Measurement of the temperature dependence of the order-parameter relaxation time of a superconducting aluminum film

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By measuring the critical dc current value of a superconducting aluminum film with a superposed ac current as a function of the frequency and amplitude of the ac current, it is shown that the order-parameter variations can be described with a time-dependent Ginzburg-Landau equation with the longitudinal relaxation time τ_Δ as the relevant time constant.

In a recent experiment[1] on the response of a superconducting aluminum strip to a current step to above the critical current it was found that there is a time delay between the beginning of the current and the onset of the voltage along the strip. These results were explained by the finite time it takes before the order parameter becomes zero as described by a time-dependent Ginzburg-Landau (TDGL) equation. In these experiments the time constant for the order-parameter variation showed no temperature dependence within measuring accuracy for the temperature range $0.76 < T/T_c < 0.92$. Theoretical work by Tinkham[2] showed that the expected time constant in the TDGL equation is the temperature-dependent longitudinal relaxation time[3] τ_Δ. More-recent pulse experiments with higher accuracy and made closer to T_c affirmed that the time constant is temperature dependent.[4]

This Communication presents results of a method that can be used to investigate the validity of the TDGL equation for describing a time-varying order parameter. The method essentially consists in determining the critical dc current of a superconducting strip as a function of the amplitude and frequency of a small superposed ac current. The use of a combination of dc and ac currents to investigate time-dependent phenomena in superconductors was already suggested by Schmid[5] and was used in a different experiment by Peters and Meissner.[6]

The TDGL equation which we use for the interpretation of our measurements is given by[2]

$$2\tau_\Delta f\frac{\partial f}{\partial t}=f-f^3-\frac{4}{27}\frac{j^2}{f^3}\ , \tag{1}$$

in which f is the order-parameter amplitude normalized to its equilibrium value and j the current density normalized to the critical current density both at the existing temperature $T < T_c$. The right-hand side of Eq. (1) is the normal form of the Ginzburg-Landau equation for a strip with a width small compared to the coherence length and a uniform value of f along the strip.[1] The time-derivative term on the left-hand side was given by Tinkham.[2] It originates from an extra term in the GL equation, the gap-control function introduced by Schmid.[7] This form is caused by the nonequilibrium distribution of the quasiparticles. If the relaxation of this distribution to equilibrium is described by a Boltzmann equation with a relaxation time constant τ_E, the inelastic electron scattering time, Eq. (1) can be derived for variations of f on a time scale long compared to τ_E and with τ_Δ given by

$$\tau_\Delta=1.2\tau_E(1-T/T_c)^{-1/2}\ . \tag{2}$$

When the total normalized current through the strip is given by a superposition of a dc and ac component

$$j=j_0-j_1\cos\omega t\ , \tag{3}$$

we want to calculate the maximum value j_{0m} of j_0 which still allows a periodic nonzero solution of Eq. (1) for f.

We therefore multiply Eq. (1) by f^3 and introduce a new variable $u=f^5$. Under the condition $j_1 \ll j_0$, j_{0m} will be close to 1, and we approximate the function $u^{4/5}-u^{6/5}$ by its Taylor expansion to second order around its maximum

$$u^{4/5}-u^{6/5}\approx\tfrac{4}{27}-\tfrac{27}{50}[u-(\tfrac{2}{3})^{5/2}]^2\ . \tag{4}$$

Equation (1) then becomes of the Riccati type, and introducing the new variables

$$\begin{aligned}\omega t&=2x,\\ \omega\tau_\Delta\frac{d}{dx}\ln y&=\tfrac{27}{10}[u-(\tfrac{2}{3})^{5/2}],\end{aligned} \tag{5}$$

we obtain

$$\frac{d^2y}{dx^2}+\left(-\frac{2(1-j_0^2)}{\omega^2\tau_\Delta^2}-\frac{4j_0j_1}{\omega^2\tau_\Delta^2}\cos2x\right)y=0\ . \tag{6}$$

With $a=-2(1-j_0^2)/\omega^2\tau_\Delta^2$ and $q=2j_0j_1/\omega^2\tau_\Delta^2$ this is the Mathieu equation in its standard form, which in general has two independent solutions of the Floquet form[8] $\exp(i\nu x)P(x)$ where $P(x)$ is a periodic function of x with period π. The characteristic exponent

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ν is real in the stability regions, while it can be chosen to be purely imaginary in the instability regions. The general solution in the stability regions always has zero points. In the instability regions the general solution becomes asymptotically equal to the exponentially growing solution of the Floquet form for large x. Because of Eq. (5), only those solutions y lead to periodic and bounded solutions of u which do not have zero points. Therefore only the first instability region, where $P(x)$ has no zeros, is acceptable. It can be shown that also the boundary of this region leads to acceptable solutions for u, so we have the condition[8]:

$$a \leq a_0(q) \quad . \tag{7}$$

For small values of q the function a_0 can be expanded as[8]

$$a_0(q) = -q^2/2 + 7q^4/128 + \cdots \quad , \tag{8}$$

and for large q we have the asymptotic expansion[8]

$$a_0(q) = -2q + 2\sqrt{q} - \tfrac{1}{4} + \cdots \quad . \tag{9}$$

With these approximations and $1-j_0 < j_1 << j_0 \approx 1$ condition (7) leads to

$$1-j_0 \geq \begin{cases} j_1 - \sqrt{j_1/2}\,\omega\tau_\Delta + \omega^2\tau_\Delta^2/16 \cdots , & \omega\tau_\Delta << 1 \quad (10) \\ j_1^2/2\omega^2\tau_\Delta^2 \cdots , & \omega\tau_\Delta >> 1. \quad (11) \end{cases}$$

For intermediate values of $\omega\tau_\Delta$ we use tabulated values of Eq. (7).[9] The value of the normalized critical current j_{0m} is given by the equality signs in Eqs. (7), (10), and (11).

The purpose of the measurements is to determine the difference in critical currents $I_{c0} - I_c = I_{c0}(1-j_{0m})$ without and with a superposed ac current of amplitude $j_1 I_{c0}$ as a function of the frequency of this current. It has to be checked whether this measured function fulfills relation (7) and if this is the case the time constant τ_Δ can be determined.

The actual measurements we will describe were made with a sample consisting of a 0.1-μm-thick, 2-μm-wide, and 40-μm-long aluminum film, evaporated on an oxidized silicon slice.[10] The sample was immersed in a temperature-regulated helium bath. The critical current was determined with a circuit which measures at a repetition rate of 100 Hz the value of a linearly increasing current through the sample at the moment when the strip switches to the normal state. A measurement of I_{c0} as a function of T showed that $I_{c0}^{2/3}$ depends linearly on T with an extrapolated critical temperature $T_c = 1.30$ K. By alternatively measuring the critical currents I_c and I_{c0} with and without a superposed ac current the difference $I_{c0} - I_c$ can be determined with a synchronous detector in much the same way as previously described.[11] The ac current is supplied by means of a 50-Ω coaxial circuit and it is checked that its amplitude is constant when the frequency is changed.

Results of measurements of $1-j_{0m}$ as a function of the frequency are given in Fig. 1 for three different values of the ac current amplitude. These

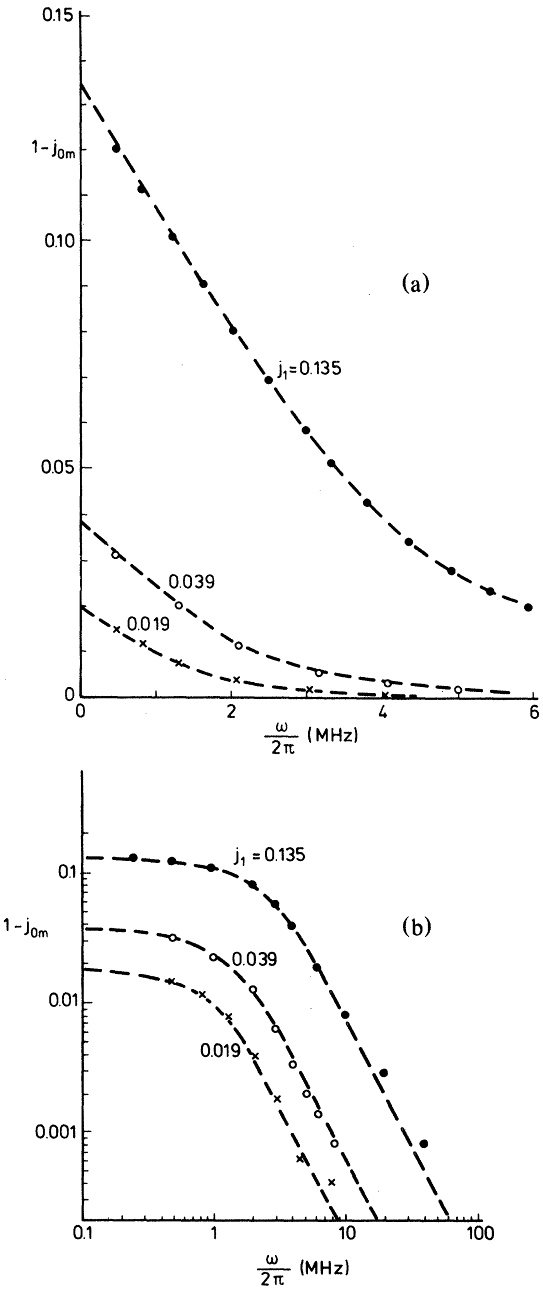

FIG. 1. Measured values of $(I_{c0}-I_c)/I_{c0} = 1-j_{0m}$ as a function of the frequency of the ac current component for three different normalized ac current amplitude values j_1. (a) As measured on a linear scale and (b) as plotted on a double logarithmic scale. The dashed curves are the curves given by Eq. (7) with $\tau_\Delta = 17.4$ nsec.

measurements are at $T = 1.146$ K where $I_{c0} = 0.52$ mA.

The dashed curves are the curves calculated with Eq. (7) with only one adjustable parameter $\tau_\Delta = 17.4$ nsec. There is very good agreement between experiments and the calculated curves. Figure 1(a) clearly illustrates the linear decrease of $1 - j_{0m}$ for small ω starting at j_1 for $\omega \rightarrow 0$ as given by Eq. (10), whereas the logarithmic plot of Fig. 1(b) shows the $1/\omega^2$ falloff at high frequencies as given by Eq. (11).

These measurements were repeated at various temperatures, and the parameter τ_Δ was determined as a function of T. The resulting values of τ_Δ are plotted in Fig. 2 versus $1 - T/T_c$. The full line is the dependence predicted by Eq. (5) with $\tau_E = 4.4$ nsec. This value for the inelastic scattering time agrees with values in the literature.[12,13]

For values $1 - T/T_c \gtrsim 0.1$ there is a tendency for τ_Δ to become less temperature dependent and this may be the reason that the previous pulse measurements[1] did not show a temperature-dependent delay time. The more-recent pulse measurements[4] show a $(1 - T/T_c)^{-1/2}$ temperature dependence of the delay time for current pulses just above the critical current

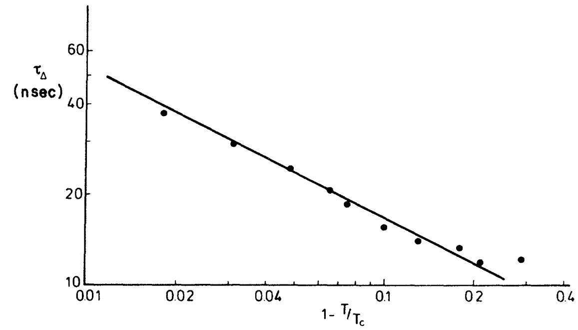

FIG. 2. Measured values of τ_Δ as a function of $1 - T/T_c$. The straight line is given by Eq. (2) with $\tau_E = 4.4$ nsec.

value, in agreement with the results of this paper.

In conclusion we propose that these measurements confirm in detail that the TDGL equation (1) describes correctly the time variations of the order parameter with the longitudinal relaxation time τ_Δ as the relevant time constant. The temperature dependence of τ_Δ agrees with the theoretical expectation.

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