

## High-field galvanomagnetic effects in compensated nonpolar semiconductors

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In the presence of shallow attractive traps, the galvanomagnetic effects of the nonequilibrium carriers are investigated for compensated nonpolar semiconductors at low lattice temperatures when the free carrier loses momentum and energy, respectively, by scattering from a system of randomly oriented dipoles and deformation acoustic phonons. Numerical calculations are made for *n*-Ge at liquid-helium temperature. It is shown that the results are useful for the determination of some capture parameters and for an estimation of the degree of compensation in the material.

### I. INTRODUCTION

The importance of the investigation of the galvanomagnetic characteristics of hot carriers is well known.<sup>1</sup> They are very sensitive to the nonequilibrium energy distribution function which depends considerably upon the momentum and energy relaxation processes for the hot carriers. The critical electric field for the onset of the hot carrier region in Ge and Si increases with the lattice temperature.<sup>1</sup> For lattice temperatures around 4.2 K, electrons may become hot in relatively weak electric fields. For example in *n*-Ge electrons become hot for a field of a few volts per centimeter.<sup>1-5</sup> In this region of lattice temperatures, the thermal energy available to excite optical-mode lattice vibrations is quite limited since the optical phonons have characteristic temperatures usually higher than 300 K.<sup>6</sup> In Si different characteristic temperatures of intervalley phonons are 190, 600, 630, and 700 K.<sup>6</sup> So both the optical-phonon scattering in Ge and the intervalley scattering in Si play an insignificant role for a lattice temperature around 4.2 K. Moreover, for a sample with carrier concentration less than  $6 \times 10^{17}$  per  $\text{cm}^3$ , one is also justified in neglecting carrier-carrier scattering when the lattice is at liquid-helium temperature.<sup>7</sup> For a covalent semiconductor at such low temperatures and with certain impurity concentrations, the energy of hot carriers is thus dominantly scattered by acoustic-mode lattice vibrations while their momentum relaxation is primarily due to the static impurities.<sup>8</sup>

In calculating the momentum relaxation time for collisions with the singly ionized Coulomb centers, it is usually assumed that the neighboring centers do not overlap, i.e.,  $\sqrt{\sigma}/d \ll 1$ ;  $\sigma$  being the effective scattering cross section and  $d (=N^{-1/3})$  the average distance between neighboring centers, where  $N$  is the concentration of such centers.<sup>9,10</sup> However, this assumption is not valid at low temperatures when the concen-

tration of the scattering centers is of the order of  $10^{15}/\text{cm}^3$  or more.<sup>9</sup> Hence since the standard expressions for the scattering cross section of these ionized centers are inapplicable a somewhat different approach is made.

Experimental observations made by Reiss, Fuller, and Morin<sup>11</sup> reveal that in compensated semiconductors, ionic pairs in the form of dipoles are formed and as a result they are removed as Coulomb scattering centers. The probability of formation of groups more complex than a dipole is not high.<sup>9</sup> Thus in a compensated semiconductor, at low temperatures, the system of scattering centers may be assumed to consist of dipoles and neutral atoms. It has been shown by Tsertsvadze<sup>9</sup> that for a particular concentration ( $N_D$ ) of the donor impurities the dipole scattering can be made to dominate over the neutral impurity scattering by suitably adjusting the degree of compensation  $C_0 (=N_A/N_D, N_A$  being the acceptor concentration). For example, in *n*-Ge with  $N_D = 10^{16}/\text{cm}^3$  and at lattice temperature of 4.2 K, the dominant momentum scattering will be due to the dipoles if  $C_0$  is  $10^{-3}$  or more.

Of the various theoretical and experimental results on the high-field galvanomagnetic effects, Conwell<sup>1</sup> and others<sup>6,12</sup> have discussed only those based on changes in the distribution function of the carriers with electric fields while the number of carriers remain constant. But the free-carrier concentration, a thermodynamic variable for semiconducting materials, depends significantly upon the external fields as a result of capture of these carriers by different trapping centers.<sup>13</sup> The trapping of the free carriers is often found to dominate over the band-to-band transitions for samples like Ge and Si.<sup>13</sup> The ionization and recombination probabilities are dependent upon the carrier velocity. If the carriers are trapped by attractive centers, the concentration increases on heating the carrier system.<sup>4</sup>

The theory of capture by shallow attractive traps described by a Coulomb potential is well

developed. The very large value of the capture coefficient observed in covalent semiconductors at low temperatures is explained by Lax's theory of cascade of one-phonon transitions.<sup>3,14</sup> Applying this theory one can calculate the field dependence of the carrier concentration. This field dependence is to be taken into account in order to investigate the galvanomagnetic characteristics of the non-equilibrium carriers in the presence of the above trapping centers.

The purpose of the present paper is to study the galvanomagnetic characteristics of the hot electrons in a compensated covalent semiconductor in the presence of shallow attractive traps and under the condition when the carrier momentum is dominantly scattered by a system of randomly oriented dipoles and the energy is scattered by deformation acoustic phonons. In Sec. II the galvanomagnetic theory for any arbitrary angle between the directions of the electric and the magnetic field vectors is developed considering the weakly heated and the strongly heated carrier systems. The field dependence of the galvanomagnetic coefficients are obtained both under the fixed-field and the fixed-current conditions. The numerical results obtained for *n*-Ge at 4.2 K are presented in Sec. III. The results are discussed in Sec. IV. It is shown that this study suggests a suitable method to determine some capture parameters and also to estimate the degree of compensation in the material.

## II. THEORY

It is most probable that the dipole impurities are distributed randomly in the crystal. Using the first Born approximation, the elastic scattering cross section and hence the isotropic momentum free path  $l_D$  can be calculated and is given by<sup>9,15</sup>

$$l_D = \frac{3\hbar^2 k_a k_B T x}{4\pi m^* e^2 c_0 n^{1/3}} = l_D^0 x, \quad (1)$$

where  $\hbar = h/2\pi$ ,  $h$  being the Planck's constant,  $k_a$  is the dielectric constant,  $k_B$  is the Boltzmann constant,  $T$  is the lattice temperature,  $x$  is the carrier energy normalized to  $k_B T$ ,  $m^*$  is the scalar effective mass,  $e$  is the electronic charge, and  $l_D^0$  is the energy-independent factor of the free path. It is to be noted that unlike the situation for the other static defects, the momentum free path for dipole scattering is explicitly dependent upon the degree of compensation (defined earlier) in the sample. The momentum relaxation time due to the dipoles will also dominate that due to deformation acoustic phonons provided  $l_a \gg l_D^0(x)$ ;  $l_a$  being the momentum free path for deformation

acoustic phonons and the angular bracket standing for an average over the distribution function.

It is well known that in the phonon equipartition approximation,  $l_a$  is independent of the carrier energy and such an approximation can be made if  $k_B T \gg m^* s^2 \langle x \rangle$ ,  $s$  being the average acoustic velocity. The energy free path  $l_a$  for the deformation acoustic phonon scattering is related to the corresponding momentum free path in the phonon equipartition approximation by the relation<sup>16</sup>  $l_a = k_B T l_a^0 / 2m^* s^2$ .

Let an electric field  $E_x$  be applied to a covalent semiconductor along the  $X$  axis simultaneously with a magnetic field  $\vec{H}$  which makes an angle  $\kappa$  with  $E_x$  in the  $X$ - $Z$  plane. Let  $\beta$  be the angle between  $\vec{H}$  and the total heating field  $\vec{E}$  ( $= \vec{E}_x + \vec{E}_y$ ;  $E_y$  being the Hall field). If the band structure is known, one can solve the Boltzmann transport equation to find the nonequilibrium energy distribution function for the carriers when the momentum and energy are scattered, respectively, by a system of randomly oriented dipoles and by deformation acoustic phonons.

Neither Ge nor Si possess parabolic band structure. But in the region of lattice temperatures of our interest, when the energy is scattered by acoustic phonons, the electron temperature in *n*-Ge assumes a value of the order of 100 times the lattice temperature for an electric field of 50 V/cm.<sup>17</sup> So under these circumstances, the energy which an electron may gain or lose in a single scattering event amounts to  $\approx 0.01$  eV. Thus the electrons will be confined to a short segment of the dispersion curve where the nature of the curve can ordinarily be regarded as parabolic. Moreover, although the complex form of the band structure is known to lead to some nontrivial results, in many cases the anisotropy and the many-valley effects give rise to just some changes in the numerical coefficients of the galvanomagnetic characteristics.<sup>4</sup> Thus in the region of liquid-helium temperature, one can assume a parabolic dispersion law without any serious loss of accuracy. But even for the parabolic band structure the transport equation is not amenable to analytical solution for any value of the electric and the magnetic field. So only some limiting cases are to be considered separately.

If we consider the carrier system in the presence of a low field that satisfies the condition  $E < (6m^* s^2 k_B T / e^2 l_D^0)^{1/2}$ , the analytical solution of the transport equation can be obtained for a magnetic field  $H < c(2m^* k_B T)^{1/2} / e l_D^0$  and is given by<sup>8</sup>

$$f_0(x) \sim \left(1 + \alpha_D \frac{l_a}{l_D^0} x\right) \exp(-x). \quad (2)$$

Here  $f_0(x)$  is the isotropic part of the nonequilibrium energy distribution function, and  $c$  is the velocity of light.  $\alpha_D$  is the square of the normalized heating electric field<sup>18</sup>:  $\alpha_D = (el_D^0 E)^2 / 6m^* s^2 k_B T$ . Using (2) one can calculate the average carrier energy and see that the carrier system is only slightly deviated from the state of thermodynamic equilibrium with the lattice. Such a system is usually termed as a weakly heated system. On the other hand, for an electric field as high as to satisfy the condition  $E > (6m^* s^2 k_B T / e^2 l_a l_D^0)^{1/2}$ , the solution in the presence of a magnetic field  $H < c(2m^* k_B T)^{1/2} / el_D^0 \langle x \rangle$  can also be obtained analytically.<sup>8</sup> This solution takes the form

$$f_0(x) \sim \left(1 - \frac{n_D \sin^2 \beta}{2\alpha_D (l_a/l_D^0) x^2}\right) \exp\left(-\frac{x}{\alpha_D (l_a/l_D^0)}\right), \quad (3)$$

where  $\eta_D$  is the square of the normalized magnetic field<sup>18</sup>:  $\eta_D = (el_D^0 H)^2 / 2m^* c^2 k_B T$ . Here one can see that the average carrier energy largely exceeds that of the lattice atoms. Hence such a system may be termed a strongly heated system.

Even for the isotropic band structure, the carrier mobility in the presence of a magnetic field becomes a second-order tensor. The total current in this case is the vector sum of the current along the heating field  $\vec{E}$ , the Hall current and the current along the magnetic field  $\vec{H}$ . The elements of the mobility tensor are easily expressible in terms of the kinetic coefficients  $K_1$ ,  $K_2$ , and  $K_3$  for the specific type of relaxation processes of our consideration. These coefficients are defined by the expression for the current density  $\vec{J}$ :

$$\vec{J} = -en \left( K_1 \vec{E} + K_2 \frac{\vec{E} \times \vec{H}}{H} + K_3 \frac{\vec{H}(\vec{E} \cdot \vec{H})}{H^2} \right), \quad (4)$$

where  $n$  is the nonequilibrium carrier concentration. Using this, one can find the expressions for the galvanomagnetic coefficients. The magnetoresistance  $\rho$  defined by  $\vec{E} \cdot \vec{J} / J^2$  is given by

$$\rho_j = \frac{K_1^2 + K_2^2 \cos^2 \kappa + K_1 K_2 \sin^2 \kappa}{en(K_1^2 + K_2^2)(K_1 + K_3)} \quad (5)$$

under the fixed current ( $j$ ) condition where all but the  $X$  component of the current density vanishes, and by

$$\rho_E = \frac{K_1 + K_2 \cos^2 \beta}{en(K_1^2 + K_2^2 \sin^2 \beta + K_3^2 \cos^2 \beta + 2K_1 K_2 \cos^2 \beta)} \quad (6)$$

under the fixed field ( $E$ ) condition where all but the component of the electric field vanishes. The Hall constant  $R$  for the case of crossed fields is given by

$$R = \frac{-K_2}{enH(K_1^2 + K_2^2)}. \quad (7)$$

Thus we see that the problem of calculating the galvanomagnetic coefficients actually leads to that of calculating the kinetic coefficients and the nonequilibrium carrier concentration. Once the energy distribution function is known, one can calculate the kinetic coefficients by the method described in Ref. 4. Let us now consider the problem of calculating the nonequilibrium carrier concentration. The carrier concentration in the presence of attractive traps of density  $N_D^+$  may be obtained from a solution of the rate equation

$$\frac{dn}{dt} = (N_D - N_D^+) \sum_M c_M A_M - n N_D^+ \sum_M c_M B_M, \quad (8)$$

where  $A_M$  and  $B_M$  are the coefficients of ionization and recombination, respectively.<sup>19</sup> The subscript "M" denotes the specific mechanism of ionization or recombination.  $c_M$  is a constant which is equal to unity for all processes but the impact one, for which it assumes the value equal to the density of the free carriers. For the cases of our consideration, only the mechanism of thermal ( $T$ ) recombination and the impact ( $I$ ) ionization need to be taken into account.<sup>20</sup> For the weakly heated system, impact ionization produces a negligible effect and the steady-state solution of this equation is given by

$$n = n_0 \left( \frac{(B_T)_0}{B_T(E, H)} \right)^{1/2}, \quad (9)$$

where the subscript "0" stands for the state of thermodynamic equilibrium. For the strongly heated system, however one can obtain

$$n = (N_D - N_A) \left( 1 - \frac{N_D}{N_D - N_A} \frac{B_T(E, H)}{A_I(E, H)} \right). \quad (10)$$

Since the recombination time is very large compared to the energy relaxation time, it can be assumed that the recombination of the free carriers takes place under the condition of equilibrium with respect to the energy and the crystal momentum. Thus, if the relevant cross sections are known, one can calculate the recombination and the ionization coefficients with the help of the stationary distribution functions (2) and (3).

The recombination coefficient  $B_T(E, H)$  will be calculated using the Lax cascade theory.<sup>14</sup> At low temperatures, electrons will have energy sufficient to emit acoustic phonons only and the effective recombination cross section is given by  $\sigma_T(x) = 4^5 \sigma_1 / 6\gamma^4 x(x + \delta_0/\gamma)$ , where  $\gamma = 2k_B T / m^* s^2$ ,  $\sigma_1$  is some characteristic cross section, and  $\delta_0$  plays the role of a binding energy whose dependence on  $\gamma$  is given in Ref. 14. The impact ionization coefficient  $A_I(E, H)$  for the strongly heated system is determined mainly by the hot carrier

distribution function and is not sensitive to the energy dependence of the impact ionization cross section  $\sigma_I$ . So one can use such an approximation as  $\sigma_I(x) = \sigma_I^0$  which simplifies the calculations considerably but does not significantly affect the value of the carrier concentration.<sup>21</sup>

Thus for the weakly heated system, using (2) one can obtain the magnetoresistance

$$\rho_E = \rho_0 \left\{ 1 - \alpha_D \frac{l_a}{l_D^0} \left[ \frac{5}{2} + \left( \frac{\delta_0}{\gamma} \right) \exp\left(\frac{2\delta_0}{\gamma}\right) + 3\eta_D \sin^2 \kappa \left( 1 - \frac{\Gamma^2(7/2)}{12} \right) \right] \right\} \quad (11)$$

and

$$\rho_j = \rho_0 \left\{ 1 - \alpha_D \frac{l_a}{l_D^0} \left[ \frac{5}{2} + \left( \frac{\delta_0}{\gamma} \right) \exp\left(\frac{2\delta_0}{\gamma}\right) + 3\eta_D \sin^2 \beta \left( 1 - \frac{\Gamma^2(7/2)}{12} \right) \right] \right\} \quad (12)$$

in the fixed field and the fixed current conditions, respectively. The inverse of the Hall constant is given by

$$R^{-1} = \frac{16}{3\sqrt{\pi} \Gamma^2(7/2) R_0} \left\{ 1 + \alpha_D \frac{l_a}{l_D^0} \left[ \frac{7}{2} + \left( \frac{\delta_0}{\gamma} \right) \exp\left(\frac{2\delta_0}{\gamma}\right) - 6\eta_D \left( 1 - \frac{\Gamma^2(7/2)}{24} \right) \right] \right\}. \quad (13)$$

Here  $\rho_0$  and  $R_0$  are, respectively, the weak electric field resistivity and the Hall constant for dipole scattering.

Considering the strongly heated system, using (3) one can obtain

$$\rho_j = \frac{\rho_0 n_0 b^{1/2}}{N_D (1 - c_0)} \left( 1 + \frac{4^5 \sigma_1 b^2 f_1(b) \exp(Z/2) (1 - b^{-1} \eta_D \varphi_j) [1 + ab^{-2} f_4(b)]}{6(1 - c_0) \gamma^4 \sigma_I^2 W_{1/2,1}(Z) Z^{1/2}} - b^{-1} \eta_D \varphi_j \right) \quad (14)$$

and

$$\rho_E = \frac{\rho_0 n_0 b^{1/2}}{N_D (1 - c_0)} \left( 1 + \frac{4^5 \sigma_1 b^2 f_1(b) \exp(Z/2) (1 - b^{-1} \eta_D \varphi_E \sin^2 \beta) [1 + ab^{-2} f_4(b)]}{6(1 - c_0) \gamma^4 \sigma_I^2 W_{1/2,1}(Z) Z^{1/2}} - b^{-1} \eta_D \varphi_E \sin^2 \beta \right). \quad (15)$$

Here  $b = l_D^0 / \alpha_D l_a$ ;  $Z = bI / R_B T$ ;  $a = (b\eta_D \sin^2 \beta) / 2$ ;

$$f_4(b) = \frac{Z W_{3/2,2}(Z)}{W_{1/2,1}(Z)} + \frac{15}{4} ab^2 Z \frac{W_{3/2,2}(Z)}{W_{1/2,1}(Z)} - \frac{15}{4} b^4 - b^2 f_3(b)$$

$$f_3(b) = f_2(b) / f_1(b)$$

$$f_2(b) = -(\delta_0 / \gamma)^2 \exp(\delta_0 b / \gamma) Ei(-\delta_0 b / \gamma) + \sum_{m=1}^2 (m-1)! (-\delta_0 / \gamma)^{2-m} (b)^{-m} + \frac{15}{4} (b)^{-2} \exp(-\delta_0 \gamma / b) Ei(-\delta_0 \gamma / b)$$

$$f_1(b) = -\exp(-\delta_0 b / \gamma) Ei(-\delta_0 b / \gamma)$$

$$\varphi_j = \frac{15}{8} + (9 \cos^2 \kappa \cos^2 \theta) / 8 + \Gamma^2(7/2) (\sin^2 \kappa) / 4$$

$$\varphi_E = \frac{15}{8} \left( 1 + \frac{15\pi}{32} \right).$$

$W(Z)$  and  $Ei(X)$  are, respectively, the Whittaker function<sup>22</sup> and the integral exponential function.<sup>23</sup>  $I$  is the ionization energy of the impurity traps. It is easy to see that the magnetoresistance is independent of the ionization energy if the trap is shallow enough to satisfy the condition  $I \ll \alpha_D l_a k_B T / l_D^0$ . The reciprocal of the Hall constant in the presence of such shallow trapping centers is given by

$$R^{-1} = \frac{128(1 - c_0) N_D}{45\pi R_0 n_0} \left[ 1 - \frac{4^5 \sigma_1 b^2}{6(1 - c_0) \sigma_I^2 \gamma^4} \ln\left(\frac{\gamma}{\delta_0 b}\right) - 2\eta_D b^{-1} \left( 1 + \frac{225\pi}{64} \right) \right]. \quad (16)$$

It should be mentioned here that the galvanomagnetic coefficients depend upon the total heating field  $E$  and upon the angle  $\beta$ . For a direct comparison of the results with experiments, one

should know the dependence of these coefficients upon the applied field  $E_x$  and upon the angle  $\kappa$ . So in order to facilitate the comparison, one should know such relations as  $E = f(E_x)$  and  $\beta = \psi(\kappa)$ . These relations are dependent upon the experimental condition. In the fixed-field condition  $E = E_x$  and  $\beta = \kappa$ , whereas in the fixed current condition  $E = E_x(1 + \tan^2 \theta)^{1/2}$  and  $\beta = \cos^{-1}(\cos \theta \cos \kappa)$ , where  $\theta$  is the Hall angle.

Lastly it should be mentioned here that on increasing the electric field when it exceeds some critical value  $E_c$ , the de Broglie wavelength of the carriers becomes smaller than the average dipole length and the carrier momentum is then scattered by the potential of the individual ions. This critical field can be obtained from the relation  $\langle x \rangle(E_c) = 2\pi^2 \hbar^2 N_D^{2/3} / m^* k_B T$ . So this imposes another limitation to the value of the applied electric field for the dipole scattering to be meaningful.

### III. DISCUSSIONS AND NUMERICAL RESULTS FOR *n*-Ge

Let us first consider the weakly heated system. For this system it is easy to see that  $E \approx E_x$  and thus the magnetoresistance both under the fixed field and fixed current condition decrease, and the inverse of the Hall constant increases with an increase of the applied electric field. The field dependence is quadratic in each case. Both the magnetoresistance,  $\rho_j$  and  $\rho_E$ , increase slowly with the compensation ratio and the angle  $\kappa$  between  $E_x$  and  $\vec{H}$ . The inverse of the Hall constant, however, slowly decreases with the increase of the compensation ratio.

For the strongly heated system, the dependence of the galvanomagnetic coefficients upon the various parameters is more involved. The above theory can be applied to get numerical results for a sample of *n*-Ge at 4.2 K. The following values of the parameters are assumed:

$$\begin{aligned} m^*/m_0 &= 0.12, \quad s = 5.4 \times 10^5 \text{ cm/sec}, \quad k_a = 15.8, \\ l_a &= 6.72 \times 10^{-4} \text{ cm}, \quad \delta_0 = 8.0, \quad \sigma_1 = 2.13 \times 10^{-9} \text{ cm}^2, \\ \sigma_I^0 &= 1.05 \times 10^{-14} \text{ cm}^2, \quad N_D = 10^{16} / \text{cm}^3, \quad H = 500.0 \text{ oersted}. \end{aligned}$$

Figures 1-3 illustrate the variation of the galvanomagnetic coefficients with different parameters. From Fig. 1 it is seen that the magnetoresistances,  $\rho_j^0$  and  $\rho_E$ , depend strongly upon the applied electric field, decreasing with the increase of the latter. For lower values of the applied field, the rate of fall is faster and at higher values the curves exhibit a trend toward lower saturation. This can be explained by the

fact that with the increase of the electric field, the carrier energy increases; as a result the thermal recombination coefficient decreases and the impact ionization coefficient increases. Thus the nonequilibrium carrier concentration in the presence of shallow attractive traps increases with the applied electric field. Initially  $B_T/A_I$  sharply decreases with the increase of the electric field and at higher values of the field when  $B_T/A_I \rightarrow 0$ , the concentration tends to saturate. So in the region of higher fields, the field dependence of the magnetoresistance is attributed only to the heating of the charge carriers, whereas in the region of low fields it is due to the capture of the free carriers by the trapping centers. The magnetoresistances are also strongly dependent upon the compensation ratio, increasing with the latter. However, the rate of fall of  $\rho$  with  $E_x$  is higher for lower values of the compensation ratio.

Since the total heating field  $E$  is greater than the applied field  $E_x$ ,  $\rho_E$  is always greater than  $\rho_j$  for any  $E_x$ . The parameter  $(\rho_E - \rho_j)$  at a particular value of the electric field depends upon the magnetic field and the compensation ratio. In the weak magnetic field limit for which the theory is developed here, the value of  $\rho_E$  differs little from that of  $\rho_j$ . At lower values of the compensation,  $(\rho_E - \rho_j)$  increases with the applied electric field, whereas for higher values  $\rho_E \sim \rho_j$  in the range of electric field of our interest.

Figure 2 illustrates that  $\rho_E$  is weakly dependent upon  $\kappa$ , decreasing slowly with the increase of the angle  $\kappa$ . It is also seen that this dependence is prominent only at higher values of the applied electric field. Moreover, with the increase of the compensation ratio, this dependence becomes prominent at still higher values of the applied electric field. The rate of change  $\partial \rho_{j,E} / \partial E_x$  at any field is determined by the degree of compensation and by the trapping parameter  $\sigma_1 / \sigma_I^0$ .

From Fig. 3 it is seen that  $-R^{-1}$  depends strongly upon the applied electric field. For lower values of  $E_x$  it increases sharply with  $E_x$ , and at higher values of the field the rate of increase slows down tending towards a region of saturation. This saturation also is due to the trend of concentration saturation at higher values of the applied electric field and the field dependence observed in this region is mainly due to the heating of the carriers, since the capture process has little effect on the galvanomagnetic coefficients as the carrier concentration approaches the saturation value  $N_D(1 - c_0)$ . It is to be noted that this saturation of the nonequilibrium carrier concentration is not characteristic of the carrier relaxation processes considered

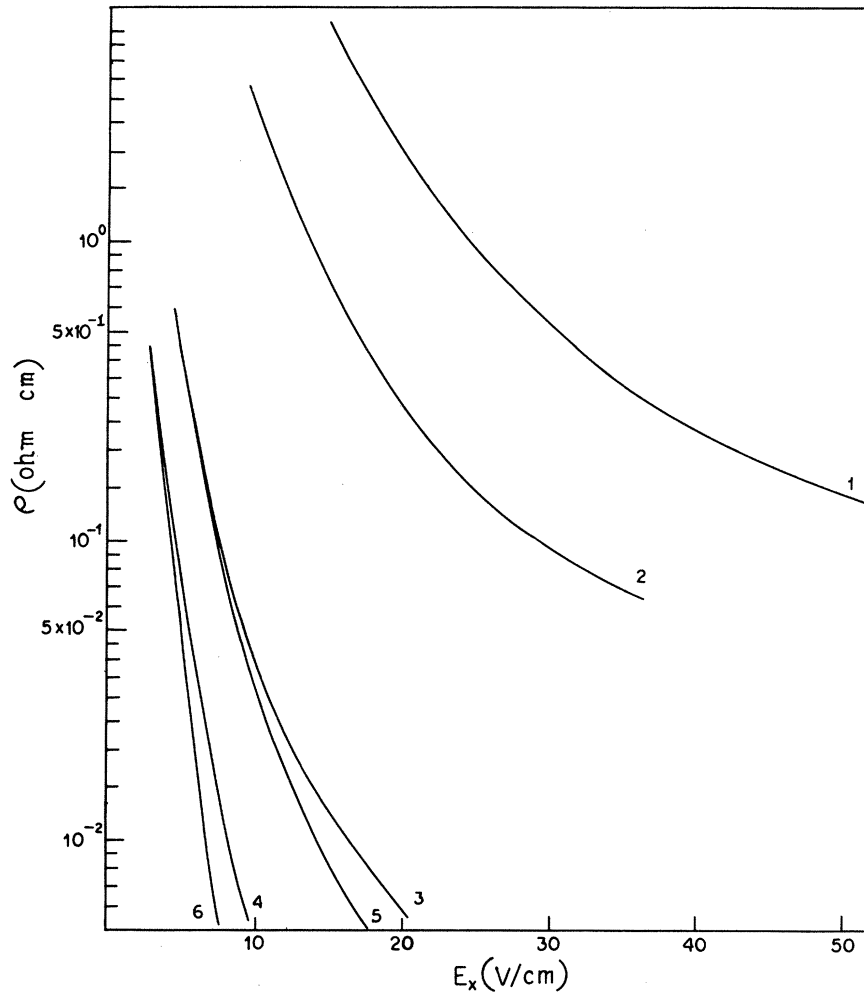


FIG. 1. Dependence of the magnetoresistance upon the applied electric field for different values of the compensation ratio when  $\kappa = 45^\circ$ . Curves 1-4 depict the dependence under the fixed-field condition for  $c_0 = 0.1, 0.05, 0.01,$  and  $0.005,$  respectively. Curves 5 and 6 give the same dependence under the fixed current condition for  $c_0 = 0.01$  and  $0.005,$  respectively.

here. But how fast this saturation is reached with the increase of the applied electric field depends upon the relaxation processes. It also depends upon the nature of the trapping centers and the degree of compensation.  $-R^{-1}$  is also strongly dependent upon the compensation ratio. It increases rapidly with  $c_0$  and approaches the saturation value at a lower value of the applied electric field for lower  $c_0$ .

#### IV. CONCLUSIONS

In the presence of shallow attractive traps, the hot-electron galvanomagnetic effects in a covalent semiconductor is studied theoretically under the condition when the carrier momentum and energy are scattered by a system of randomly

oriented dipoles and deformation acoustic phonons, respectively. Expressions for the magnetoresistance are obtained for the fixed field and also for the fixed current regime. Numerical results for  $n$ -Ge show that for the types of relaxation processes considered dominant here, the non-linear effects can be observed even for a field of a few volts per cm. The galvanomagnetic coefficients and their derivatives with respect to the applied electric field are found to be strongly dependent upon the applied electric field and the degree of compensation. They are also dependent upon the capture parameter  $\sigma_1/\sigma_T^0$ . However, their dependence upon the angle  $\kappa$  between  $E_x$  and  $\vec{H}$  is rather weak.

The observed field dependence of the galvanomagnetic coefficients is the simultaneous effect

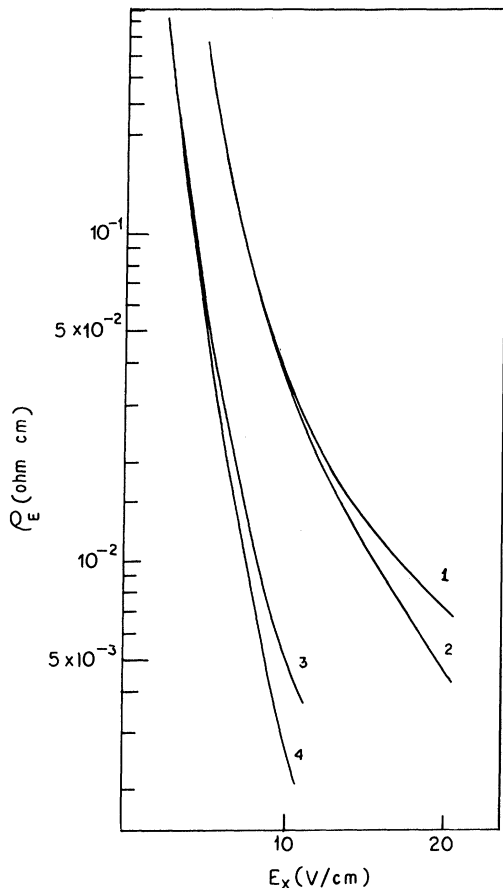


FIG. 2. Dependence of the magnetoresistance upon the applied electric field for different values of  $\kappa$  at a fixed value of the compensation ratio. Curves 1 and 2 show the dependence under the fixed-field condition at  $c_0 = 0.01$  for  $\kappa = 20^\circ$  and  $70^\circ$ , respectively. Curves 3 and 4 depict the same dependence at  $c_0 = 0.005$  for  $\kappa = 20^\circ$  and  $70^\circ$ , respectively.

of the capture and the heating of the carriers. The former effect dominates in the low-field region whereas the latter dominates in the high-field region. The theory developed here is valid for the strongly heated system in  $n$ -Ge in the applied electric field range of a few volts per cm to a few tens of volts per cm. Outside this range, in the lower-field region the condition of heating suffers whereas in the high-field region the condition of phonon equipartition approximation becomes invalid.

Lastly it should be mentioned that the theory has been developed for a carrier system in the presence of a weak magnetic field for which the quantization effect can be neglected. For the sample  $n$ -Ge at 4.2 K, the calculations are per-

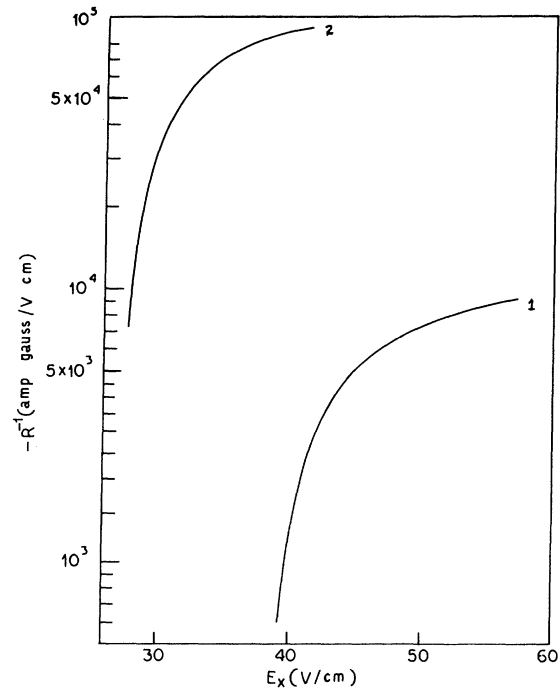


FIG. 3. Dependence of  $-R^{-1}$  upon the applied electric field for different values of compensation ratio. Curve 1 is for  $c_0 = 0.1$  and curve 2 is for  $c_0 = 0.05$ .

formed for  $H = 500.0$  oersted, whereas it is easy to see that the quantization effect should be considered for magnetic fields largely exceeding  $5 \times 10^3$  oersted.

The above results could not be checked since there is a dearth of experimental data,<sup>1,6,12</sup> although the approximations and the region of external fields for which the theory is developed seem to be easily accessible experimentally. However it is evident that if experiments are performed within the limits of the approximations used here, then from a comparison of these results with the experimental curves giving the applied field dependence of the galvanomagnetic coefficients of their derivatives, one can find the value of the capture parameter  $\sigma_1/\sigma_I^0$  and can also estimate the degree of compensation in the sample.

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- <sup>1</sup>E. M. Conwell, *High Field Transport in Semiconductors* (Academic, New York, 1967).
- <sup>2</sup>A. Zylbersztejn, *Phys. Rev.* **127**, 744 (1962).
- <sup>3</sup>S. H. Koeing, R. D. Brown, and W. Schillinger, *Phys. Rev.* **128**, 1668 (1962).
- <sup>4</sup>Z. S. Kachlishvili, *Phys. Status Solidi A* **33**, 15 (1976).
- <sup>5</sup>D. P. Bhattacharya, *Phys. Status Solidi B* **101**, issue 1 (1980).
- <sup>6</sup>B. R. Nag, *Theory of Electrical Transport in Semiconductors* (Pergamon, New York, 1972).
- <sup>7</sup>R. Stratton, in *Solid State Physics in Electronics and Telecommunications*, Proceedings of an International Conference, Brussels, 1958, edited by M. Desirant and J. L. Michiels, Vol. 1 (Academic, New York, 1960).
- <sup>8</sup>Z. S. Kachlishvili, *Phys. Status Solidi* **40**, 471 (1970).
- <sup>9</sup>A. A. Tsertsvadze, *Fiz. Tekh. Poluprovodn.* **1**, 1820 (1967) [*Sov. Phys.—Semicond.* **1**, 1505 (1968)].
- <sup>10</sup>V. I. Kogan and V. M. Galitskii, *Collection of Problems of Quantum Mechanics* (Prentice-Hall, Englewood Cliffs, 1963).
- <sup>11</sup>H. Reiss, C. S. Fuller, and F. J. Morin, *Bell Syst. Tech. J.* **35**, 535 (1956).
- <sup>12</sup>*International Conference on Hot Electronics in Semiconductors, North Texas State University, 1977*, edited by D. G. Seiler and A. E. Stephens [*Solid-State Electron.* **21** (1978)].
- <sup>13</sup>V. L. Bonch-Bruevich and E. G. Landsberg, *Phys. Status Solidi* **29**, 9 (1968).
- <sup>14</sup>M. Lax, *Phys. Rev.* **119**, 1502 (1960).
- <sup>15</sup>J. Appel and W. B. Teutsch, *J. Phys. Chem. Solids* **23**, 1521 (1962).
- <sup>16</sup>T. B. Levinson, thesis, Vilnius, 1966 (unpublished).
- <sup>17</sup>Z. S. Kachlishvili, *Phys. Status Solidi B* **48**, 65 (1971).
- <sup>18</sup>D. P. Bhattacharya and Z. S. Kachlishvili, *J. Phys. Chem. Solids* **41**, 83 (1980).
- <sup>19</sup>J. S. Blakemore, *Semiconductor Statistics* (Pergamon, New York, 1962).
- <sup>20</sup>N. Sclar and E. Burstein, *Phys. Rev.* **98**, 1757 (1955).
- <sup>21</sup>Z. S. Kachlishvili, *Fiz. Tekh. Poluprovodn.* **2**, 580 (1968) [*Sov. Phys.—Semicond.* **2**, 478 (1968)].
- <sup>22</sup>H. Batemann, in *Tables of Integral Transforms*, edited by A. Erdélyi, Vols. I and II (McGraw-Hill, New York, 1954).
- <sup>23</sup>A. Kratser and F. Frants, *Transtsendentalnie Funktsii* (Izd. in. Lit., Moskva, 1963).