients used are those reported in literature.[10] The ξ range, for which solutions $F(\xi) = 0$ exist, is (0–1).

As a first result we can see that there is always a solution at $\xi = 1$ which represents the electromagnetic field propagating with velocity v_a and dispersion $\omega(k) = \pm v_a k$, i.e., the Alfvén waves. This solution is obtained in the limit $\Gamma_q \to 0$ and gives the classic field. The fact that this solution is always present for every value of H and ϑ is a first check of the theory.

The Alfvén field is experimentally shown by means of the Fabry-Perot–Alfvén–type oscillations measured for small widths of the samples[11-14] also in the quantum conditions for the magnetic field H and temperature T. For larger widths of samples the Fabry-Perot conditions are no longer held, hence the Alfvén oscillations disappear even if the Alfvén field is present. With reference to our experimental results reported in I [see case (vi) of Sec. III] we can see that the

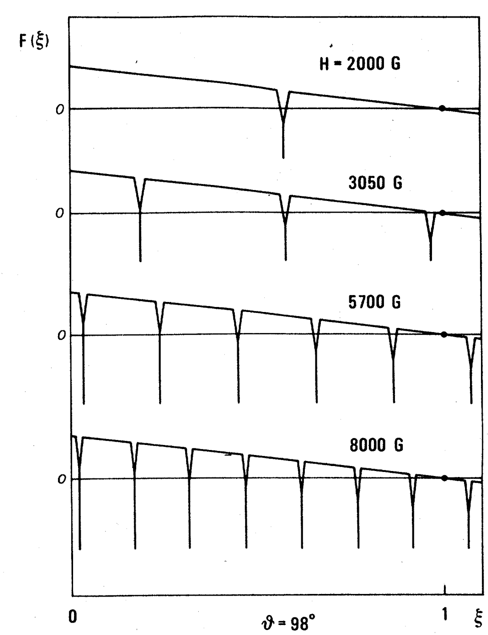

FIG. 3. Computer solution of equation $F(\xi) = 0$ for different values of the magnetic field H rotating in the trigonal-bisector plane for bismuth in the case $\vartheta = 98°$ (this angle corresponds to the angle $\vartheta = 86°$ in Fig. 4 of paper I).

Alfvén oscillations exist for the sample widths $l<4.5$ mm; for $4.5<l<5.0$ the Alfvén oscillations modulated by the quantum electromagnetic oscillations (QEO) still exist (as already seen for electrons by Dinger and Lawson[15]); for $l>5.0$ mm only QEO are present. As a conclusion, according to the provision that the solution $\xi=1$ of Eqs. (25) is always present, we have measured both the Alfvén field and QEO for the same value of ϑ and the same range of H. The width l of the sample is the physical parameter which controls the presence either one or the other type of oscillations. The theory of Kaner and Skobov[16] did not give the presence of both solutions for classic and quantum fields.

Now let us consider in Fig. 3 $F(\xi)$ for relatively low magnetic fields. In addition to the solution $\xi=1$ there is another solution connected to a particular Landau level n which, as H increases, exists for values of ξ closer and closer to zero. The solution $\xi=0$ is obtained at a certain value H_n of the magnetic field when the nth Landau level crosses the Fermi surface. For $H>H_n$ but closer to it no quantum solution of $F(\xi)=0$ exists. With H still increasing, the $(n-1)$th level comes into the solution range (0–1) for ξ, and so on. This picture clearly explains the microwave absorption as the quantum ultrasound oscillations, in particular the initial oscillating part of our measurements (Fig. 1 of I).

For higher magnetic field more solutions for $F(\xi)=0$ exist in the range (0–1); in other words, different quantum electromagnetic fields are simultaneously present in the metal. As a model, hence, one can think that from a certain value of H the quantum oscillations become less and less resonant (as opposed to the other quantum effects that always increase with the magnetic field) because many levels take part in the absorption.

The absorption β obtained from Eqs. (25) is

$$\beta=-\frac{c_1G(\xi,\beta)}{2[1+c_2G^2(\xi,\beta)]}\,. \qquad (31)$$

In Eq. (31), ξ enters as a parameter whose values are found by means of the graphic method above described. Because the calculation of β is difficult, we assume that the linewidth of the spikes related to Landau levels of Fig. 3 is small. The values for $F(\xi)=0$ are then approximated to those for which the function $F(\xi)$ assumes a relative minimum. The computer calculation of β as function of H for different values of ϑ is shown in Fig. 4.

As a conclusion the experimental results that are correctly explained by the theory are the following.

(i) In Fig. 4, as one can see, with increasing

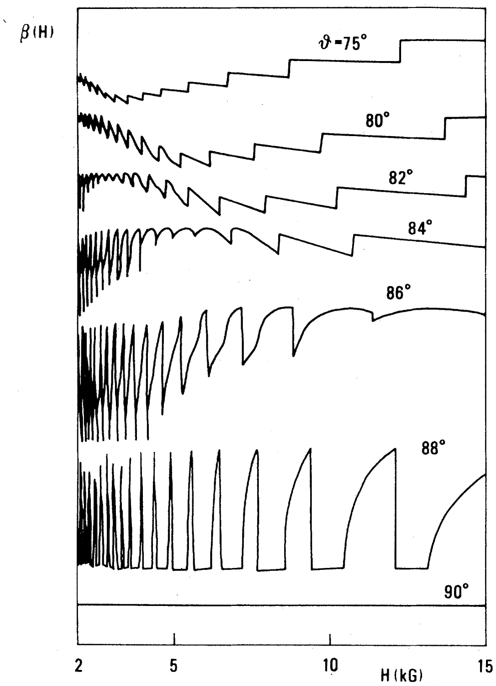

FIG. 4. Computer solution of microwave absorption $\beta(H)$ versus the magnetic field for different values of the angle ϑ.

H, oscillations disappear. We want to stress that this is the goal of our theory because it is able to explain the peculiar aspect of QEO we observed in Bi.

(ii) The reducing of the amplitude of oscillations as a function of the angle $\alpha=\pi/2-\vartheta$ between H and the sample surface. The resonances in Fig. 4 reduce 90% for $|\alpha|>15°$, according to measures reported at (iii) of Sec. III of I.

(iii) The linewidth of resonaces in Fig. 4 is proportional to $1/a$, where a is given by Eq. (30). For $\vartheta\to\pi/2$ we have $1/a\to 0$. The quantity β, and so the resonance amplitude, remains finite for $\vartheta\to\pi/2$, and, as a consequence, a resonance with finite amplitude and a linewidth zero is not observable. This is in agreement with measurements reported in (ii) of Sec. III of I.

VI. CONCLUSION

The theory presented for the quantum microwave absorption, correctly taking into account that many values of the wave vector $\vec{k}$ exist for the electromagnetic field in a metal in the presence of a static magnetic field, is able to explain some experimental results previously found for bismuth. However, there are some discrepancies between theory and measurements. In particular, the behavior of $n_{\min}$ (that is, the last level for which resonances are measured) as a function of the cross sectional area S_F of the Fermi surface is not completely in agreement with the experimental results of Fig. 11 of I. The increasing of $n_{\min}$ with S_F is not logarithmic but almost linear. Further, when the magnetic field H is parallel to the sample surface within ±1° the value of $n_{\min}$ goes to zero. We think that these discrepancies have to be ascribed to the drastic approximation used to obtain Eq. (24).

APPENDIX

We discuss the following.

(a) To write Eq. (2) in the frame ϵ, p_z, and φ one must remember that φ is the angular variable that locates the point representative, in momentum space, of the carrier state on the orbit given by the intersection of the energy constant surface $\epsilon(\vec{p})=\epsilon$ with the plane $p_z=\text{const}$ orthogonal to the magnetic field $\vec{H}$ (Fig. 2). If one uses the translation

$$p_x=p_x'-a, \quad p_y=p_y'-b,$$

and, subsequently, the rotation

$$p_x'=\bar{p}_x\cos\alpha-\bar{p}_y\sin\alpha, \quad p_y'=\bar{p}_x\sin\alpha+\bar{p}_y\cos\alpha,$$

Eq. (2) becomes

$$2m_0(\epsilon-\bar{\alpha}_z p_z^2)=\bar{\alpha}_x\bar{p}_x^2+\bar{\alpha}_y\bar{p}_y^2,$$

provided that

$$a=\frac{2a_{22}a_{13}-a_{23}a_{12}}{4a_{11}a_{22}-a_{12}^2}\,p_z=\gamma_x p_z,$$

$$b=\frac{2a_{11}a_{23}-a_{13}a_{12}}{4a_{11}a_{22}-a_{12}^2}\,p_z=\gamma_y p_z,$$

$$\tan\alpha=\frac{-(a_{11}-a_{22})\pm[(a_{11}-a_{22})^2+a_{12}^2]^{1/2}}{a_{12}},$$

$$\bar{\alpha}_z=a_{33}+a_{22}\gamma_y^2+a_{12}\gamma_x\gamma_y+a_{11}\gamma_x^2-a_{13}\gamma_x-a_{23}\gamma_y,$$

$$\bar{\alpha}_x=a_{11}\cos^2\alpha+a_{22}\sin^2\alpha+a_{12}\sin\alpha\cos\alpha,$$

$$\bar{\alpha}_y=a_{11}\sin^2\alpha+a_{22}\cos^2\alpha-a_{12}\sin\alpha\cos\alpha.$$

To introduce the angular variable $\varphi=\Omega t$, it is sufficient to set

$$\bar{p}_x=\frac{r}{(\bar{\alpha}_x)^{1/2}}\cos\varphi, \quad p_y=\frac{r}{(\bar{\alpha}_y)^{1/2}}\sin\varphi,$$

where $r^2=2m_0[\epsilon-\bar{\alpha}_z p_z^2/(2m_z)]$. Then, the quantities v_x, v_y, and v_z are given by Eq. (4) with

$$u_x=\frac{r(\bar{\alpha}_x)^{1/2}}{m_0}, \quad u_y=\frac{r(\bar{\alpha}_y)^{1/2}}{m_0},$$

$$u_1=\gamma_x\cos\alpha+\gamma_y\sin\alpha, \quad u_2=\gamma_y\cos\alpha-\gamma_x\sin\alpha.$$

Further, it is easy to see that for the cyclotron mass one obtains

$$m_c=\frac{1}{2\pi}\frac{\partial S(\epsilon,p_z)}{\partial\epsilon}=\frac{m_0}{(\bar{\alpha}_x\bar{\alpha}_y)^{1/2}},$$

where S is the area of the ellipse Γ of Fig. 2.

(b) The expressions of $f^2(\vartheta)$, $\bar{\delta}$, a_x, and a_y are the following:

$$f^2(\vartheta)=[(\bar{\alpha}_x)^{1/2}\sin\alpha\sin\vartheta+(\bar{\alpha}_x)^{1/2}\cos\vartheta(\gamma_x\cos\alpha+\gamma_y\sin\alpha)]^2+[(\bar{\alpha}_y)^{1/2}\cos\alpha\sin\vartheta+(\bar{\alpha}_y)^{1/2}\cos\vartheta(\gamma_y\cos\alpha-\gamma_x\sin\alpha)]^2/m_0^2,$$

$$\bar{\delta}=\left(\frac{2a_y\sin^2\vartheta+a_x}{\bar{\alpha}_x\bar{\alpha}_y}\right)_{\text{electrons}}+\cdots,$$

where the ellipsis represents an analogous term for holes,

$$a_x=\frac{\bar{\alpha}_x\cos^2\alpha+\bar{\alpha}_y\sin^2\alpha}{\bar{\alpha}_x\bar{\alpha}_y},$$

$$a_y=\frac{\bar{\alpha}_x\sin^2\alpha+\bar{\alpha}_y\cos^2\alpha}{\bar{\alpha}_x\bar{\alpha}_y}.$$

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