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# Theoretical explanation of observed quantum-limit cyclotron resonance linewidth in InSb

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A theoretical explanation of the minimum observed in the cyclotron resonance (CR) linewidth as a function of magnetic field is presented by including electron scattering via acoustic phonons and ionized impurities. Quantum-limit CR linewidth broadens for phonon scattering while it narrows for ionized-impurity scattering with the increasing magnetic field. The minimum arises due to the competition between the two scattering mechanisms.

The last decade has seen a large number of investigations, both theoretical and experimental, to study cyclotron resonance (CR) linewidth in semiconductors as a potential source of unscrambling various scattering interactions. Unfortunately, such studies have produced a bewildering variety of sometimes contradictory results. McCombe *et al.*,[1,2] have systematically studied, from an experimental point of view, the quantum-limit CR linewidth in InSb as a function of total ionized-impurity concentration, free-carrier concentration, temperature, and magnetic field. A minimum in the linewidth as a function of magnetic field is observed in all samples. The magnitude of the linewidth and the magnetic field position of the minimum linewidth are independent of free-carrier concentration, indicating that free-carrier screening of the ionized impurities does not play a role. At high fields, the linewidths are proportional to $\sqrt{n_i}$, with $n_i$ the total number of ionized impurities. The experimental results so obtained were analyzed and interpreted in the light of existing theories.

In spite of a large number of theoretical works (see Refs. 1–5 for other useful references) directed specifically toward ionized-impurity scattering, no satisfactory, explicit or implicit, explanation of the linewidth minimum has been given. It is believed[1,2] that the linewidth minimum is indeed related to matching the cyclotron orbit size with the effective range of potential $a$ ($\lambda/a \approx 1$). According to the interpretation given,[2] the minimum results from a subtle interplay between inter- and intra-Landau-level scattering. But in the quantum limit when most electrons populate the lowest Landau level, interlevel scattering is quite small, indicating the importance of intralevel scattering in the lowest Landau level.[4,5]

Heuser and Hajdu,[6] by using the self-consistent perturbation theory, have attempted to explain the experimentally observed CR linewidth. They claim to have obtained a good agreement with results of other experiments[7,8] on InSb for realistic values of the strength and the range of the impurity potential taken to be of Gaussian form. The minimum of the linewidth found by McCombe *et al.*,[1,2] could not be explained by their theory as long as realistic values of the parameters were inserted into the conductivity formula.

Arora[3] developed a quantum theory of ac magnetoconductivity for isotopic scattering interactions by using an iterative solution of the Liouville equation for the density matrix, a technique which is equivalent to the Mori formalism used by Kawabata.[9] This theory was further elaborated by Arora and Spector[5] by including the anisotropic nature of the ionized-impurity scattering. When applied to combined acoustic-phonon and ionized-impurity scattering in the quantum limit, this theory yields a simple expression for the linewidth $\Gamma$ as a function of magnetic field $B$:

$$\Gamma = \Gamma_a + \Gamma_i \;, \tag{1}$$

$$\Gamma_a = 0.96\,\tau_a^{-1}\hbar\omega_c/k_B T \;, \tag{2}$$

$$\Gamma_i = 23.2\,\tau_i^{-1}C^{-1}(T)k_B T/\hbar\omega_c \;, \tag{3}$$

with

$$\tau_a^{-1} = 3(2m^* k_B T)^{3/2}E_1^2/8\pi^{1/2}\rho u^2\hbar^4 \;, \tag{4}$$

$$\tau_i^{-1} = \pi^{3/2}e^4 n_i C(T)/8(2m^*)^{1/2}\chi^2(k_B T)^{3/2} \;. \tag{5}$$

Here $\omega_c = eB/m^*c$ is the angular cyclotron frequency for an electron of effective mass $m^*$ in a magnetic field $B$. $E_1$ is the deformation potential constant, $\rho$ is the crystal density, $u$ is the sound velocity, $\chi$ is the dielectric constant, and $n_i$ is the number of ionized impurities per unit volume. $\tau_i^{-1}$ defined here differs from that given in Ref. 5 by a factor of 2 to make it consistent with the zero-field ionized-impurity relaxation time.[10] $C(T)$ is a slowly varying function of temperature $T$ given by[10]

 

$$C(T)=\ln[1+(7\chi k_B T/2e^2 n_i^{1/3})^2]\,, \tag{6}$$

whose value at $T=4.5$ K for InSb is 1.9.

The simple relationship of Eq. (1) arises from the scattering probability per unit time by an electron from two independent scattering mechanisms, namely, the acoustic-phonon and impurity scattering centers. In strongly degenerate samples, the quasiparticle effect described by Lodder and Fujita[11] may become important. Then, the collision broadening of the Landau level needs to be taken into account in the density of states. In the strong-damping limit, this quasiparticle effect leads to CR linewidth proportional to $\sqrt{n_i}$. But, for nondegenerate samples, this quasiparticle effect can be neglected leading to "collision dephasing" which yields the simple relation given by Eq. (1).

The magnetic field position $B^*$ of the linewidth minimum, as obtained by setting $d\Gamma/dB=0$, is given by

$$B^*=4.92(\tau_a/\tau_i)^{+1/2}C^{-1/2}(T)m^*ck_BT/\hbar e\,, \tag{7}$$

and the minimum linewidth $\Gamma^*$ corresponding to this magnetic field is given by

$$\Gamma^*=9.4(\tau_a\tau_i)^{-1/2}C^{-1/2}(T)\,. \tag{8}$$

Transmission linewidths at several frequencies studied by McCombe *et al.*,[2] as a function of temperature between 4.2 and 15 K were found to be independent of temperature. This led them to conclude that phonon scattering is unimportant in this temperature range, and the linewidth is determined exclusively by ionized-impurity scattering. As is clear from the numerical results presented below, the zero-field ionized-impurity scattering as compared to zero-field phonon scattering is predominant at low temperatures. But it is suppressed by the magnetic field, in the quantum limit, by a factor $\hbar\omega_c/k_BT\sim 30$ at the CR linewidth minimum, while the phonon scattering is enhanced by the same factor. At the minimum, these two competing processes produce a temperature-independent linewidth, as $\Gamma^*$ of Eq. (8) is independent of temperature, in agreement with the experimental observations which were done near the minimum. On the other hand, $B^*$ depends on temperature ($B^*\sim T^{-1/2}$) which has not been indicated in any of the published works. A similar minimum is expected in CR linewidth as a function of temperature; the temperature at which minimum occurs is proportional to $B^{-2}$.

Both $B^*$ and $\Gamma^*$ are proportional to $\sqrt{n_i}$. $n_i^{1/2}$ behavior is, therefore, expected to hold well for measurements made near the minimum. At low magnetic fields, well below the minimum, linear-in-$n_i$ behavior of CR linewidth should be expected. At sufficiently high magnetic fields, well above the minimum, the CR linewidth should be independent of $n_i$. This tends to be consistent with actual observations. It may be noted that at low temperatures and high magnetic fields, the above behavior may be masked, to some extent, by the freeze-out effect, but this freeze-out was taken into account in the experimental work in the determination of $n_i$. The derivation[5] of Eq. (3) also indicates that the screening tends to be ineffective at high magnetic fields, consistent with actual observations.[2]

For InSb at 4.5 K, because of low effective mass, the quantum limit ($\hbar\omega_c \gtrsim k_BT$) sets in at magnetic fields of the order of 0.5 kG. The quantum-limit expression for $\Gamma$ thus holds extremely well for measurements made between 3 and 23 kG by McCombe *et al.*[2] For Ge at 77 K, the quantum limit sets in at much higher fields, and hence the scattering at low magnetic fields is still in the classical limit, where it can be assumed to be approximately independent of magnetic fields, while at sufficiently high magnetic fields above

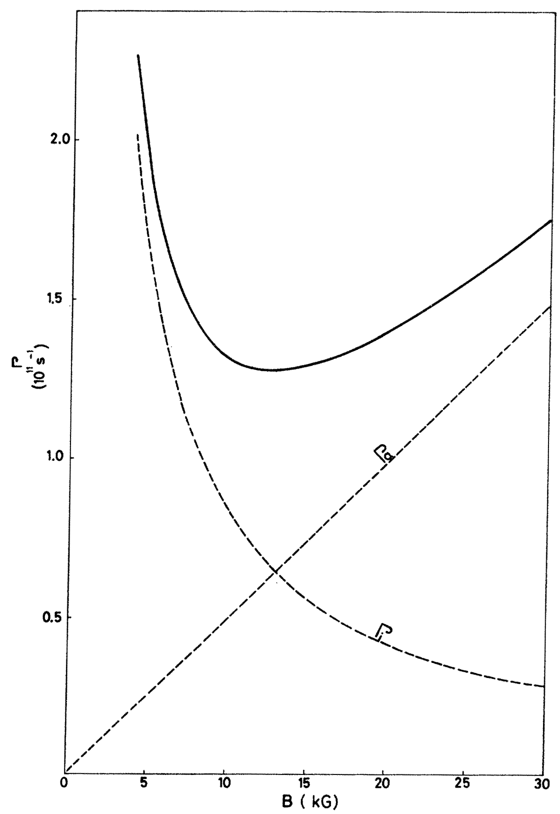


FIG. 1. CR linewidth $\Gamma=\Gamma_a+\Gamma_i$ as a function of magnetic field for InSb at 4.5 K (solid curve). Dashed curve marked $\Gamma_a$ is for acoustic-phonon scattering, and that marked $\Gamma_i$ is for ionized-impurity scattering.

the quantum limit, the acoustic-phonon scattering dominates and a linear behavior is observed, consistent with the experimental observation of Ito *et al.*[12] At low magnetic fields and low temperatures, in the quantum limit, when ionized-impurity scattering dominates, a line narrowing is normally observed[8] (see Fig. 1). The line narrowing in $n$-GaAs observed by Chamberlain *et al.*,[13] is an example where lines narrowed drastically at the lowest temperatures available. The present theory simply incorporates the above observation into the effect of attributing the total linewidth arising due to the ionized-impurity scattering alone. No specific numerical estimates seem available, but it is conjectured that, due to the difference in sample and temperature, phonon scattering is too inefficient to compete with the impurity one. Perhaps by changing the field, the minimum could be detected at higher fields when phonon scattering becomes important. This may provide a further check on the present theory. It is interesting to note from the work of Chamberlain *et al.*,[13] that, with CdTe, a slight broadening was apparent at a wavelength of 119 $\mu$m which was attributed to strong polaron nonparabolicity. Thus the observation of narrowing, broadening, or minimum depends very much on the material and the experimental conditions under which the measurements are made. But at sufficiently high magnetic fields, the CR linewidth should be an approximately linear function of magnetic field, at least for parabolic semiconductors with isotropic mass, neglecting the spin splitting (see Fig. 1).

In Fig. 1, we present some numerical results to study CR linewidth as a function of magnetic field. $\tau_a$ and $\tau_i$ were chosen to fit the values of experimentally observed[1] $B^*$ and $\Gamma^*$ and were found to be $\tau_a = 4.2\times10^{-10}$ s, $\tau_i = 6.9\times10^{-12}$ s. The value of $\tau_a$ agrees with that obtained from mobility at 77 K and extrapolated to 4.2 K by following the $T^{-3/2}$ law. As said earlier, $\tau_i^{-1} \gg \tau_a^{-1}$, which indicates that zero-field ionized-impurity scattering is predominant, consistent with zero-field experiments. But the value of $\tau_i$ is an order of magnitude different from that obtained from mobility data or Eq. (5). This may be due to the neglect of either or all of nonparabolicity, carrier freeze-out, or heavy-hole scattering.[14] The present theory of ionized-impurity scattering has been described by Rode[14] as the weakest part of the present-day theory of electron transport due to failure of the Born approximation and the occurrence of multiple scattering. It is also stated by Rode[14] that the nonparabolicity and the electron freezeout may affect the mobility and hence the relaxation time. The model proposed by Rode[14] could not be included in the above theoretical framework because of its complexity. But in spite of these shortcomings, our theory presented above explains very well the minimum in the observed CR linewidth.

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